Stanford Exploration Project, Report 115, May 22, 2004, pages 305–315

Target-oriented shot-profile one-way wave equation inversion

Alejandro A. Valenciano and Biondo Biondi[1]

ABSTRACT

Least-squares shot-profile inversion could improve image amplitudes while remaining consistent with the data. This could be done efficiently by approximating the scalar two-way wave-equation operator by two one-way wave-equation operators: one for the source wavefield and the other for the receiver wavefield. In addition, a target-oriented scheme could help reduce the computation time. Instead of recursively computing the Green functions at each depth step for each inversion iteration, the Green functions from the surface to a target and from the target to the surface could be calculated and stored in the first iteration. In the subsequent iterations, the Green functions are retrieved from the disk.

INTRODUCTION

The uneven illumination of reflectors can lead to misinterpretation of their amplitudes after migration. This can be due to the seismic experiment acquisition geometry (incomplete data), or to energy focusing or defocusing caused by obstacles in the wave path. In the worst case, this problem can create shadow zones at reservoir depths (Muerdter et al., 1996; Prucha et al., 1998).

Some attempts to solve this subsurface imaging problem have used the power of geophysical inverse theory (Tarantola, 1987). It compensates for the experimental deficiencies (acquisition geometry, obstacles, etc.) while being consistent with the acquired data. Usually the inversion is implemented by using iterative least-squares algorithms (Nemeth et al., 1999; Duquet and Marfurt, 1999; Ronen and Liner, 2000; Prucha et al., 2000; Kuehl and Sacchi, 2001). The main inconvenient of this approach is that it is computationally expensive since it iteratively apply the seismic modeling and migration operators to build the whole image.

Since reflection amplitudes are more important at reservoir depths, we propose to apply a target-oriented shot-profile one-way wave equation inversion strategy. Instead of recursively computing the Green functions at each depth step and at each inversion iteration, they could be computed from the surface to the target and from the target to the surface during the first iteration, stored, and reused in subsequent iterations. In that way, the computational cost could be kept reasonable.

In this paper, we first discuss how a target-oriented least-squares inversion could help to

[1]**email:** valencia@sep.stanford.edu, biondo@sep.stanford.edu

improve the image amplitudes while keeping computational cost reasonable. We also review target-oriented modeling by shot-profile extrapolation with the one-way wave equation.

LINEAR LEAST SQUARES INVERSION

Tarantola (1987) formalizes the geophysical inverse problem from a Bayesian point of view. He gives a theoretical background to compensate for the reflection experiment's deficiencies (acquisition geometry, obstacles, etc.), while being consistent with the acquired data.

Under this scheme, data and model (subsurface image) are assumed to have Gaussian distribution (*a priori* data and model probability density are Gaussian). The forward operator is assumed to be linear (or weakly non-linear). The resulting subsurface image is the mean of a posterior Gaussian probability density in the model space. Its expression for a general linear (forward modeling) operator $\mathbf{L}$ is

$$\hat{\mathbf{m}} = (\mathbf{L}^t \mathbf{C}_D^{-1} \mathbf{L} + \mathbf{C}_M^{-1})^{-1} (\mathbf{L}^t \mathbf{C}_D^{-1} \mathbf{d}_{obs} + \mathbf{C}_M^{-1} \mathbf{m}_{prior}), \quad (1)$$

where $\hat{\mathbf{m}}$ is the mean of a posterior Gaussian probability density, $\mathbf{L}^t$ is the transpose of the linear forward operator, $\mathbf{C}_D$ is the data covariance, $\mathbf{C}_M$ is the model covariance, $\mathbf{d}_{obs}$ is the measured data and $\mathbf{m}_{prior}$ is the prior model.

Equation (1) can be solved by using gradient-based methods like steepest descent. This iterative algorithm can be written in the form $m_{n+1} = m_n + \delta m_n$ (Tarantola, 1987), where the term δm_n depends on the gradient of the function to be optimized, a metric in the model space, and an *ad-hoc* constant.

Target-oriented least squares inversion strategy

In the case of wave equation migration or inversion, the operator $\mathbf{L}$ is expensive to apply. Thus, iteratively applying this operator and its transpose is sometimes prohibitive. The computational cost is proportional to the number of depth steps the wavefields need to be propagated (Audebert, 1994), and the number of iterations, among other factors.

