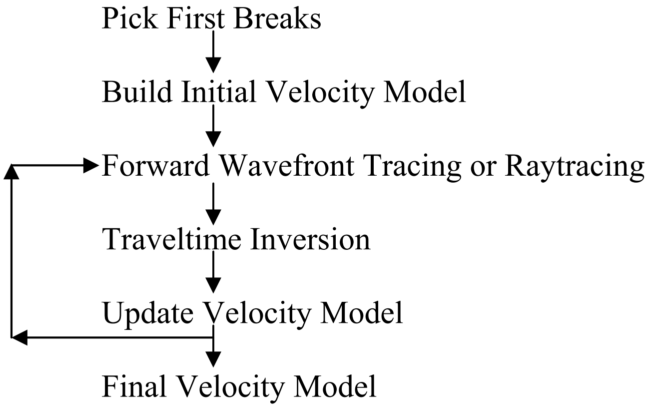
GEOPHYS 242: Near Surface Geophysical Imaging

Class 4: First-Arrival Traveltime Tomography
Mon, April 11, 2011

- Wavefront tracing methods - speed versus accuracy
- Inversion algorithms - any magic approach?
- Model regularization - continuum inverse theory
- Review of tomography case histories

From this class and later, we shall introduce high end near-surface imaging technologies. During today's class we are going to focus on the traveltime tomography approach, looking into the details inside this method, and discussing key technical issues.

Traveltime Tomography Workflow:



Forward Traveltime Calculation:

Purpose: calculate theoretical traveltimes T and also calculate derivatives (raypath l_{ij})

$\partial T/\partial m_{ij} = l_{ij}$ T: traveltime between S and R; m_{ij}: cell slowness; l_{ij}: ray length in the cell
i,j: cell index in 2D.

Basic Theories:

Snell's law: $\sin\theta_1/\sin\theta_2 = V_1/V_2$

(*About a ray crossing an interface*)

Fermat's principle:

(*About a ray between two points*)

In Optics, Fermat's principle states that the path light takes between two points is the path that has the minimum Optical Path Length (OPL).

$$OPL=\int_c n(s)ds$$

where $n(s)$ is the local refractive index as a function of distance, s, along the path C.

Huygens' principle (Christiaan Huygens, 1629-1695):

(*About wavefront expansion starting from a point*)

Every point of a wavefront may be considered the source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the waves.

Huygens' principle → Fermat's principle → Snell's law

Raytracing:

Shooting Method (Andersen and Kak, 1982)

From a source point to a receiver, given an initial value, shoot rays following the equation:

$$\frac{d}{ds}\left(n\frac{d\mathbf{x}}{ds}\right) = \nabla n,$$

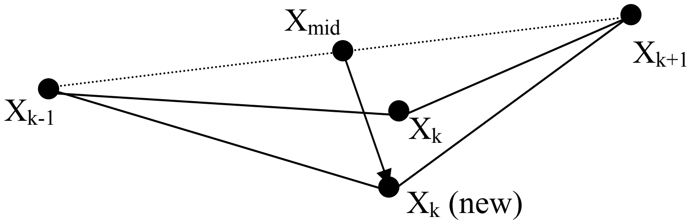
where ds is the differential distance along the ray, n is the refractive index (slowness), and X is the position along the ray.

For undershooting or overshooting results, repeat or interpolate.

Two-Point Perturbation Method (Um and Thurber, 1987)

$$T = \int_{\mathbf{x}'}^{\mathbf{x}''} \frac{1}{v(\mathbf{x})} ds,$$

Where X′ and X″ are the two-point positions.



Wavefront Raytracing (Wavefront Tracing)
Calculate first-arrival wavefront traveltimes and associated raypaths

1) Solving the eikonal equation by finite-difference extrapolation
Eikonal equation follows Fermat's principle, Vidale's approach also applies Huygens' principle.

2) Graph method by following Huygens' principle.

