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Equivalence between shot-profile and source-receiver migration

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ABSTRACT

Shot-profile migration and source-receiver migration seem different, but the image and Common Image Gather they obtain is the same. In this paper, we prove that shot-profile migration and source-receiver migration are equivalent, assuming that the imaging condition is cross-correlation and the method for propagating the source and receiver wavefields is a one-way wave equation. This is achieved after generalizing source-receiver migration to arbitrary sources.

INTRODUCTION

At first glance, shot-profile migration and source-receiver migration seem to be completely different. They are performed on different data geometry and use different equations to extrapolate wavefields. Shot-profile migration downward continues source and receiver wavefields independently, and produces an image through a cross-correlation between these two wavefields along the time axis. Source-receiver migration extrapolates the CMP gathers with the Double Square Root equation, and creates an image by extracting the wavefield at zero time and zero subsurface offset.

However, shot-profile migration and source-receiver migration obtain the same migration result. Wapenaar and Berkhout (1987) proves that identical stacked images will be obtained from these two methods. Biondi (2002) proves that an equivalent image cube will be obtained, given the assumptions that the source is an impulse function, the imaging condition is cross-correlation, and the source and receiver wavefields are downward propagated by a one-way wave equation. The cross-correlation imaging condition and one-way wave equation downward continuation are the key points for the equivalence between shot-profile migration and source-receiver migration. In this paper, we give a new proof for the equivalence between shot-profile migration and source-receiver migration for an arbitrary source.

Before demonstrating the equivalence between them, we first present an overview of shot-profile migration and source-receiver migration.

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SHOT-PROFILE MIGRATION

In shot-profile migration, each shot is treated as an independent physics experiment. Each shot is migrated separately and the images of all the shots are then stacked to generate the final image. The source for shot-profile migration is not necessarily an impulse function. Actually, it is a wavelet for a point shot in practice. The source can also be a plane wave, a primary reflection (Guitton, 2002), or some other configurations. We assume that the source wavefield is a down-going wavefield and the receiver wavefield is an up-going wavefield. These two wavefields are downward continued independently. Let $U(x_U, z = 0, \omega, s)$ be the receiver wavefield and $D(x_D, z = 0, \omega, s)$ be the source wavefield at the surface for shot s. The source wavefield at the subsurface $U(x_U, z, \omega, s)$ can be obtained by extrapolating $U(x_U, z = 0, \omega, s)$ with the up-going wave equation

$$\frac{\partial}{\partial z} U(x_U, z, \omega, s) = \frac{i\omega}{v(x_U, z)} \sqrt{1 + \frac{v^2(x_U, z)}{\omega^2} \frac{\partial^2}{\partial x_U^2}} U(x_U, z, \omega, s), \tag{1}$$

and the source wavefield at subsurface $D(x_D, z, \omega, s)$ can be obtained by extrapolating $D(x_D, z = 0, \omega, s)$ with the down-going wave equation

$$\frac{\partial}{\partial z} D(x_D, z, \omega, s) = \frac{-i\omega}{v(x_D, z)} \sqrt{1 + \frac{v^2(x_D, z)}{\omega^2} \frac{\partial^2}{\partial x_D^2}} D(x_D, z, \omega, s). \tag{2}$$

where $\sqrt{1 + \frac{v^2(x_D, z)}{\omega^2} \frac{\partial^2}{\partial x_D^2}}$ is a pseudo-partial differential operator (Zhang, 1993). The image for shot s is formed by cross-correlating the source wavefield $D(x_D, z, \omega, s)$ and receiver wavefield $U(x_U, z, \omega, s)$ along the time axis at all depths and evaluating this at zero time lag (Claerbout, 1971). Stacking the images of all the shots, we can get the image of frequency ω

$$I(x, z, \omega) = \sum_s U(x_U = x, z, \omega, s) \bar{D}(x_D = x, z, \omega, s), \tag{3}$$

and the image of all frequencies

$$I(x, z) = \sum_\omega I(x, z, \omega). \tag{4}$$

To perform a velocity analysis, Rickett and Sava (2002) developed the Common Image Gather (CIG) for shot-profile migration, calculated by

$$I(x, h, z) = \sum_\omega I(x, h, z, \omega) = \sum_\omega \sum_s U(x_U = x + h, z, \omega, s) \bar{D}(x_D = x - h, z, \omega, s). \tag{5}$$

SOURCE-RECEIVER MIGRATION

Traditional source-receiver migration is based on the concept of survey sinking (Claerbout, 1985). It sorts the recorded data into CMP gathers $P(x, h, z = 0, \omega)$ and propagates the CMP gather to the subsurface $P(x,h,z,\omega)$ with the Double Square Root (DSR) equation

$$\frac{\partial}{\partial z}P(x,h,z,\omega) = \left(\frac{i\omega}{v(x_s,z)}\sqrt{1+\frac{v^2(x_s,z)}{\omega^2}\frac{\partial^2}{\partial x_s^2}} + \frac{i\omega}{v(x_r,z)}\sqrt{1+\frac{v^2(x_r,z)}{\omega^2}\frac{\partial^2}{\partial x_r^2}} \right) P(x,h,z,\omega), \tag{6}$$

where $x_s = x - h$ is the shot location, $x_r = x + h$ is the receiver location. The wavefield $P(x,h,z,\omega)$ at each depth z is equivalent to the data that would have been recorded when the shots and receivers were located at that depth. Source-receiver migration produces an image by extracting the wavefield at zero subsurface offset $P(x,h=0,z,\omega)$, and stacking over all frequencies. Correspondingly, the stack of $P(x,h,z,\omega)$ along the frequencies is its CIG.

Traditional source-receiver migration assumes that the source is an impulse function at the source location x_s. But in fact, source-receiver migration works for arbitrary sources, such as wavelets, plane waves, and primary reflections as well. For arbitrary sources, the surface wavefield $P(x,h,z=0,\omega)$ is not a simple CMP gather of the recorded data, but the stack of the cross-correlation between the source wavefield and the receiver wavefield at the surface

$$P(x,h,z=0,\omega) = \sum_s U(x_U = x+h, z=0,\omega,s)\bar{D}(x_D = x-h, z=0,\omega,s). \tag{7}$$

Then $P(x,h,z=0,\omega)$ is downward continued to the subsurface with the DSR equation (6) and the image is formed by the same method as the traditional source-receiver migration. In fact, traditional source-receiver migration is a special case of source-receiver migration when the source is an impulse function at the source location, and the CMP gather of the recorded data at the surface is the cross-correlation between the impulse function source and the receiver wavefield, which is the recorded data for each shot.

DEMONSTRATION OF EQUIVALENCE

Although shot-profile migration and source-receiver migration look totally different, they obtain both the same image and CIG. In this section, we prove that the mono-frequency image $I(x,z,\omega)$ and CIG $I(x,h,z,\omega)$ in the shot-profile migration are exactly the same mono-frequency image $P(x,h=0,z,\omega)$ and CIG $P(x,h,z,\omega)$ in the source-receiver migration, respectively.

We define a new wavefield $Q_s(x_U,x_D,z,\omega)$, which is the cross-correlation between the source wavefield $D(x_D,z,\omega,s)$ and the receiver wavefield $U(x_U,z,\omega,s)$ in the shot-profile migration for shot s, that is

$$Q_s(x_U,x_D,z,\omega) = U(x_U,z,\omega,s)\bar{D}(x_D,z,\omega,s), \tag{8}$$

and the wavefield $Q(x_U,x_D,z,\omega)$ is the stack of $Q_s(x_U,x_D,z,\omega)$ along all the shots,

$$Q(x_U,x_D,z,\omega) = \sum_s Q_s(x_U,x_D,z,\omega). \tag{9}$$

Obviously, from equation (7), $Q_s(x_U, x_D, z=0, \omega)$ is the surface data in source-receiver migration. We will demonstrate that the wavefield $Q_s(x_U, x_D, z, \omega)$ satisfies the DSR equation,

$$\frac{\partial}{\partial z} Q_s = \left(\frac{i\omega}{v(x_U,z)} \sqrt{1 + \frac{v^2(x_U,z)}{\omega^2} \frac{\partial^2}{\partial x_U^2}} + \frac{i\omega}{v(x_D,z)} \sqrt{1 + \frac{v^2(x_D,z)}{\omega^2} \frac{\partial^2}{\partial x_D^2}} \right) Q_s, \tag{10}$$

where $Q_s = Q_s(x_U, x_D, z, \omega)$. By extension, $Q(x_U, x_D, z, \omega)$ also satisfies the DSR equation. Thus shot-profile migration and source-receiver migration are two different ways to obtain wavefield Q at the subsurface. In shot-profile migration, source and receiver wavefields are downward continued into the subsurface with the one-way wave equation, and the wavefield $Q(x_U, x_D, z, \omega)$ is formed by cross-correlating the source wavefields and receiver wavefields and stacking over all shots at all depths. But in source-receiver migration, the wavefield $Q(x_U, x_D, z, \omega)$ at the surface $Q(x_U, x_D, z=0, \omega)$ is obtained by cross-correlating the source wavefield and the receiver wavefield at the surface, and $Q(x_U, x_D, z, \omega)$ is formed by extrapolating $Q(x_U, x_D, z=0, \omega)$ to all depths with the DSR equation.

From the Leibniz rule, we have

$$\frac{\partial Q_s}{\partial z} = \frac{\partial U}{\partial z} \bar{D} + U \frac{\partial \bar{D}}{\partial z}, \tag{11}$$

where $U = U(x_U, z, \omega, s)$ and $D = D(x_D, z, \omega, s)$. Since $U(x_U, z, \omega, s)$ is an up-going wavefield, it satisfies the up-going wave equation(1), so we have

$$\frac{\partial U}{\partial z} \bar{D} = \left(\frac{i\omega}{v(x_U,z)} \sqrt{1 + \frac{v^2(x_U,z)}{\omega^2} \frac{\partial^2}{\partial x_U^2}} U \right) \cdot \bar{D}. \tag{12}$$

$\bar{D}(x_D, z, \omega, x)$ is not dependent on x_U, so it is constant with respect to the operator $\sqrt{1 + \frac{v^2(x_U,z)}{\omega^2} \frac{\partial^2}{\partial x_U^2}}$, and we have

$$\left(\sqrt{1 + \frac{v^2(x_U,z)}{\omega^2} \frac{\partial^2}{\partial x_U^2}} U \right) \cdot \bar{D} = \sqrt{1 + \frac{v^2(x_U,z)}{\omega^2} \frac{\partial^2}{\partial x_U^2}} (U\bar{D}) = \sqrt{1 + \frac{v^2(x_U,z)}{\omega^2} \frac{\partial^2}{\partial x_U^2}} Q_s \tag{13}$$

Summarizing equation (12) and (13), we have

$$\frac{\partial U}{\partial z} \bar{D} = \frac{i\omega}{v(x_U,z)} \sqrt{1 + \frac{v^2(x_U,z)}{\omega^2} \frac{\partial^2}{\partial x_U^2}} Q_s. \tag{14}$$

It is easy to prove that

$$\frac{\partial \bar{D}}{\partial z} = \overline{\frac{\partial D}{\partial z}}. \tag{15}$$

So the second term of equation (11) changes to

$$U \frac{\partial \bar{D}}{\partial z} = U \overline{\frac{\partial D}{\partial z}}. \tag{16}$$

Since $D(x_D,z,\omega,s)$ is a down-going wavefield, it satisfies the down-going wave equation (2), and we have

$$U\cdot\overline{\frac{\partial D}{\partial z}} = U\cdot\overline{\frac{-i\omega}{v(x_D,z)}\sqrt{1+\frac{v^2(x_D,z)}{\omega^2}\frac{\partial^2}{\partial x_D^2}}D} \tag{17}$$

$$= U\cdot\left(\frac{i\omega}{v(x_D,z)}\sqrt{1+\frac{v^2(x_D,z)}{\omega^2}\frac{\partial^2}{\partial x_D^2}}\bar{D}\right). \tag{18}$$

