Double-difference time-lapse FWI with a total-variation regularization

Overview and Synthetic Example

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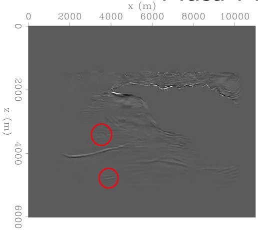

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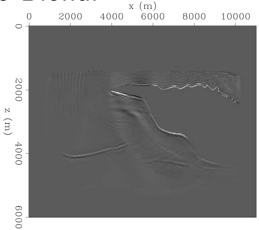


Motivation: improved reservoir monitoring

- Joint 4D FWI with total-variation regularization $\Rightarrow$ recovery of large-scale features
- Model-space methods are less sensitive to repeatability, but what about starting velocity?
- Goal: design a robust 4D FWI less sensitive to errors in the starting velocity

Find a model $\mathbf{m}$ that minimizes misfit between the true $\mathbf{d}$ and predicted $\mathbf{u}(\mathbf{m})$ data (Lailly, 1983; Tarantola, 1984)

$$\text{Misfit} \;=\; \|\mathbf{W}_d\left[\mathbf{d}-\mathbf{u}(\mathbf{m})\right]\|_2^2 \;\rightarrow\; \min, \tag{1}$$

with optional model regularization

$$\beta\|\mathbf{R}\mathbf{W}_m\mathbf{m}\|_2^2, \tag{2}$$

where $\mathbf{m}$ is a subsurface velocity model, $\mathbf{W}_d$ and $\mathbf{W}_m$ are data and model weighting operators (masks), $\mathbf{R}$ is a model regularization operator.

Total-variation difference regularization

- NEW: *Simultaneous FWI* of baseline and monitor with the *total-variation difference regularization*:

$$\text{Baseline Misfit } + \text{ Monitor Misfit} + \tag{3}$$

$$\alpha \| |\nabla \mathbf{W}_m \left[\mathbf{m}_2 - \mathbf{m}_1 \right] | \|_1 . \tag{4}$$

- The Total-variation (TV) seminorm (4) provides **edge-preserving** regularization that promotes model **"blockiness"** and helps to reduce spurious oscillations (Rudin et al., 1992).

Joint FWI Total-variation difference regularization

The most general formulation may include a double difference term (DD) (Maharramov and Biondi, 2014):

$$\alpha\|\mathbf{u}(\mathbf{m}_b) - \mathbf{d}_b\|_2^2 + \beta\|\mathbf{u}(\mathbf{m}_m) - \mathbf{d}_m\|_2^2 + \tag{JOINT}$$

$$\gamma\|\left(\mathbf{u}(\mathbf{m}_m) - \mathbf{u}(\mathbf{m}_b)\right) - \left(\mathbf{d}_m - \mathbf{d}_b\right)\|_2^2 + \tag{DD}$$

$$\delta\|\mathbf{W}\mathbf{R}(\mathbf{m}_m - \mathbf{m}_b)\|_1 \rightarrow \min, \tag{TVREG}$$

Regularization operator for Total Variation (TV):

$$\mathbf{R}f(x,y,z) = |\nabla f|. \tag{5}$$

Double difference with Steering Total Variation (STV)

1) Invert the baseline model $\mathbf{m}_b$:

$$\|\mathbf{u}(\mathbf{m}_b) - \mathbf{d}_b\|_2^2 \;\rightarrow\; \min\,. \tag{6}$$

2) Generate new synthetic monitor survey data $\mathbf{d}_2$ by adding the observed data difference $\mathbf{d}_m - \mathbf{d}_b$ to the forward-modeled baseline data $\mathbf{u}(\mathbf{m}_b)$:

$$\mathbf{d}_2 \;=\; \mathbf{u}(\mathbf{m}_b) \;+\; (\mathbf{d}_m \;-\mathbf{d}_b)\,, \tag{7}$$

3) Invert the monitor model $\mathbf{m}_m$ from the new synthetic data $\mathbf{d}_2$:

$$\|\,(\mathbf{u}(\mathbf{m}_m) - \mathbf{d}_2)\,\|_2^2 + \tag{8}$$

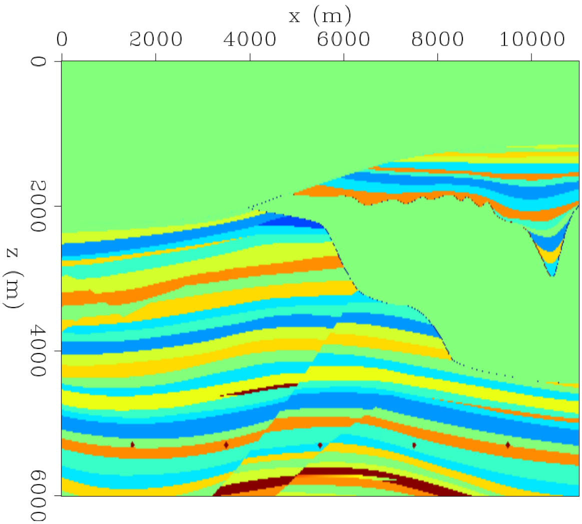
$$\delta\|\mathbf{W}\mathbf{R}(\mathbf{m}_m - \mathbf{m}_b)\|_2^2 \;\rightarrow\; \min\,. \tag{9}$$