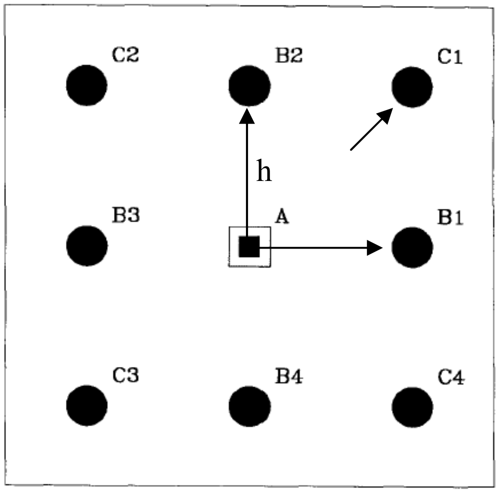
Solving eikonal equation (Vidale, 1988, cited by 553)

$$\left(\frac{\partial t}{\partial x}\right)^2 + \left(\frac{\partial t}{\partial z}\right)^2 = s(x, z)^2. \qquad (1)$$

Finite-difference extrapolation method

Assuming source at A, to time points at B_1, B_2, B_3, and B_4:

$t_i = (s_{Bi} + s_A)h/2$ (*h* is cell size, *s* is slowness) (2)



Given t_0 at A, t_1 at B_1, t_2 at B_2, to calculate t_3 at C_1:

Plane-wave approximation

$$t_3 = t_0 + \sqrt{2(hs)^2 - (t_2 - t_1)^2}.$$

Circular wavefront

$$t_3 = t_s + s\sqrt{(x_s + h)^2 + (z_s + h)^2}.$$

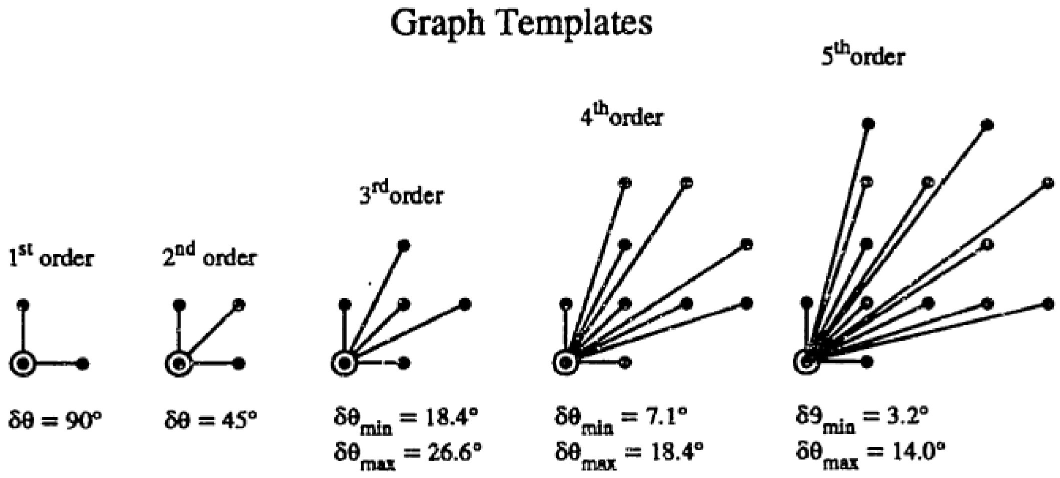
Plane-wave approximation due to:

$$\frac{\partial t}{\partial x} = \frac{1}{2h}\,(t_0 + t_2 - t_1 - t_3)$$

$$\frac{\partial t}{\partial z} = \frac{1}{2h}\,(t_0 + t_1 - t_2 - t_3).$$

Graph method

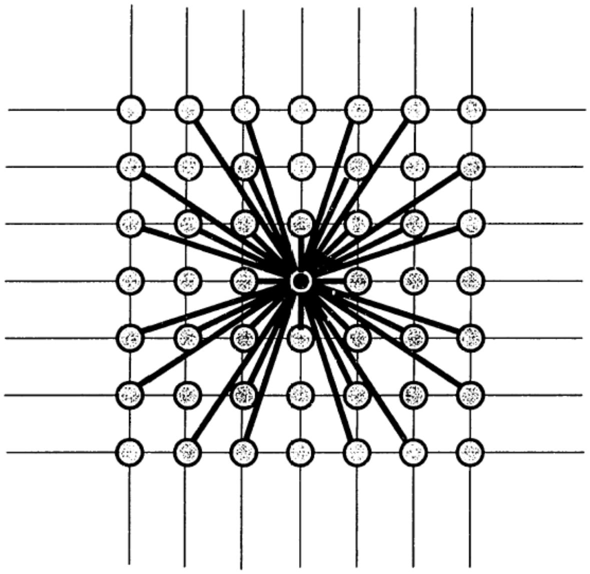
Hideki Saito and T. J. Moser presented the same approach in SEG meeting in 1989 independently. Moser's paper was published in *Geophysics* in 1991 (cited 231).

