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Amplitude balanced PEF estimation

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ABSTRACT

Inverse theory teaches us that the residual, or misfit function, should be weighted by the inverse covariance matrix of the noise. Because the covariance operator is often difficult to estimate, we can approximate it with a diagonal weight that can be more easily computed. This paper investigates the possible choices of weighting functions for the data residual when prediction-error filters are estimated. Examples with 2-D and 3-D field data prove that it is better to weight the residual than to weight the data before starting the inversion.

INTRODUCTION

In seismic processing, we often have to estimate filters in order to, for example, improve the result of tomography (Clapp, 2001), interpolate data (Spitz, 1991; Crawley et al., 1999) or perform signal-noise separation (Soubaras, 1994; Abma and Claerbout, 1995). The filter estimation can be done in the time or frequency domain. In general, we first start by estimating the filters and then use them for a particular geophysical task. One general assumption when we estimate filters is that the time series from which we estimate the filters is wide-sense stationary. Often with real data, however, this assumption is violated. The non-stationarity can be related to a different moveout or a different amplitude behavior throughout the seismic record. The filter estimation in the first case can be improved by introducing patches. We then estimate one filter per patch assuming that the data are locally stationary. The boundary conditions for patching technology are rather difficult to handle and a better solution is to use non-stationary filters (Crawley et al., 1999). The amplitude aspect of non-stationarity is usually tackled before processing by applying, for example, an Automatic Gain Control (AGC) on the data. One problem with this approach is that it is a non-linear process that can damage important amplitude information. In a setting where more attributes are extracted from seismic data to derive rock properties, treating amplitudes accurately becomes a major challenge in seismic processing.

In this paper, I show strategies for estimating filters when amplitude problems exist with seismic data. My claim is that (1) any weight on the data should be applied as late as possible in the processing work-flow, preferably for display purposes only and (2) if weights are needed, they should be incorporated inside our processing scheme without letting any footprint in the final image (Claerbout, 1992). Inverse theory provides us with tools to handle amplitude problems which are usually not used by the industry. Theoretically, from a statistical view-

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point (Tarantola, 1987), the residuals should be weighted during the filter estimation by the inverse covariance matrix of the noise. This requirement might be quite difficult to meet because we might not know the noise a-priori and/or the covariance operator might be too expensive to compute. In the context of filter estimation, I will assume that the covariance operator is a diagonal weight. Here I investigate the filter estimation problem for signal/noise separation. I will show with 2-D and 3-D field data that it is important to weight the residual during the filter-estimation step to obtain the best noise attenuation results.

In the first section following this introduction, I will describe the theory of prediction-error filters (PEF) estimation when the data are weighted and when the residuals are weighted. Then, I will show with a 2-D example with stationary filters and a 3-D example with non-stationary filters that a weighted-residual PEF estimation leads to the best noise attenuation results, as predicted by inverse theory.

THEORY OF PEF ESTIMATION

This section is largely inspired by chapter 7.5 in Claerbout (1992). I show that the proper way of dealing with amplitude problems with seismic data for the PEF estimation is to weight the residual and *not* the data itself.

I will now describe the theory of PEF estimation for a three coefficients filter $\mathbf{a} = (1, a_1, a_2)'$ from a time series $\mathbf{y} = (y_0, y_1, y_2, ..., y_6)'$. Helical boundary conditions (Claerbout, 1998) make this analysis valid for multidimensional PEFs as well. What we want is to minimize the energy of the residual $\mathbf{r}$ according to

$$\mathbf{0} \quad \approx \quad \mathbf{r} = \begin{bmatrix} y_2 & y_1 & y_0 \\ y_3 & y_2 & y_1 \\ y_4 & y_3 & y_2 \\ y_5 & y_4 & y_3 \\ y_6 & y_5 & y_4 \end{bmatrix} \begin{bmatrix} 0 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} \tag{1}$$

which can be rewritten

$$\mathbf{0} \approx \mathbf{r} = \mathbf{YKa} + \mathbf{r}_0 \tag{2}$$

where $\mathbf{Y}$ is the convolution matrix with $\mathbf{y}$, $\mathbf{K}$ is a masking operator and $\mathbf{r}_0$ the first column of the convolution matrix $\mathbf{Y}$. The next step is to find $\mathbf{a}$ such that

$$f(\mathbf{a}) = \|\mathbf{r}\|^2 = \|\mathbf{YKa} + \mathbf{r}_0\|^2 \tag{3}$$

is minimum. We can solve this problem with an iterative solver such as a conjugate-direction method (Claerbout and Fomel, 2002). The least-squares inverse of $\mathbf{a}$ becomes

$$\hat{\mathbf{a}} = -(\mathbf{K}'\mathbf{Y}'\mathbf{YK})^{-\mathbf{1}}\mathbf{K}'\mathbf{Y}'\mathbf{r}_0. \tag{4}$$