Since reflection amplitudes are more important in the neighborhood of the reservoir, it makes sense to apply a target-oriented strategy to reduce the number of depth steps. A way to achieve this objective is to write the modeling operator $\mathbf{L}$ in a target-oriented fashion. Instead of recursively computing the Green functions at each depth step and at each inversion iteration, we can compute them from the surface to the target and from the target to the surface during the first iteration, store them, and reuse them in subsequent iterations. By storing the Green functions to the target and from the target, we add a new problem, since we then require approximately twice the disk space for data storage; however, we save computing time.

In the next sections, we write a target-oriented, one-way approximation to the scalar wave equation operator.

FORWARD OPERATOR

Ideally, the elastic wave equation should be used to model the propagation of seismic waves in the earth. But in exploration reflection seismology, except in very specific cases, only the vertical component of the propagated wavefield is recorded at the surface. This recorded wavefield is often assumed to be a scalar (converted modes projected in the vertical wavefield component are treated as noise). Thus a scalar wave equation is a good approximation of the physics involved in an exploration reflection seismology experiment.

Additionally, modeling or migrating with the scalar wave equation is computationally expensive. That is why it is rarely used for migration (Claerbout, 1985; Stolt and Benson, 1986). Instead, two one-way wave equations are used to mimic the scalar wave equation solution (Claerbout, 1971; Stolt and Benson, 1986). This approximation is reasonable in computational time and accuracy.

In the next three subsections we review the three steps involved in shot-profile modeling with the one-way wave equation: first, downward continuation of the source wavefield, second, upward continuation of the receiver wavefield to the surface, and third, truncation of the receiver wavefield at zero depth.

Source wavefield downward extrapolation

The first step of shot-profile modeling with the one-way wave equation consists of downward continuation of the source wavefield (for each shot, at each frequency), which is done by using the following equation:

$$p^{+}(z_{i+j}) = w^{+}(z_{i+j}, z_i)\; p^{+}(z_i), \tag{2}$$

initialized by the wavefield at the surface, as follows:

$$p^{+}(z_0) = f, \tag{3}$$

where $p^{+}(z_i)$ is the source wavefield at depth z_i, $p^{+}(z_0)$ is the source wavefield at zero depth, $w^{+}(z_{i+j}, z_i)$ is the downward continuation operator (Green function) that downward propagates the source wavefield from depth z_i to z_{i+j}, and f is the source wavefield at the surface.

The Green functions can be computed recursively at each depth step (from z_i to z_{i+1}) (Figure 1), or precomputed and stored to allow jumps (from z_i to z_{i+j}) (Figure 2). The equations (2) and (3) can be written also in matrix form (see **APPENDIX A**).

Including all the frequencies and all the shot positions in the data, it follows from equation (A-3) that the source wavefield ($\mathbf{P}^{+}$) can be computed as a function of the source signature ($\mathbf{f}$), as follows:

$$\mathbf{P}^{+} = \left(\mathbf{I} - \mathbf{W}^{+}\right)^{-1} \mathbf{f}. \tag{4}$$

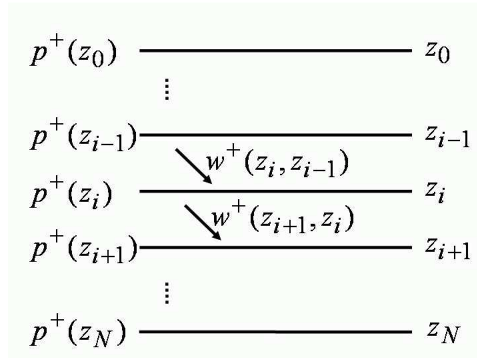

Figure 1: Downward continuation of the source wavefield by recursive computation of the Green functions at each depth step. alejandro1-figure1 [NR]

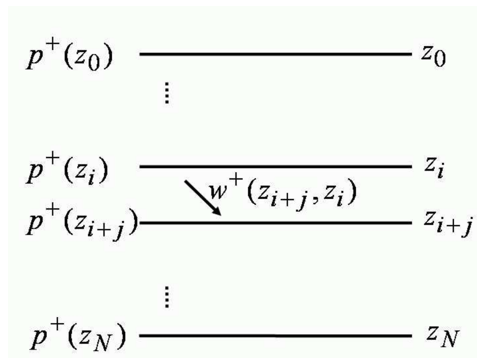

Figure 2: Downward continuation of the source wavefield with the precomputed Green functions. alejandro1-figure2 [NR]

Defining the the multi-frequency, multi-shot, downward propagation operator as $\mathbf{B}^+ = \left(\mathbf{I} - \mathbf{W}^+\right)^{-1}$, equation (4) can be written as

$$\mathbf{P}^+ = \mathbf{B}^+ \mathbf{f}. \tag{5}$$

Receiver wavefield extrapolation

The second step of shot-profile modeling consists of upward continuation of the receiver wavefield to the surface, which is done by using the following equation:

$$p^-(z_i) \quad = \quad w^-(z_i, z_{i+j})\, p^-(z_{i+j}) + p^+(z_i)\, r(z_i) + p_r^-(z_i) \tag{6}$$

where

$$p_r^-(z_i) = \begin{cases} \sum_{m=1}^{j-1} w^-(z_i, z_{i+m}) p^+(z_{i+m})\, r(z_{i+m}) & \text{if} \quad j \neq 1, \\ 0 & \text{otherwise,} \end{cases} \tag{7}$$

with the final condition

$$r(z_0) = 0, \tag{8}$$

where $p^-(z_i)$ is the receiver wavefield at depth z_i, $p^+(z_i)$ is the source wavefield at depth z_i, $w^-(z_i, z_{i+j})$ is the upward continuation operator (Green function) from depth z_{i+j} to z_i, $r(z_i)$ is the reflectivity at depth z_i, and $p_r^-(z_i)$ is the contribution by the source wavefield and the reflectivity when jumps bigger than a depth step are used.

The Green functions can be computed recursively at each depth step (from z_{i+1} to z_i) (Figure 3), or precomputed and stored to allow jumps (from z_{i+j} to z_i) (Figure 4). In addition, the third term in equation (6) must also be stored. Equation (6) can be written also in matrix form (see **APPENDIX B**).

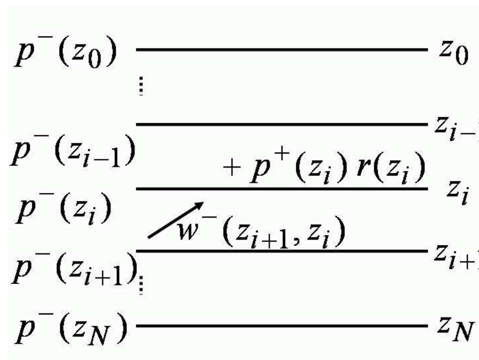

Figure 3: Upward continuation of the receiver wavefield by recursive computation of the Green functions at each depth step. alejandro1-figure3 [NR]

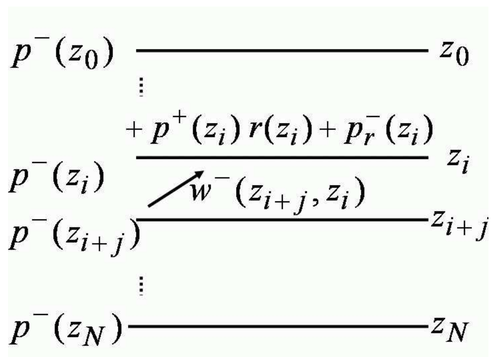

Figure 4: Upward continuation of the receiver wavefield with the precomputed Green functions. alejandro1-figure4 [NR]

Including all the frequencies and all the shot positions in the data, it follows from equation (B-4) that the receiver wavefield ($\mathbf{P}^-$) can be computed as follows:

$$\mathbf{P}^- = [\mathbf{I} - \mathbf{W}^-]^{-1}\mathbf{P}^+\, \Sigma^t_{\omega s}\, \mathbf{r}. \tag{9}$$

Defining the the multi-frequency, multi-shot, upward propagation operator as $\mathbf{B}^- = \left(\mathbf{I} - \mathbf{W}^-\right)^{-1}$, equation (4) can be written as

$$\mathbf{P}^- = \mathbf{B}^-\mathbf{P}^+\, \Sigma^t_{\omega s}\, \mathbf{r}. \tag{10}$$

Linear forward operator (L)

Since in the conventional surface seismic experiment, geophones are located only at the surface, the data is the receiver wavefield at $z = 0$. This can be represented in equation (9) as a truncation operation (transpose of zero padding **Z**):

$$\mathbf{d} = \mathbf{Z}^t\mathbf{P}^- = \mathbf{Z}^t\mathbf{B}^-\mathbf{P}^+\, \Sigma^t_{\omega s}\, \mathbf{r}. \tag{11}$$

Then the expression for linear shot-profile modeling one-way wave equation operator is

$$\mathbf{L} = \mathbf{Z}^t\mathbf{B}^-\mathbf{P}^+\, \Sigma^t_{\omega s}, \tag{12}$$

a chain of linear operators.

SUMMARY

Target-oriented shot-profile least-squares inversion could help improve amplitudes while keeping computational cost reasonable. We propose a methodology that combines first, an approximation of the scalar two-way wave equation operator by two one-way wave equation operators, and second, precomputed Green functions to the target. This methodology will allow to perform the shot-profile least-squares inversion without having to recursively compute the Green functions for all the depth steps at each iteration.

REFERENCES

Audebert, F., 1994, 3-D prestack depth migration: Why Kirchhoff?: SEP–**80**, 191–208.

Claerbout, J. F., 1971, Toward a unified theory of reflector mapping: Geophysics, **36**, no. 3, 467–481.

Claerbout, J. F., 1985, Imaging the Earth's Interior: Blackwell Scientific Publications.

Duquet, B., and Marfurt, K. J., 1999, Filtering coherent noise during prestack depth migration: Geophysics, **64**, no. 4, 1054–1066.

Kuehl, H., and Sacchi, M., 2001, Generalized least-squares DSR migration using a common angle imaging condtion: Soc. of Expl. Geophys., 71st Ann. Internat. Mtg, 1025–1028.

Muerdter, D. R., Lindsay, R. O., and Ratcliff, D. W., 1996, Imaging under the edges of salt sheets: a raytracing study: Soc. Expl. Geophys., 66th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 578–580.

Nemeth, T., Wu, C., and Schuster, G. T., 1999, Least-squares migration of incomplete reflection data: Geophysics, **64**, no. 1, 208–221.

Prucha, M. L., Clapp, R. G., and Biondi, B. L., 1998, Imaging under the edges of salt bodies: Analysis of an Elf North Sea dataset: SEP–**97**, 35–44.

Prucha, M. L., Clapp, R. G., and Biondi, B., 2000, Seismic image regularization in the reflection angle domain: SEP–**103**, 109–119.

Ronen, S., and Liner, C. L., 2000, Least-squares DMO and migration: Geophysics, **65**, no. 5, 1364–1371.

Stolt, R. H., and Benson, A. K., 1986, Seismic migration theory and practice: Geophysical Press.

Tarantola, A., 1987, Inverse problem theory: Elsevier.

APPENDIX A

Source wavefield downward extrapolation

The recursion in equations (2) and (3) can be also written in matrix form as

$$(I - W^+)\ P^+ = F, \tag{A-1}$$

$$\begin{bmatrix} 1 & 0 & 0 & ... & 0 & 0 \\ -w^+(z_i, z_0) & 1 & 0 & ... & 0 & 0 \\ 0 & -w^+(z_{i+j}, z_i) & 1 & ... & 0 & 0 \\ ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & -w^+(z_{N_z}, z_{N_z - 1}) & 1 \end{bmatrix} \begin{bmatrix} p^+(z_0) \\ p^+(z_i) \\ p^+(z_{i+j}) \\ ... \\ p^+(z_{N_z}) \end{bmatrix} = \begin{bmatrix} f \\ 0 \\ 0 \\ ... \\ 0 \end{bmatrix},$$

where

- W^+ is a lower bidiagonal matrix containing the downward continuation operator for all depth levels,
- P^+ is a column vector containing the source wavefield at all depth levels, and
- F is a column vector containing the source signature.