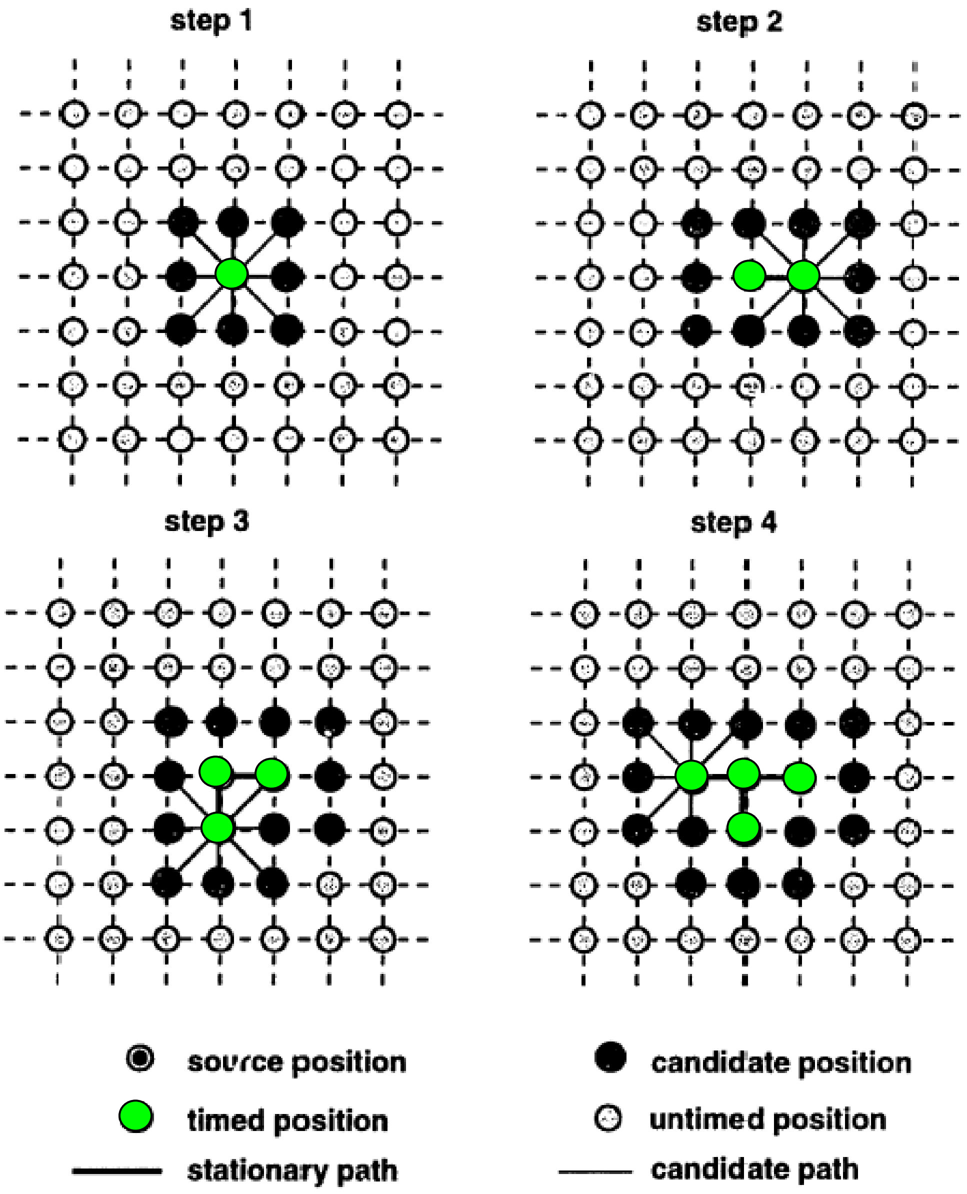


Dijkstra Algorithm (1959):

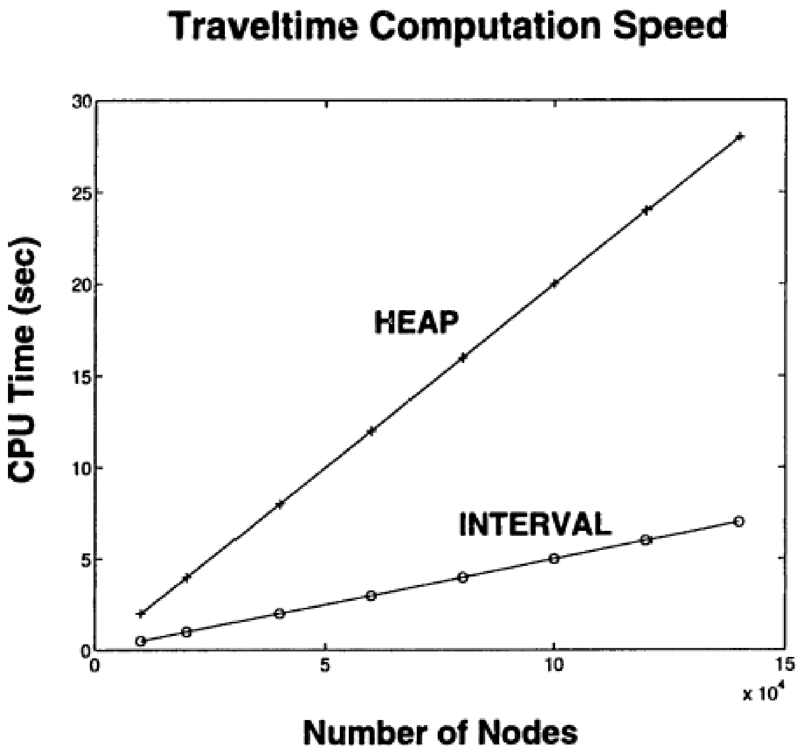
1) Define a graph template
2) Time points near the source by the graph template
3) Find the node with minimum time, and it becomes a new source, apply the graph template to time again until every node becoming a source.

4^{th} order graph template:



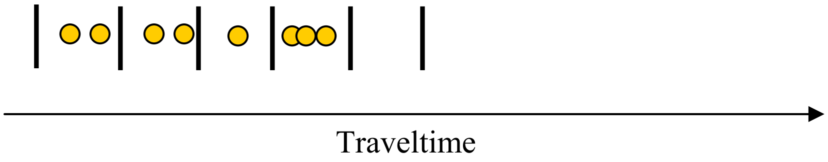


Extrapolation Process

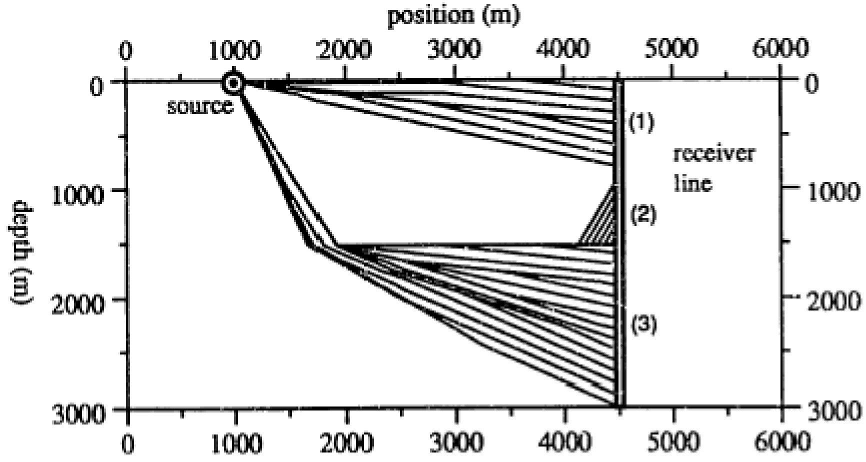


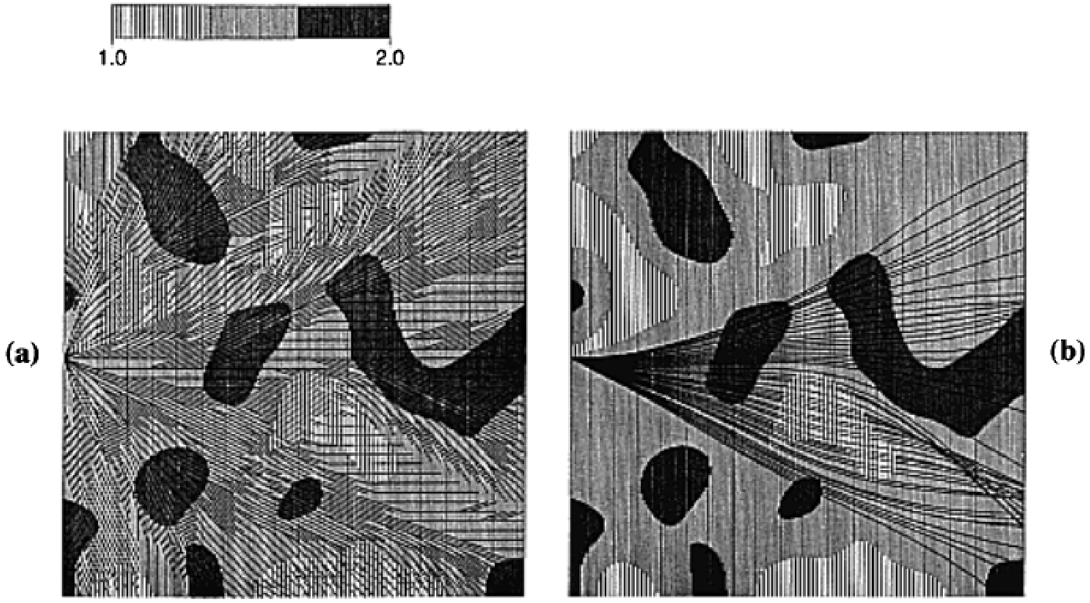
HEAP Sort:

Interval Approach (Klimes and Kvasnicka, 1994): interval=h/V_{max}



Tree-shape raypaths:





Tomographic Inversion Method:

Objective Function: $\psi = ||d - G(m)||^2 + \tau ||L(m)||^2$

Where d: data; $G(m)$: synthetics; L: Laplacian operator; τ: smoothing control; m: velocity model.

