Lab 7 - Madagascar

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ABSTRACT

The Madagascar satellite data set provides images of a spreading ridge off the coast of Madagascar. This data set has two regions: the southern half is densely sampled and the northern half is sparsely sampled. The sparsely sampled region presents a missing data problem. In this exercise, you will only view figures created by various fitting goals and answer questions.

Here are some definitions: Let components of $\mathbf{d}$ be the data, altitude measured along a satellite track. The model space is $\mathbf{h}$, altitude in the $(x, y)$-plane. Let $\mathbf{L}$ denote the 2-D linear interpolation operator from the track to the plane. Let $\mathbf{H}$ be the helix derivative, a filter with response $\sqrt{k_x^2 + k_y^2}$. Except where otherwise noted, the roughened image $\mathbf{b}$ is the preconditioned variable $\mathbf{p} \equiv \mathbf{Hh}$. The derivative along a track in data space is $\frac{d}{dt}$. A weighting function that vanishes when any filter hits a track end or a bad data point is $\mathbf{W}$.

All of the raw Madagascar data is displayed on the left of Figure 1. It is one long 1D array. You can see the sudden jumps at the track ends where the satellite completes one pass and begins the next. Sporadic noise events can be seen as spikes. The right side of the figure contains the weight $\mathbf{W}$ that identifies both the track ends and noise values.

If we merely take the raw data and interpolate it onto a grid, we get the result on the left of Figure 2. As stated above, the operator $\mathbf{L}$ is 2D linear interpolation and is therefore initialized with the sampling coordinates of the satellite. Applying the adjoint $\mathbf{L}'$ interpolates the data onto the grid. The grid cells have irregular amounts of data values placed in them therefore it is helpful to normalize by the fold as shown on the right side of Figure 2. Its roughened result is shown in Figure 3. Notice that the image seems to suffer from a strong acquisition footprint related to low frequency shifts from one pass of the satellite to the next.

One way we can remove the low frequency shifts between tracks is to apply a low cut filter the raw data. A very simple low cut filter is the derivative $\frac{d}{dt}$ along the

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Figure 1: All the data \( \mathbf{d} \) (left). Marker of track ends and bad points (right). The long tracks are the ones that are sparse in the north.

Figure 2: Left is the adjoint \( \mathbf{L}'\mathbf{d} \). Everything is dominated by the satellite tracks. Right is the adjoint normalized by the bin count, \( \mathbf{L}'\mathbf{d}/\text{diag} (\mathbf{L}'\mathbf{1}) \). You might notice a few empty bins—they are tiny. Overall, the topographic function is too smooth, suggesting we need a roughener.
Figure 3: The roughened normalized adjoint, $HL'\text{d/\text{diag}}(L1)$. Some interesting topography is perceptible through the track noise.

Figure 4: With a simple roughening derivative in data space, model space shows two nice topographic images stronger than the data acquisition tracks. Let $n$ denote ascending tracks. Let $s$ denote descending tracks. Left is $L'n$. Right is $L's$. We have two attractive images (with noticeable tracks) that cannot be added directly because the shadows go in different directions.

How do we combine the information of both ascending and descending tracks into one image? (rhetorical) We can use conjugate gradients to solve the following fitting goal:

$$W_d \frac{d}{dt}(Lh - d) \approx 0$$

(1)

Here, we estimate a model $h$ that is sampled into data-space by $L$ then its weighted derivative is matched to the weighted derivative of the data $d$. The results are seen on the left of Figure 5. Its roughed results are shown on the right.

A useful way to judge the quality of an inversion result is to look at the residual. As explained in the textbook, the residual should be white. Sometimes, it is interesting to not just look at the exact residual but possibly at an unweighted residual. The left
Figure 5: All data merged into a track-free image (hooray!) by applying the track derivative, not to the data, but to the residual. Left is $h$ estimated by $0 \approx \mathbf{W} \frac{d}{dt} (Lh - d)$. Right is the roughened altitude, $p = Hh$.

Figure 6: Fifty thousand of a half million (537,974) data points of Figure 5. Left is the residual $Lh - d$. Right is the residual $\mathbf{W} \frac{d}{dt} (Lh - d)$.
figure in Figure 6 shows the residual $\mathbf{Lh} - \mathbf{d}$ after $\mathbf{h}$ was found using $0 \approx \mathbf{W} \frac{d}{dt}(\mathbf{Lh} - \mathbf{d})$. How would you interpret the results of this figure? In other words, what is causing the shapes in that figure?

YOUR ANSWER:

There are numerous gaps in Figure 5. These gaps are in areas (bins) where the satellite did not pass. Now we have a missing data problem. One way we can fill the missing spaces is to use SEPLIB routines. We can first use the program $\text{Pef}$ to estimate a PEF on the left of Figure 5. Then we can use $\text{Miss}$ to fill in the missing gaps. See SEPDOC for information on either of these two programs. The results are shown in Figure 7.

Figure 7: Holes filled with a model space PEF. Starting from the $\mathbf{h}$ in Figure 5 and filling holes with GEE program $\text{Miss}$ using a PEF $\mathbf{A}$ found by $\text{Pef}$ we get $\mathbf{h}$ on the left. The ridge topography is building in the northern region. Right is the roughened altitude $\mathbf{Hh}$. The northern ridge cannot stand the roughener and the north again becomes dominated by tracks.

Figure 8: The 2-D prediction error $\mathbf{Ah}$ of model space.

Figure 8 is $\mathbf{Ah}$. What result would you expect to see? Is Figure 8 what you expect?
An alternative way to fill in the missing data is to add a regularization term to our original fitting goal so we now have:

\[
W \frac{d}{dt} (Lh - d) \approx 0 \quad \text{(2)}
\]

\[
e \nabla h \approx 0 \quad \text{(3)}
\]

The result of these goals are in Figure 9. Notice the inversion tends to spend most of its effort fitting the lower half of the map and ignoring the upper half. In order to insure that the upper half is filled in a fairly large epsilon parameter is used. This large epsilon causes blurring of the lower half. Why is this and can you suggest a way to eliminate this blurring?

How was Figure 10 created? See if you can guess the fitting goals used.

Figure 11 was created by preconditioning with the helical derivative by substituting in \( h = H^{-1} p \). Convergence is achieved with far fewer iterations and the cost of each iteration is only one additional matrix-vector operation. Figure 12 shows the result of preconditioning with the PEF estimated by PEF.
Figure 10: Using the track derivative in residual space and helix preconditioning in model space we start building topography in the north. Left is $h = H^{-1}p$ where $p$ is estimated by $0 \approx W^{d/dt}(LH^{-1}p - d)$ for only 10 iterations. Right is $p = Hh$.

Figure 11: Given a PEF $A$ estimated on the densely defined southern part of the model, $p$ was estimated by $0 \approx W^{d/dt}(LA^{-1}p - d)$ for 50 iterations. Left is $h = A^{-1}p$. Right is $p = Hh$. This is about as good as we are going to get. Our fractured ridge continues nicely into the north.
In Figure 7 the tracks are still visible in Hh but in Figure 12 they are not. Can you explain why?

YOUR ANSWER: