Stanford Exploration Project, Report 113, July 8, 2003, pages 1–454

Dynamic permeability in poroelasticity

James G. Berryman[1]

ABSTRACT

The concept of dynamic permeability is reviewed. Modeling of seismic wave propagation using dynamic permeability is important for analyzing data as a function of frequency. In those systems where the intrinsic attenuation of the wave is caused in large part by viscous losses due to the presence of fluids, the dynamic permeability provides a very convenient and surprisingly universal model of this behavior.

INTRODUCTION

This article will review some of what is known about the concept of "dynamic permeability," as it is now commonly called, within the subject of poroelasticity. There are competing terms that are used sometimes. In particular, Biot (1956b) introduced the concept of dynamic viscosity, and this concept is incorporated into the dynamic permeability concept as we shall see. Dynamic tortuosity is closely related to dynamic permeability since the electric tortuosity is related (inversely) to the high frequency limit of the dynamic permeability, whereas the usual Darcy permeability is exactly the low frequency limit of the dynamic permeability. The tortuosity is also related to the effective mass of the fluid in the presence of the solid frame or to the effective mass of the solid in the presence of the pore fluid. We will discuss all of these concepts at greater length in the following sections.

EQUATIONS OF POROELASTICITY

For long-wavelength disturbances ($\lambda >> h$, where h is a typical pore size) propagating through such a porous medium, we define average values of the (local) displacements in the solid and also in the saturating fluid. The average displacement vector for the solid frame is $\mathbf{u}$ while that for the pore fluid is $\mathbf{u}_f$. The average displacement of the fluid relative to the frame is $\mathbf{w} = \phi(\mathbf{u} - \mathbf{u}_f)$, where ϕ is the porosity. For small strains, the frame dilatation is

$$e = e_x + e_y + e_z = \nabla \cdot \mathbf{u}, \tag{1}$$

where e_x,e_y,e_z are the Cartesian strain components. Similarly, the average fluid dilatation is

$$e_f = \nabla \cdot \mathbf{u}_f \tag{2}$$

[1]**email:** berryman@sep.stanford.edu

(e_f also includes flow terms as well as dilatation) and the increment of fluid content is defined by

$$\zeta = -\nabla \cdot \mathbf{w} = \phi(e - e_f). \tag{3}$$

With these definitions, Biot (1962) obtains the stress-strain relations in the form

$$\delta\tau_{xx} = He - 2\mu(e_y + e_z) - C\zeta, \tag{4}$$

and similarly for $\delta\tau_{yy}$,$\delta\tau_{zz}$,

$$\delta\tau_{zx} = \mu\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right), \tag{5}$$

and again similarly for $\delta\tau_{yz}$,$\delta\tau_{xy}$, and finally

$$\delta p_f = M\zeta - Ce. \tag{6}$$

The $\delta\tau_{ij}$ are deviations from equilibrium of average Cartesian stresses in the saturated porous material and δp_f is similarly the isotropic pressure deviation in the pore fluid.

With time dependence of the form $\exp(-i\omega t)$, the coupled wave equations that incorporate (4)-(6) are of the form

$$-\omega^2(\rho\mathbf{u} + \rho_f\mathbf{w}) = (H - \mu)\nabla e + \mu\nabla^2\mathbf{u} - C\nabla\zeta, \tag{7}$$

$$-\omega^2(\rho_f\mathbf{u} + q\mathbf{w}) = -M\nabla\zeta + C\nabla e, \tag{8}$$

where $\rho = \phi\rho_f + (1-\phi)\rho_s$, with ρ_f being the fluid density, ρ_s being the solid density, and ρ being the overall bulk-density of the material, while

$$q(\omega) = \rho_f\left[\alpha/\phi + iF(\xi)\eta/\kappa_0\omega\right] \tag{9}$$

is the frequency dependent effective density of the fluid in relative motion. This expression is also the one that we study later in order to introduce the concepts of dynamic permeability and tortuosity (Johnson *et al.*, 1987). So we want to emphasize now how important this equation for $q(\omega)$ will be to our later analyses. The kinematic viscosity of the liquid is η; the low frequency permeability of the porous frame is κ_0; the 3D dynamic viscosity factor [as it was first called by Biot (1956b)] is given, for our choice of sign for the frequency dependence, by $F(\xi) = \frac{1}{4}\{\xi T(\xi)/[1 + 2T(\xi)/i\xi]\}$, where $T(\xi) = \frac{\mathrm{ber}'(\xi) - i\,\mathrm{bei}'(\xi)}{\mathrm{ber}'(\xi) - i\,\mathrm{bei}'(\xi)}$ and $\xi = (\omega h^2/\eta)^{\frac{1}{2}}$. The functions $\mathrm{ber}(\xi)$ and $\mathrm{bei}(\xi)$ are the real and imaginary parts of the Kelvin function. The dynamic parameter h is a characteristic length generally associated with and comparable in magnitude to the steady-flow hydraulic radius. (For a model calculation we discuss later, $h = a$, the radius of a cylindrical pore.) The tortuosity $\alpha \geq 1$ is a pure number related to the frame inertia which has been measured by Johnson *et al.* (1982) and has also been estimated theoretically by Berryman (1980). We discuss α and its various interpretations in greater detail later in this review.

The coefficients H, C, and M are given (Gassmann, 1951; Biot and Willis, 1957; Biot, 1962) by

$$H = K + \frac{4}{3}\mu + (K_s - K)^2/(D - K), \tag{10}$$

$$C = K_s(K_s - K)/(D - K), \tag{11}$$

and

$$M = K_s^2/(D - K), \tag{12}$$

where

$$D = K_s[1 + \phi(K_s/K_f - 1)], \tag{13}$$

with K_f being the fluid bulk modulus and K_s being the solid bulk modulus. The frame (porous solid without liquid in the pores) constants are K for bulk and μ for shear. Equations (10)-(13) are correct as long as the porous material may be considered homogeneous on the microscopic scale as well as the macroscopic scale.

Eq. (7) is essentially the equation of elastodynamics of the solid frame with coupling terms (involving $\mathbf{w}$ and ζ) to the fluid motion. Eq. (8) reduces exactly to Darcy's equation when the solid displacement $\mathbf{u}$ and frame strain e are zero, since the right hand side of the equation is just $-\nabla p_f$.

To decouple (and subsequently solve) the wave equations in (7) and (8) into Helmholtz equations for the three modes of propagation, note that the displacements $\mathbf{u}$ and $\mathbf{w}$ can be decomposed as

$$\mathbf{u} = \nabla\Upsilon + \nabla\times\boldsymbol{\beta}, \qquad \mathbf{w} = \nabla\psi + \nabla\times\boldsymbol{\chi}, \tag{14}$$

where Υ, ψ are scalar potentials and $\boldsymbol{\beta}$, $\boldsymbol{\chi}$ are vector potentials. Substituting (14) into (7) and (8), the two equations are solved if two pairs of equations are satisfied:

$$(\nabla^2 + k_s^2)\boldsymbol{\beta} = 0, \qquad \boldsymbol{\chi} = -\rho_f\boldsymbol{\beta}/q \tag{15}$$

and

$$(\nabla^2 + k_\pm^2)A_\pm = 0. \tag{16}$$

The wavenumbers in (15) and (16) are defined by

$$k_s^2 = \omega^2(\rho - \rho_f^2/q)/\mu \tag{17}$$

and

$$k_\pm^2 = \tfrac{1}{2}\left\{b + f \mp [(b - f)^2 + 4cd]^{\frac{1}{2}}\right\}, \tag{18}$$

$$b = \omega^2(\rho M - \rho_f C)/\Delta,\ c = \omega^2(\rho_f M - qC)/\Delta,$$
$$d = \omega^2(\rho_f H - \rho C)/\Delta,\ \ f = \omega^2(qH - \rho_f C)/\Delta, \tag{19}$$

with $\Delta = MH - C^2$. The linear combination of scalar potentials has been chosen to be $A_\pm = \Gamma_\pm \Upsilon + \psi$, where

$$\Gamma_\pm = d/(k_\pm^2 - b) = (k_\pm^2 - f)/c. \tag{20}$$

With the identification (20), the decoupling is complete.

Equations (15) and (16) are valid for any choice of coordinate system, not just Cartesian coordinates, and they are therefore very useful in all applications of the theory.

INDUCED MASS EFFECT

Biot (1956a) defines the kinetic energy T of the porous medium by

$$2T = \rho_{11}\dot{\mathbf{u}}\cdot\dot{\mathbf{u}} + 2\rho_{12}\dot{\mathbf{u}}\cdot\dot{\mathbf{u}}_f + \rho_{22}\dot{\mathbf{u}}_f\cdot\dot{\mathbf{u}}_f, \tag{21}$$

where $\dot{\mathbf{u}}$ and $\dot{\mathbf{u}}_f$ are the solid and fluid velocities at a point in the medium. In the standard way, the overdots indicate a time derivative. Then he shows that the inertial coefficients satisfy two sum rules:

$$\rho_{11} + \rho_{12} = (1-\phi)\rho_s \tag{22}$$

and

$$\rho_{22} + \rho_{12} = \phi\rho_f. \tag{23}$$

Furthermore, as a matter of definition for the “structure factor” α, we also have

$$\rho_{12} = -(\alpha - 1)\phi\rho_f. \tag{24}$$

And Biot’s discussion makes it clear, furthermore, that $-\rho_{12}$ should be thought of as the added or induced mass of the solid when it oscillates in the presence of the fluid. So we have

$$\rho_{11} = (1-\phi)\rho_s - \rho_{12} \equiv (1-\phi)(\rho_s + r\rho_f), \tag{25}$$

where r is a measure of the geometry of the solid. In particular, it is well-known [see Lamb (1936)] that the factor $r = \frac{1}{2}$ for well-separated spheres and that it can vary from zero to unity, depending on the shape for simple objects such as spheroids. Combining these results, we find in general that

$$\alpha = 1 + r(\phi^{-1} - 1), \tag{26}$$

or, if we take a conservative value for r and set it to $r = \frac{1}{2}$, then we have

$$\alpha = \frac{1}{2}\left(1 + \phi^{-1}\right). \tag{27}$$

The formula (27) was derived by Berryman (1980, 1981), and it is frequently used with success when fitting real data.

ANOTHER INTERPRETATION OF α AS TORTUOSITY

It has been shown by Berryman (1980, 1981) that the speed of the slow wave [*i.e.*, the second compressional wave predicted by Biot (1956a)] can be written as

$$v_-^2 = \frac{K_f}{\alpha\rho_f}, \tag{28}$$

where K_f is the bulk modulus of the pore fluid. Or, first noting that the sound speed in the fluid is given by $v_f = \sqrt{K_f/\rho_f}$, we also have

$$v_- = \frac{v_f}{\sqrt{\alpha}}. \tag{29}$$

Thus, since the slow wave is a compressional wave through the pore space which presumably travels at approximately the speed v_f locally, the formula (29) can be interpreted as showing that the path length between two points in the pore space a distance L apart in space are a distance $\sqrt{\alpha}L$ apart when the travel path is restricted to the pore space. Thus, $\sqrt{\alpha}$ is a measure of the tortuosity of the pore space.

That α is a measure of the tortuosity has also been shown using a very different argument by Brown (1980). He shows that α is related to the electric formation factor for a porous system. In particular $\alpha = \phi\mathcal{F}$, where $\mathcal{F}$ is the electrical formation factor [which should not be confused with the dynamic viscosity factor $F(\xi)$ to be introduced later] defined by $\mathcal{F} = g_f/g \geq 1$, with g_f being the electrical conductivity of the pore fluid and g being the effective overall conductivity of the system when the solid is nonconducting. The formation factor and the tortuosity are both equal to unity when $\phi = 1$. For some systems, it is known both from Archie's law and from theory too that $\mathcal{F} \simeq \phi^{-3/2}$ is often a reasonable estimate for pore systems. Thus, $\alpha \simeq \phi^{-1/2}$. Both this expression and (27) have the same repesentation as $\alpha \simeq (3-\phi)/2$ when ϕ is large, showing that the two approaches using electrical tortuosity and the induced mass do in fact give similar results in this limit, even though it might seem hard to understand physically just exactly why this should be true.

DYNAMIC PERMEABILITY AND TORTUOSITY

Now we will return to the expression (9) and make the following two definitions: First, the dynamic tortuosity is given by

$$\alpha(\omega) \equiv \phi q(\omega)/\rho_f = \alpha + iF(\xi)\eta\phi/\kappa_0\omega \tag{30}$$

and, second, the inverse of the dynamic permeability is given by

$$\frac{1}{\kappa(\omega)} \equiv \frac{\omega q(\omega)}{i\eta\rho_f}. \tag{31}$$

This second expression can also be rewritten as

$$\kappa(\omega) = \frac{\kappa_0}{F(\xi) - i\kappa_0\alpha\omega/\eta\phi}. \tag{32}$$

The motivations for both definitions can be seen in their limiting values:

$$\alpha(\infty) = \alpha \qquad \text{and} \qquad \kappa(0) = \kappa_0. \tag{33}$$

These results follow once it is recognized that the dynamic viscosity factor $F(\xi)$ has unity as its the limit when $\omega \to 0$ and, although it can have differing numerical factors, it always goes like $(-i\omega/\omega_0)^{1/2}$ for large $\omega \to \infty$. We will give some examples of this behavior in the next section.

ANALYTICAL EXAMPLES OF THE DYNAMIC VISCOSITY FUNCTION

Solid spheres

Chase (1979), using some results from Landau and Lifshitz (1959), shows that, for well-separated spheres of radius R, the factor

$$q(\omega) = \frac{\rho_f}{\phi}\left[1 + i\,\Delta(\omega)\right], \tag{34}$$

where

$$\Delta(\omega) = \frac{9}{4}(\phi^{-1} - 1)\left[1 + z - iz(1 + 2z/9)\right]z^{-2} \tag{35}$$

with $z = 2^{-1/2}\xi$. Here $\xi = (\omega R^2/\eta)^{1/2}$, and η is the viscosity of the fluid, as usual. Comparing the main terms, we find that this formula shows

$$\alpha = 1 + \frac{1}{2}(\phi^{-1} - 1), \tag{36}$$

which is in complete agreement with (27), and

$$F(\xi) = 1 + (-i)^{1/2}\xi. \tag{37}$$

The result (37) is the first of several results showing that F behaves like $\omega^{1/2}$ for large ω.

2D duct

Biot (1956b) shows that for a 2D duct, *i.e.*, fluid between two planes separated by a distance $2a_1$, $F(\xi) \to 1$ as $\omega \to 0$ and

$$F(\xi) \to (-i)^{1/2}\frac{\xi}{3} \tag{38}$$

as $\omega \to \infty$. Here $\xi = (\omega a_1^2/\eta)^{1/2}$ with a_1 being half the separation distance between the planes.

3D duct

Biot (1956b) also shows that for a 3D circular cylinder duct, *i.e.*, fluid inside a circular cylindrical pore of radius a, $F(\xi) \to 1$ as $\omega \to 0$ and

$$F(\xi) \to (-i)^{1/2}\frac{\xi}{4} \tag{39}$$

as $\omega \to \infty$. Here $\xi = (\omega a^2/\eta)^{1/2}$, with a being the cylinder radius.

Discussion of analytical examples

All three of these examples, and indeed any other example as well, show that $F(\xi) \to 1$ as $\omega \to 0$. This result is universal. The other limit for $\omega \to \infty$ gives somewhat different results for the three cases considered, but they can all be approximated by using the form

$$F(\xi) \simeq \left(1 - iP\xi^2\right)^{1/2}, \tag{40}$$

where P is a number that depends on the duct model. Assuming for the moment that $R = a_1 = a$: For spherical particles, $P = 1$. For the 2D duct, $P = 1/9$. For the 3D duct, $P = 1/16$.

Since one of the implicit goals of the dynamic permeability analysis is to determine a universal form for the function $F(\xi)$ and thereby determine a universal form for the dynamic permeability, it is important to consider how these problems differ from each other. The radius R is a measure of the particle size, but this size is not easy to relate to the radius of the cylindrical pore in the third case, or to the duct height in the second case. It would make more sense to relate these quantities in some more general way since the typical rock sample will not have any of these geometries. Perhaps the obvious choice is to use κ_0 itself, since we need a pertinent measure of length squared, and that is exactly what the low frequency permeability is.

In fact, if we first consider the form of (37) and take the point of view that the factor $\kappa_0\alpha\omega/\eta\phi$ in the denominator determines a "natural" characteristic frequency for the problem given by

$$\omega_0 = \eta\phi/\alpha\kappa_0, \tag{41}$$

then, we can choose to approximate the dynamic viscosity factor by

$$F(\xi) \simeq \left(1 - iP\xi^2\right)^{1/2} \tag{42}$$

where now $\xi \equiv (\omega/\omega_0)^{1/2}$ and P is a real numerical factor that is at least approximately problem independent. Experimental and computational results show (Sheng and Zhou, 1988; Johnson, 1989; Sheng *et al.*, 1989) that many rocks and other porous systems can be successfully modeled this way using P's such that $0.4 \le P \le 0.5$. So the range of values for P is really quite small in many cases.

There are some exceptions to these rules but they are beyond our present scope, so the reader is encouraged to see the paper by Pride *et al.* (1993) for an extended discussion.

NUMERICAL EXAMPLES OF THE DYNAMIC PERMEABILITY FUNCTION

From (32) and the preceding discussion, we conclude that a reasonable choice of the functional form for dynamic permability is

$$\frac{\kappa(\omega)}{\kappa_0} = \frac{1}{(1 - iP\omega/\omega_0)^{1/2} - i\omega/\omega_0}, \quad (43)$$

where $0 \le P \le 1$ and $\omega_0 = \eta\phi/\alpha\kappa_0$. We plot this function in the complex plane for five choices of P in Figure 1. The polar angle θ is displayed as a function of ω/ω_0 in Figure 2. Then, the real and imaginary parts are plotted in Figures 3 and 4 as a function of the quantity ω/ω_0.

Figure 1 shows that all these choices give very similar behavior in the complex plane. The function looks much like a semi-circle centered at the point $(1/2, 0)$ and having radius $1/2$ in all cases. But this observation is only exactly true for the case $P = 0$. For that case it is also true that the relationship between ω and the polar angle θ in the complex plane is given exactly by $\omega/\omega_0 = \tan(\theta/2)$. For the other values of P, this relationship holds approximately true for $\omega/\omega_0 \le 1$ and $P \le 1/2$, but deviations become substantial in all cases for higher values of ω, as is observed in Figure 2.

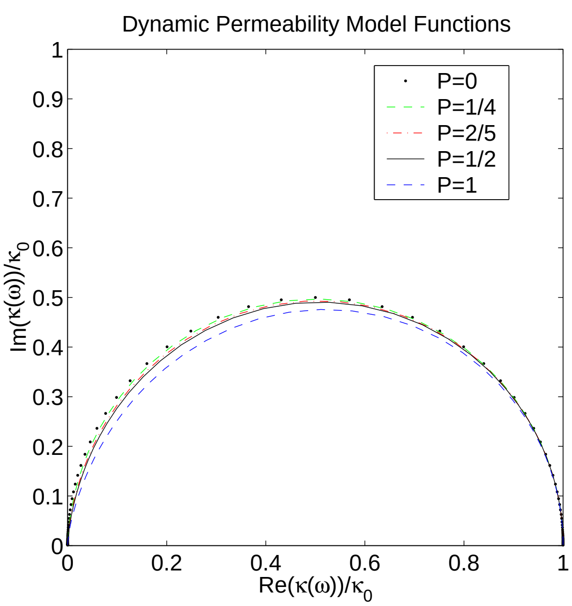

Figure 1: lustration of the behavior in the complex plane of five model functions for $\kappa(\omega)/\kappa_0$ of the form found in Eq. (43) for the different values of $P = 0$, 0.25, 0.4, 0.5, 1. Note that the case $P = 0$ is exactly a semi-circle of radius $1/2$ centered at the point $(1/2, 0)$ in the complex plane. jim1-fivemodels [NR]

Figure 3 shows that the real part of the dynamic permeability is essentially flat for a wide range of the smaller frequencies, but then the function falls off rapidly as the frequency gets close to the resonance frequency ω_0. Figure 4 shows that all the action occurs over six decades of frequency and the main region of the deviation (half maximum and above in the imaginary part) lies approximately in the range $0.2 \le \omega/\omega_0 \le 3.0$, which is just slightly over one decade in width.

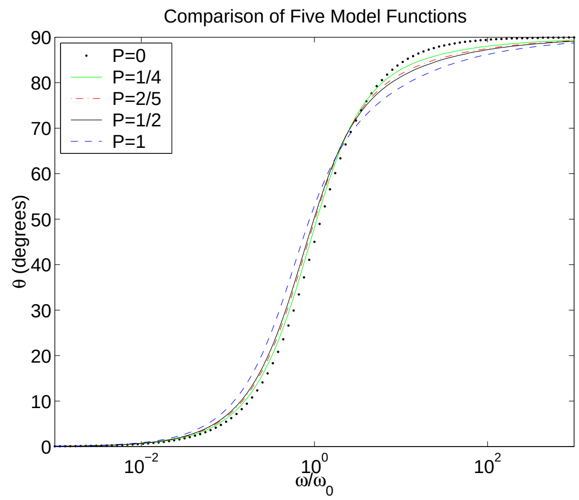

Figure 2: Polar angle θ in degrees in the complex plane of points of five model functions of the form found in Eq. (43) for the different values of $P = 0$, 0.25, 0.4, 0.5, 1 (see Figure 1). Note that the case $P = 0$ is exactly a semi-circle of radius $1/2$ centered at the point $(1/2, 0)$ in the complex plane, and for this case $\omega/\omega_0 = \tan(\theta/2)$. All other cases are observed to deviate from this behavior. jim1-angledeg [NR]

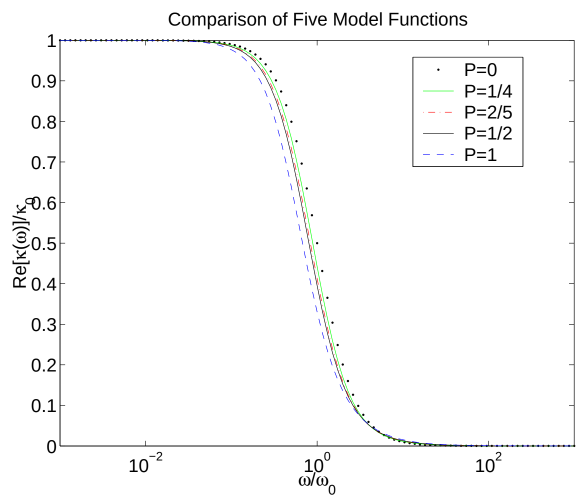

Figure 3: Illustration of the behavior of the real part of five model functions for different values of $P = 0$, 0.25, 0.4, 0.5, 1 as a function of the argument ω/ω_0. Note that the real part of the dynamic permeability is essentially flat for a wide range of the smaller frequencies, but then falls off rapidly as the frequency approaches the resonance frequency ω_0 from below. jim1-fivemodsre [NR]

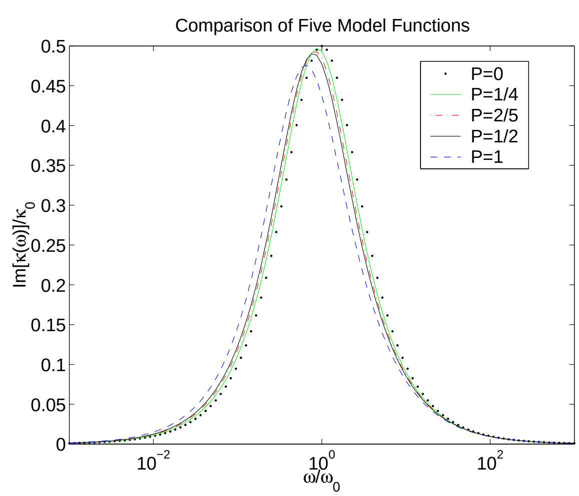

Figure 4: Illustration of the behavior of the imaginary part of five model functions for different values of $P =$ 0, 0.25, 0.4, 0.5, 1 as a function of the argument ω/ω_0. Note the main region of the deviation (half maximum and above) lies approximately in the range $0.2 \le \omega/\omega_0 \le 3.0$, which is just slightly over one decade in width. jim1-fivemodsim [NR]

CONCLUSIONS

The dynamic permeability is a useful concept that is important for analyzing seismic data as a function of frequency. In those systems where the intrinsic attenuation of the wave is caused in large part by viscous losses due to the presence of fluids, the dynamic permeability provides a convenient model of this behavior. Perhaps surprising is the fact that most systems can be modeled with a nearly universal function, depending only weakly on a scalar parameter P whose range of variation has been shown to be quite small ($0.4 \leq P \leq 0.5$) for a very wide range of systems relevant to seismic exploration.