1) **Gauss-Newton method**:

$(A^TA + \tau L^TL)\Delta m = A^T (d - G(m)) - \tau L^TL(m)$

where A: sensitivity matrix, with elements $\partial G(m)/\partial m_{ij} = l_{ij}$

   a) Apply Conjugate Gradient method to the above matrix problems
   b) Apply Greenfield algorithm to partition the matrix

2) **Nonlinear Conjugate-Gradient Method**:

   a) Compute the gradient: $g_0 = -A^T (d - G(m_0)) + \tau L^TL(m_0)$
   b) Precondition the gradient $p_0 = Pg_0$, $P = (A^TA + \tau L^TL)^{-1} \approx (\tau L^TL)^{-1}$
   c) Initial model update: $c_0 = -p_0$
   d) $m_{K+1} = m_K + \alpha_K c_K$, compute $g_{K+1} = -A^T (d - G(m_K)) + \tau L^TL(m_K)$
   e) Precondition the gradient $p_{K+1} = Pg_{K+1}$
   f) $c_{K+1} = -p_{K+1} + \beta_K c_K$, where $\beta_K = ((g_{K+1} - g_K)^T p_{K+1)/(}\, p_K^T g_{K)}$

Inversion Theories

1) Creeping versus Jumping Methods

Objective functions:

$$Creeping: \quad \Phi(m) = \|d - G(m)\|^2 + \tau\|R(\Delta m)\|^2$$

$$Jumping: \quad \Phi(m) = \|d - G(m)\|^2 + \tau\|R(m)\|^2$$

Inversion Problem:

$$Creeping: \quad (A^T A + \tau L^T L)\Delta m_k^c = A^T(d - G(m_{k-1}^c)),$$

$$Jumping: \quad (A^T A + \tau L^T L)m_k^j = A^T(d - G(m_{k-1}^j) + A(m_{k-1}^j)).$$

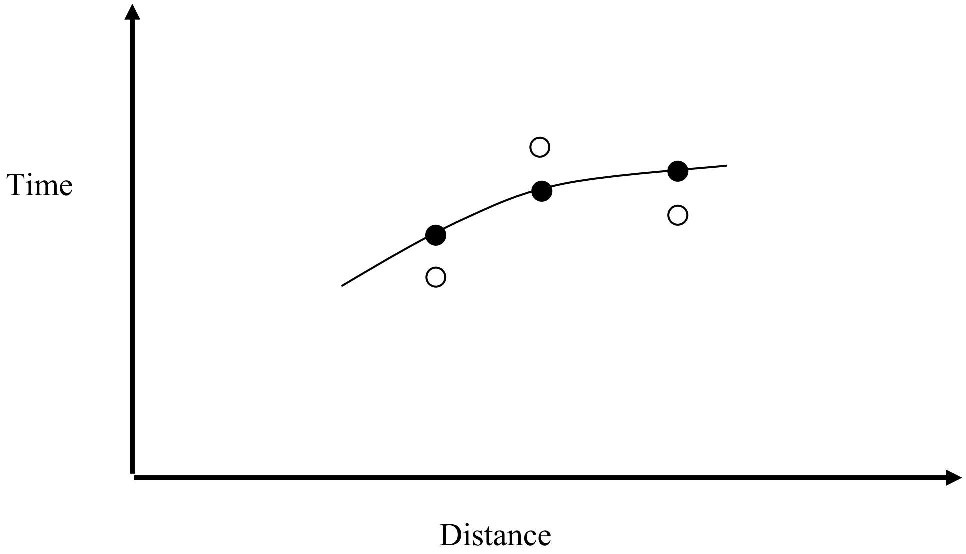
Difference in Solutions:

$$m_1^j - m_1^c = -(A^T A + \tau L^T L)^{-1}(\tau L^T L m_0).$$

2) Continuum Inverse Theory

Objective Function: ψ = Misfit of traveltime curves + model derivatives

Continuum data
Continuum model



Reading Materials:

Vidale, J., 1988, Finite-difference calculation of travel times, BSSA, Vo. 78, No. 6, 2062-2076.

Moser, T.J., 1991, Shortest path calculation of seismic rays, Geophysics, Vol. 56, No. 1, 59-67.

Zhang, J., and Toksoz, M. N., 1998, Nonlinear refraction traveltime tomography, *Geophysics,* Vol. 63, No. 5, 1726-1737.

Zhu et al, 2008, Recent applications of turning-ray tomography, Geophysics, Vol 73, No.5, 243-254.