Again, $U(x_U,z,\omega,s)$ does not depend on x_D, so we have

$$U\left(\frac{i\omega}{v(x_D,z)}\sqrt{1+\frac{v^2(x_D,z)}{\omega^2}\frac{\partial^2}{\partial x_D^2}}\bar{D}\right) = \frac{i\omega}{v(x_D,z)}\sqrt{1+\frac{v^2(x_D,z)}{\omega^2}\frac{\partial^2}{\partial x_D^2}}(U\bar{D}) \tag{19}$$

$$= \frac{i\omega}{v(x_D,z)}\sqrt{1+\frac{v^2(x_D,z)}{\omega^2}\frac{\partial^2}{\partial x_D^2}}Q_s \tag{20}$$

Summarizing equations (15-20), we have

$$U\frac{\partial\bar{D}}{\partial z} = \frac{i\omega}{v(x_D,z)}\sqrt{1+\frac{v^2(x_D,z)}{\omega^2}\frac{\partial^2}{\partial x_D^2}}Q_s \tag{21}$$

Finally, from equation (11), equation (14) and equation (21), we know Q_s satisfies the DSR equation (10). Q is the stack of Q_s over all shots, so by extension Q satisfies the DSR equation also.

It is obvious that the image of shot-profile migration in equation (3) is

$$I(x,z,\omega) = Q(x_U = x, x_D = x, z,\omega), \tag{22}$$

and the corresponding CIG in equation (5) is

$$I(x,h,z,\omega) = Q(x_U = x+h, x_D = x-h, z,\omega). \tag{23}$$

In traditional source-receiver migration, $Q(x_U = x+h, x_D = x-h, z = 0,\omega)$ is the stack of the cross-correlation between the impulse source and the recorded data along shots, which is the CMP gather $P(x,h,z=0,\omega)$ of the recorded data at the surface. Since both $Q(x_U = x+h, x_D = x-h, z,\omega)$ and $P(x,h,z,\omega)$ are obtained by propagating $Q(x_U = x+h, x_D = x-h, z = 0,\omega)$ to the subsurface with the DSR equation (10), they are equivalent. If the source in source-receiver migration is not an impulse function, $Q(x_U = x+h, x_D = x-h, z = 0,\omega)$ is the stack of the cross-correlation between the source wavefield and the receiver wavefield, and the same conclusion is reached. Thus we have

$$I(x,z,\omega) = P(x,h=0,z,\omega) = Q(x_U = x, x_D = x, z,\omega), \tag{24}$$

and

$$I(x,h,z,\omega) = P(x,h,z,\omega) = Q(x_U = x+h, x_D = x-h, z,\omega). \tag{25}$$

CONCLUSION

In this paper, we generalize the conception of source-receiver migration. Generalized source-receiver migration works for arbitrary sources. It is not restricted to a point source and it is not survey-sinking, but rather the downward continuation of the cross-correlation between the source and the receiver wavefield. We prove the equivalence between shot-profile migration and source-receiver migration for arbitrary sources, given the assumption that the imaging condition is cross-correlation and the wavefields are propagated by one-way wave equation downward continuation.

REFERENCES

Biondi, B., 2002, Equivalence of source-receiver migration and shot-profile migration: Geophysics, accepted for publication.

Claerbout, J. F., 1971, Toward a unified theory of reflector mapping: Geophysics, **36**, no. 03, 467–481.

Claerbout, J. F., 1985, Imaging the Earth's Interior: Blackwell Scientific Publications.

Guitton, A., 2002, Shot-profile migration of multiple reflections: SEP–**111**, 17–33.

Rickett, J., and Sava, P., 2002, Offset and angle-domain common image-point gathers for shot-profile migration: Geophysics, **67**, 883–889.

Wapenaar, C. P. A., and Berkhout, A. J., 1987, Full prestack versus shot record migration: 69th Ann. Internat. Meeting, Soc. of Expl. Geophys., Expanded Abstracts, Session:S15.7.

Zhang, G., 1993, System of coupled equations for upgoing and downgoing waves: Acta Math. Appl. Sinica, **16**, 251–263.