Regularization operator for Steering Total Variation (Ma et al., 2015):

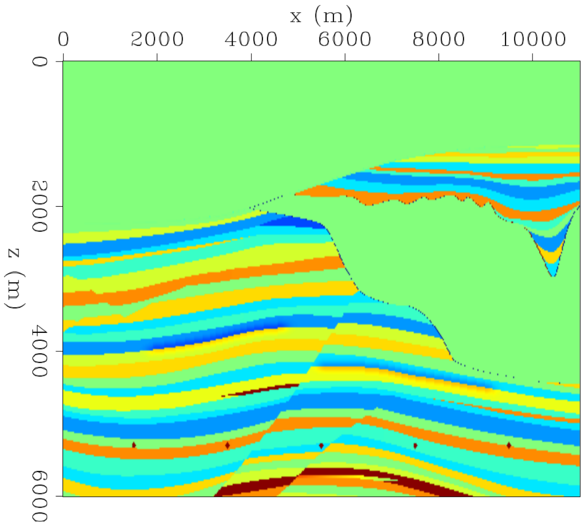
$$\mathbf{R}f(x,y,z) \;=\; |\nabla_{\boldsymbol{\xi}} f|,\ \boldsymbol{\xi} \text{ is parallel to dip.} \tag{10}$$

Test: Linearized Waveform Inversion with Steering Total Variation (STV) regularization

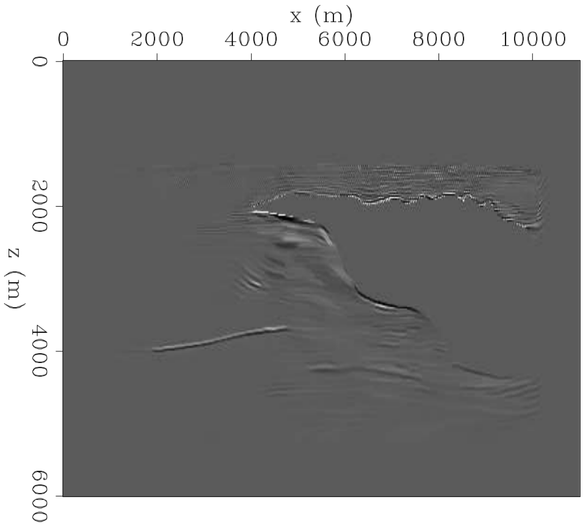
True baseline model



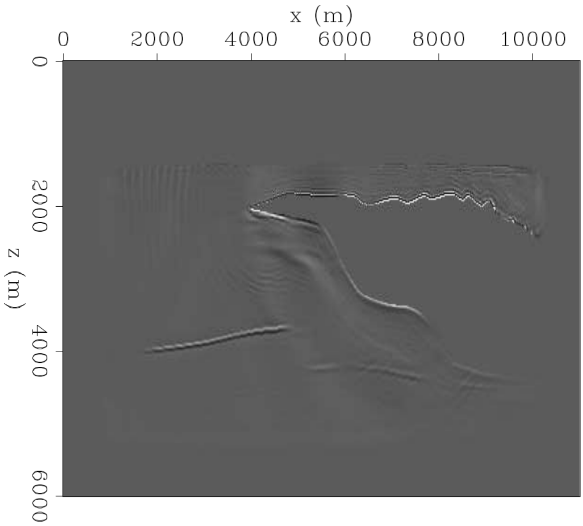
True monitor model



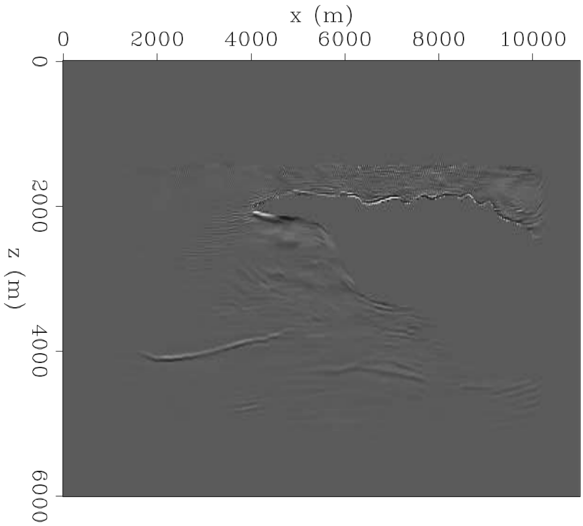
Simultaneous LWI with STV using true velocity



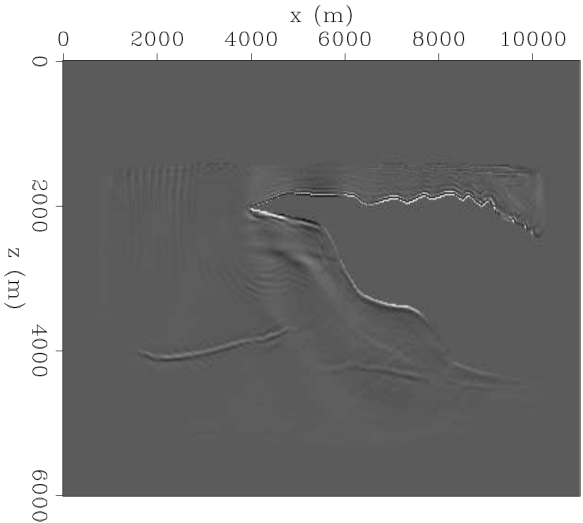
Double Difference using the true velocity



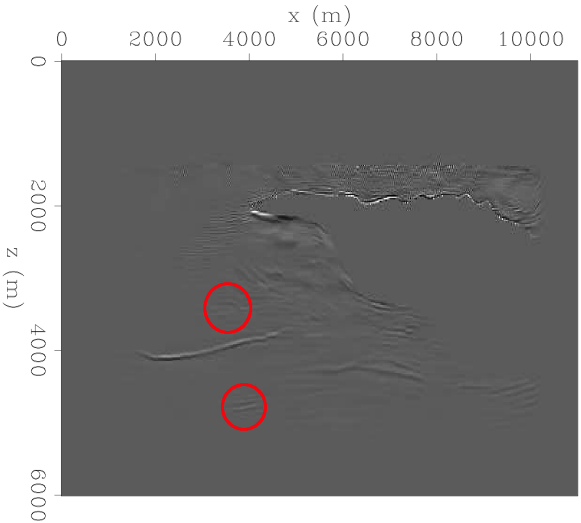
Simultaneous LWI with STV using 10% too-high velocity



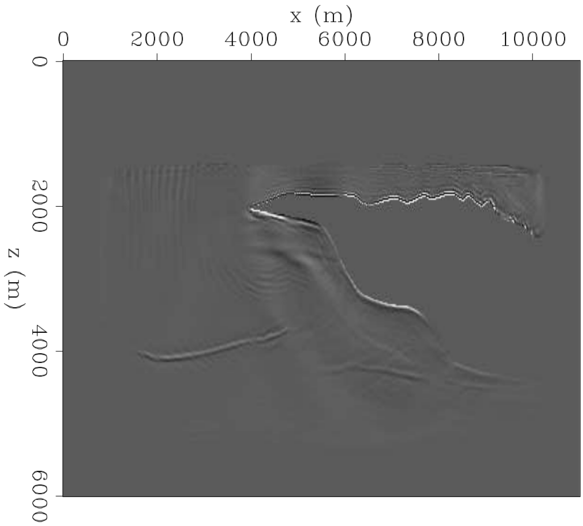
Double Difference using the wrong velocity



Simultaneous LWI—production or inversion artifacts?



Double Difference using the wrong velocity



Conclusions and perspectives

- Double difference is sensitive to repeatability issues but more stable with respect to wrong model
- Joint double-difference + simultaneous 4D FWI (JOINT, DD, TVREG)—best of both methods?
- Next: TV-regularized double-difference experiments

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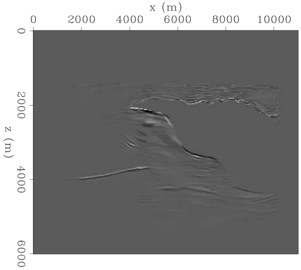
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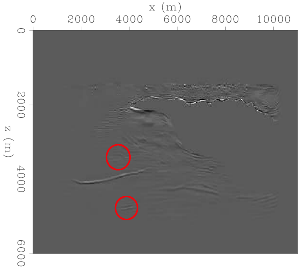
z (m)
0 2000 4000 6000

Q&A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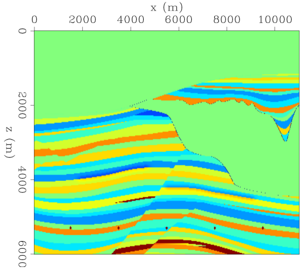
x (m)
0 2000 4000 6000 8000 10000
z (m)
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