Equation (A-1) represents the downward continuation recursion written for a given frequency. We can write a similar relationship for each of the frequencies in the data, and group them all in a matrix relationship:

$$\left(\mathcal{I} - \mathcal{W}^+\right) \mathcal{P}^+ = \mathcal{F}, \tag{A-2}$$

$$\begin{bmatrix} I - W^+(\omega_1) & 0 & ... & 0 \\ 0 & I - W^+(\omega_2) & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & I - W^+(\omega_{N_\omega}) \end{bmatrix} \begin{bmatrix} P^+(\omega_1) \\ P^+(\omega_2) \\ ... \\ P^+(\omega_{N_\omega}) \end{bmatrix} = \begin{bmatrix} F(\omega_1) \\ F(\omega_2) \\ ... \\ F(\omega_{N_\omega}) \end{bmatrix},$$

where

- $\left(\mathcal{I} - \mathcal{W}^+\right)$ is a lower bidiagonal matrix containing the downward continuation operators for all the frequencies in the data,
- $\mathcal{P}^+$ is a column vector containing the wavefield data for all the frequencies, and
- $\mathcal{F}$ is a column vector containing the source signature for all the frequencies.

Equation (A-2) represents the downward continuation recursion written for a given shot position. We can write a similar relationship for each of the shot positions in the data, and group them all in a matrix relationship:

$$\left(\mathbf{I}-\mathbf{W}^{+}\right)\mathbf{P}^{+}=\mathbf{f}, \tag{A-3}$$

$$\begin{bmatrix} \mathcal{I}-\mathcal{W}^{+}(s_1) & 0 & ... & 0 \\ 0 & \mathcal{I}-\mathcal{W}^{+}(s_2) & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \mathcal{I}-\mathcal{W}^{+}(s_{N_s}) \end{bmatrix} \begin{bmatrix} \mathcal{P}^{+}(s_1) \\ \mathcal{P}^{+}(s_2) \\ ... \\ \mathcal{P}^{+}(s_{N_s}) \end{bmatrix} = \begin{bmatrix} \mathcal{F}(s_1) \\ \mathcal{F}(s_2) \\ ... \\ \mathcal{F}(s_{N_s}) \end{bmatrix},$$

where

- $\left(\mathbf{I}-\mathbf{W}^{+}\right)$ is a lower bidiagonal matrix containing the downward continuation operators for all the shots in the data,
- $\mathbf{P}^{+}$ is a column vector containing the wavefield data for all the shots, and
- $\mathbf{f}$ is a column vector containing the source signature for all the shots.

APPENDIX B

Receiver wavefield extrapolation

The recursion in equation (6) can be also written in matrix form as

$$(I-W^{-})\,P^{-}=P^{+}\,\mathbf{r}, \tag{B-1}$$

$$\begin{bmatrix} 1 & -w^{-}(z_0,z_i) & 0 & ... & 0 & 0 \\ 0 & 1 & -w^{-}(z_1,z_{i+j}) & ... & 0 & 0 \\ 0 & 0 & ... & ... & ... & ... \\ ... & ... & ... & ... & 1 & -w^{-}(z_{N_z-1},z_{N_z}) \\ 0 & 0 & 0 & ... & 0 & 1 \end{bmatrix} \begin{bmatrix} p^{-}(z_0) \\ p^{-}(z_i) \\ p^{-}(z_{i+j}) \\ ... \\ p^{-}(z_{N_z}) \end{bmatrix} = \tag{B-2}$$

$$\begin{bmatrix} p^{+}(z_0) & 0 & 0 & ... & 0 \\ 0 & p^{+}(z_i) & 0 & ... & 0 \\ 0 & 0 & p^{+}(z_{i+j}) & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & p^{+}(z_{N_z}) \end{bmatrix} \begin{bmatrix} r(z_0) \\ r(z_i) \\ r(z_{i+j}) \\ ... \\ r(z_{N_z}) \end{bmatrix} + \begin{bmatrix} 0 \\ p_r^{-}(z_i) \\ 0 \\ ... \\ 0 \end{bmatrix},$$

where

- W^{-} is a upper bidiagonal matrix containing the upward continuation operator for all depth levels,

- P^- is a column vector containing the receiver wavefield at all depth levels,
- P^+ is a diagonal square matrix containing the source wavefield at all depth levels, and
- **r** is the reflectivity at all depth levels.