ACKNOWLEDGMENTS

These notes were generated to supplement a lecture given at University of California – Berkeley. Questions from Professor Chi-Yuen Wang and other members of the audience played a significant role in the choice of topics presented here.

REFERENCES

Berryman, J. G., 1980, Confirmation of Biot's theory: *Appl. Phys. Lett.* **37**, 382–384.

Berryman, J. G., 1981, Elastic wave propagation in fluid-saturated porous media: *J. Acoust. Soc. Am.* **69**, 416–424.

Berryman, J. G., 1982, Elastic waves in fluid-saturated porous media: in *Macroscopic Properties of Disordered Media*, Springer-Verlag, New York, pp. 38–50.

Berryman, J. G., 1988, Seismic wave attenuation in fluid-saturated porous media: *PAGEOPH* **128**, 423–432.

Berryman, J. G., 1992, Effective stress for transport properties of inhomogeneous porous rock: *J. Geophys. Res.* **97**, 17409–17424.

Biot, M. A., 1956a, Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low-frequency range: *J. Acoust. Soc. Am.* **28**, 168–178.

Biot, M. A., 1956b, Theory of propagation of elastic waves in a fluid-saturated porous solid. II. Higher frequency range: *J. Acoust. Soc. Am.* **28**, 179–191.

Biot, M. A., 1962, Mechanics of deformation and acoustic propagation in porous media: *J. Appl. Phys.* **33**, 1482–1498.

Biot, M. A., and D. G. Willis, 1957, The elastic coefficients of the theory of consolidation: *J. Appl. Mech.* **24**, 594–601.

Brown, R. J. S., 1980, Connection between formation factor for electrical resistivity and fluid-solid coupling factor in Biot's equations for acoustic waves in fluid-filled porous media: *Geophysics* **45**, 1269–1275.

Chase, D. M., 1979, Wave propagation in liquid-saturated open-cell foam: *J. Acoust. Soc. Am.* **65**, 1–8).

Churchill, R. V., 1960, *Complex Variables and Applications*, McGraw-Hill, New York, Section 34: The linear fractional transformation, pp. 73–79. Gassmann, F., 1951, Über die Elastizität poröser Medien: Vierteljahrsschrift der Naturforschenden Gesellschaft in Zurich, **96**, 1–23.

Johnson, D. L., 1989, Scaling function for dynamic permeability in porous media – Comment: *Phys. Rev. Lett.* **63**, 580.

Johnson, D. L., J. Koplik, and R. Dashen, 1987, Theory of dynamic permeability and tortuosity in fluid-saturated porous media: *J. Fluid Mech.* **176**, 379–402.

Johnson, D. L., T. J. Plona, C. Scala, F. Pasierb, and H. Kojima, 1982, Tortuosity and acoustic slow waves: *Phys. Re. Lett.* **49**, 1840–1844 (1982).

Lamb, H., 1932, *Hydrodynamics*, Dover, New York, pp. 76–79, 122–125, 152–157.

Landau, L. D., and E. M. Lifshitz, 1959, *Fluid Mechanics*, Pergamon Press, Reading Massachusetts, pp. 95–96.

Landau, L. D., and E. M. Lifshitz, 1958, *Statistical Physics*, Addison-Wesley, Reading Massachusetts, pp. 384–391.

Milton, G. W., 1981, Bounds on complex permittivity of a two-component composite material: *J. Appl. Phys.* **52**, 5286–5293.

Milton, G. W., 2002, *The Theory of Composites*, Cambridge University Press, Cambridge, UK, pp. 369–395.

Pride, S. R., F. D. Morgan, and A. F. Gangi, 1993, Drag forces of porous media acoustics: *Phys. Rev. B* **47**, 4964–4978.

Sheng, P., and M.-Y. Zhou, 1988, Dynamic permeability in porous media: *Phys. Rev. Lett.* **61**, 1591–1594 (1988).

Sheng, P., M.-Y. Zhou, E. Charlaix, A. P. Kushnick, and J. P. Stokes, 1989, Scaling function for dynamic permeability in porous media – Reply: *Phys. Rev. Lett.* **63**, 581.