Rayleigh-wave dispersion curve inversion (shallow engineering applications)
Guided-wave dispersion curve inversion (seismic exploration, 3C processing)
Production implementation of dispersion-curve inversion

Surface-wave inversions have been extensively applied for imaging the Earth’s interior with earthquake data. However, Rayleigh-wave inversion with multichannel data is applied for shallow engineering imaging with only limited efforts. On the other hand, guided-wave dispersion-curve inversion seems more promising for the industry applications, and it becomes a new tool that produces near-surface P- and S-wave statics needed for 3C converted-wave imaging. We shall review these details including theories and production implementation.

Concepts:

According to Sheriff and Geldart (1995):
**Guided Wave**: near surface trapped wave, including Rayleigh wave.
**Rayleigh Wave** is also called **Ground Roll**. Some others say Ground Roll is a particular type of Rayleigh wave that travels along the ground surface (Sheriff, 1991).

This Figure shows elastic synthetics for two velocity models with different layer thickness.

However, sometimes people also refer “guided wave” as the dispersive wave before Rayleigh–wave arrivals as a way to separate it from Rayleigh wave (Herman et al., 2000; Ernst, 2007). In this case, “guided wave” does not include Rayleigh wave.
The following figure shows an earthquake seismogram recorded in Missouri with epicenter under Alabama. Where is “guided wave?”

![Seismogram](image)

Note the facts:

1) Industry seismic data is from P-wave source.
2) The phase velocity of guided wave is much faster than Rayleigh wave.
3) High-frequency guided wave is strongly attenuated during propagation.

References

*(Rayleigh-wave dispersion curve inversion)*

*(Guided-wave dispersion curve inversion)*

**Rayleigh-wave dispersion curve inversion**
*(Shallow engineering applications)*

**Rayleigh wave**: retrograde particle motion, longer wavelength, deeper penetration, greater phase velocity.

**Goal**: Invert for S-wave velocities, and layer thickness, high frequency (>5 Hz).

**Multichannel data:**
1) Signal processing, remove noise
2) Time-variant data analysis
3) Cover a broad depth range from a single survey
Field Data

Channel Number

- 780 m/s (15 Hz)
- 220 m/s (50 Hz)

a)

Measured
Initial A
Final A
Initial B
Final B

Phase velocity (m/s)

Frequency (Hz)
Multichannel Dispersion Curve Calculation ($f$-$v$ domain)

**Step 1**: frequency decomposition – offset-time domain to stretch impulsive data into pseudo-vibroseis data or frequency-swept data (time to frequency)

**Step 2**: Slant stacking – complete the transform from offset to velocity
Guided-wave dispersion curve inversion

Wave propagation in layered media:

\[ V(x, t) = \int dk \int d\omega \exp(i(\omega t - kx)) \frac{N(k, \omega)}{W(k, \omega)}, \tag{1} \]

where \( x \) is offset, \( t \) is traveltime, \( \omega \) is angular frequency, \( k \) is spatial wavenumber, and \( W \) is called Wronskian. \( W=0 \) leads to singularities in the integrand which give rise to surface waves and guided waves.

Calculated Wronskian: blue areas correspond to zeros in Wronskian function.

### Model 1
- \( V_p=750 \text{ m/s} \)
- \( V_s=315 \text{ m/s} \)
- \( D=50 \text{ m} \)

### Model 2
- \( V_p=500 \text{ m/s} \)
- \( V_s=315 \text{ m/s} \)
- \( D=32 \text{ m} \)

### Model 3
- \( V_p=2000 \text{ m/s} \)
- \( V_s=1200 \text{ m/s} \)

### Model 4
- \( V_p=2000 \text{ m/s} \)
- \( V_s=1200 \text{ m/s} \)
Dispersion curves: Model 1 (purple) versus Model 2 (black)

Procedures:
Summary:

1) We are inverting both Vp and Vs, mostly Vs from guided waves.
2) Dispersion curve picking is automatic, whole procedure is automatic.
3) Rayleigh wave is not sensitive to the velocity variations over frequencies
4) Higher modes are the important constraints
5) We are talking about huge amount of data!

Production Implementation Design:

1) Data handling and selection for large dataset (for example: 50 files, and 500Gb to 1.0Tb in total).

2) x-t to $\omega$-$p$ transform and dispersion curve picking 
3) Inversion of picked dispersion curves to a layered near-surface model 
4) Interpolate into a large model