Equation (B-1) represents the upward continuation recursion written for a given frequency. We can write a similar relationship for each of the frequencies in the data, and group them all in a matrix relationship:

$$(\mathcal{I} - \mathcal{W}^-)\, \mathcal{P}^- = \mathcal{P}^+\, \Sigma_\omega^t\, \mathbf{r}, \tag{B-3}$$

where

- $\left(\mathcal{I} - \mathcal{W}^-\right)$ is a upper bidiagonal matrix containing the upward continuation operators for all the frequencies in the data,

$$(\mathcal{I} - \mathcal{W}^-) = \begin{bmatrix} I - W^-(\omega_1) & 0 & ... & 0 \\ 0 & I - W^-(\omega_2) & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & I - W^-(\omega_{N_\omega}) \end{bmatrix},$$

- $\mathcal{P}^-$ is a column vector containing the receiver wavefield for all the frequencies,

$$\mathcal{P}^- = \begin{bmatrix} P^-(\omega_1) \\ P^-(\omega_2) \\ ... \\ P^-(\omega_{N_\omega}) \end{bmatrix},$$

- $\mathcal{P}^+$ is a diagonal square matrix containing the source wavefield for all the frequencies,

$$\mathcal{P}^+ = \begin{bmatrix} P^+(\omega_1) & 0 & ... & 0 \\ 0 & P^+(\omega_2) & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & P^+(\omega_{N_\omega}) \end{bmatrix},$$

- and Σ_ω is the sum over frequency matrix with dimensions $N_Z \times (N_Z \times N_\omega)$ (the transpose of the spreading over frequencies),

$$\Sigma_\omega = \left[\left| \begin{matrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & 1 \end{matrix} \right| \left| \begin{matrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & 1 \end{matrix} \right| \begin{matrix} ... \\ ... \\ ... \\ ... \end{matrix} \left| \begin{matrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & 1 \end{matrix} \right| \right].$$

Equation (B-3) represents the upward continuation recursion written for a given shot position. We can write a similar relationship for each of the shot positions in the data, and group them all in a matrix relationship:

$$(\mathbf{I} - \mathbf{W}^-)\, \mathbf{P}^- = \mathbf{P}^+\, \Sigma_{\omega s}^t\, \mathbf{r}, \tag{B-4}$$

where

- $\left(\mathbf{I}-\mathbf{W}^-\right)$ is a upper bidiagonal matrix containing the upward continuation operators for all the shots in the data,

$$\left(\mathbf{I}-\mathbf{W}^-\right) = \begin{bmatrix} \mathcal{I}-\mathcal{W}^-(s_1) & 0 & ... & 0 \\ 0 & \mathcal{I}-\mathcal{W}^-(s_2) & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \mathcal{I}-\mathcal{W}^-(s_{N_s}) \end{bmatrix},$$

- $\mathbf{P}^-$ is a column vector containing the receiver wavefield for all the shots,

$$\mathbf{P}^- = \begin{bmatrix} \mathcal{P}^-(s_1) \\ \mathcal{P}^-(s_2) \\ ... \\ \mathcal{P}^-(s_{N_s}) \end{bmatrix},$$

- $\mathbf{P}^+$ is a diagonal square matrix containing the source wavefield for all the shots,

$$\mathbf{P}^+ = \begin{bmatrix} \mathcal{P}^+(s_1) & 0 & ... & 0 \\ 0 & \mathcal{P}^+(s_2) & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \mathcal{P}^+(s_{N_s}) \end{bmatrix},$$

- and $\Sigma_{\omega s}$ is the sum over the frequency matrix with dimensions $N_Z \times (N_Z \times N_\omega \times N_s)$ (the transpose of the spreading over frequencies),

$$\Sigma_{\omega s} = \begin{bmatrix} \begin{vmatrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & 1 \end{vmatrix} & \begin{vmatrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & 1 \end{vmatrix} & \begin{matrix} ... \\ ... \\ ... \\ ... \end{matrix} & \begin{vmatrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & 1 \end{vmatrix} \end{bmatrix}.$$