It is common practice to weight the data before doing any processing in order to correct for amplitude anomalies. Doing so, the fitting goal in equation (1) becomes

$$\mathbf{0} \quad \approx \quad \mathbf{r} = \begin{bmatrix} w_2y_2 & w_1y_1 & w_0y_0 \\ w_3y_3 & w_2y_2 & w_1y_1 \\ w_4y_4 & w_3y_3 & w_2y_2 \\ w_5y_5 & w_4y_4 & w_3y_3 \\ w_6y_6 & w_5y_5 & w_4y_4 \end{bmatrix} \begin{bmatrix} 0 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} w_2y_2 \\ w_3y_3 \\ w_4y_4 \\ w_5y_5 \\ w_6y_6 \end{bmatrix} \tag{5}$$

where $\mathbf{w} = (w_1, w_2, ..., w_6)'$ are weights for a particular point of the time series $\mathbf{y}$. This is the wrong approach to correct for amplitude anomalies but it has practical values. First, equation (5) keeps the Toeplitz structure of the Hessian in equation (4), allowing fast computation of $\hat{\mathbf{a}}$. Then, the PEF can be easily estimated in the Fourier domain. However, inverse theory teaches us that we should be weighting the residual instead such that

$$\mathbf{0} \quad \approx \quad \mathbf{Wr} = \begin{bmatrix} w_2y_2 & w_2y_1 & w_2y_0 \\ w_3y_3 & w_3y_2 & w_3y_1 \\ w_4y_4 & w_4y_3 & w_4y_2 \\ w_5y_5 & w_5y_4 & w_5y_3 \\ w_6y_6 & w_6y_5 & w_6y_4 \end{bmatrix} \begin{bmatrix} 0 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} w_2y_2 \\ w_3y_3 \\ w_4y_4 \\ w_5y_5 \\ w_6y_6 \end{bmatrix} \tag{6}$$

with

$$\mathbf{W} = \begin{bmatrix} w_2 & 0 & 0 & 0 & 0 \\ 0 & w_3 & 0 & 0 & 0 \\ 0 & 0 & w_4 & 0 & 0 \\ 0 & 0 & 0 & w_5 & 0 \\ 0 & 0 & 0 & 0 & w_6 \end{bmatrix}. \tag{7}$$

The difference between equations (5) and (6) is that in the first case, we weight the data points and that in the second case, we weight the regression equations. In equation (6), we weight each row independently and leverage the PEF estimation globally. With the weighted residual, the Hessian loses its Toeplitz structure making the estimation of **a** less computationally efficient. In addition, the estimation of **a** in the Fourier domain becomes much more difficult.

It might not be clear why one method is better than the other. The choice of a weighting function can also be tricky. In the next section, I present guidelines and results on how to choose **W** and what it changes for two signal/noise separation examples.

SIGNAL-NOISE SEPARATION WITH WEIGHTED PEFS

In this section, I test the idea of weighting the residual for the PEF estimation as it is done in equation (6). I present guidelines on how to choose the weighting function. I illustrate my method with a 2-D and a 3-D data example for stationary and non-stationary PEFs estimation, respectively.

A brief theory of signal/noise separation

I present one possible formulation of the signal/noise separation problem as exemplified by Soubaras (1994); Abma and Claerbout (1995); Brown and Clapp (2000); Guitton et al. (2001). To simplify, we have the two following fitting goals:

$$\begin{array}{rcccl} \mathbf{0} & \approx & \mathbf{r_d} & = & \mathbf{N}(\mathbf{s}-\mathbf{d}) \\ \mathbf{0} & \approx & \epsilon\mathbf{r_s} & = & \epsilon\mathbf{Ss}, \end{array} \tag{8}$$

where the data $\mathbf{d}$ is equal to the sum of the noise $\mathbf{n}$ and signal $\mathbf{s}$, $\mathbf{N}$ is the PEF for the noise ($\mathbf{Nn} \approx \mathbf{0}$), $\mathbf{S}$ is the PEF for the signal ($\mathbf{Ss} \approx \mathbf{0}$), and ϵ a constant. Solving for $\mathbf{s}$ we have

$$\hat{\mathbf{s}} = (\mathbf{N}'\mathbf{N} + \epsilon^2\mathbf{S}'\mathbf{S})^{-1}\mathbf{N}'\mathbf{Nd}. \tag{9}$$

The signal-noise attenuation is then separated into two distinct steps: first we estimate the noise and signal filters $\mathbf{N}$ and $\mathbf{S}$, then we estimate the signal $\mathbf{s}$ based on the fitting goals in equation (8).

A 2-D example

Now I show how to estimate PEFs with data having amplitude anomalies. Figure 1a display a 2-D shot gather from a land survey. This gather shows high amplitudes at short offsets as indicated by the red color. The noise we want to attenuate is the low velocity/low frequency event visible throughout the section. I estimated a noise model in Figure 1b by transforming the data in the Radon domain and muting the velocities corresponding to the fastest events. The signal model in Figure 1c is obtained by subtracting Figure 1b to Figure 1a. This gather is particularly interesting because it illustrates very well the problem of amplitude balancing with real data. Indeed, it is clear from Figure 1 that the amplitude is not uniform across offset and time. That will cause problems for the PEF estimation and the signal-noise separation steps.

I show in Figure 2 the result of the signal noise separation when no weight is applied, i.e, equation (1). Figures 2a and 2c show the estimated signal and noise, respectively. The noise attenuation clearly failed here. Figures 2b and 2d display the impulse responses of the signal PEF and the noise PEF, respectively. In theory, Figure 2b should look like the signal (or at least have the same spectrum) and Figure 2d should look like the noise. The signal PEF is clearly wrong here since we do not retrieve the signal spectrum very well. This mismatch comes from the high amplitudes at short offset in Figure 1a that bias the estimation of the filter.

To make things right, a weighting function is needed for the residual. My choice of a weighting function is based on statistical considerations. I apply a weighting function on the data and look at the histogram. If the histogram has a Gaussian distribution, I keep it and use it during the estimation of the noise and signal PEFs. I keep this weighting function unchanged during the iterations, although I could reestimate it as it is done with Iteratively Reweighted Least Squares methods. Figure 3 shows four histograms corresponding to different weighting functions. The Amplitude Gain Control (AGC) produces the most satisfying result in term of

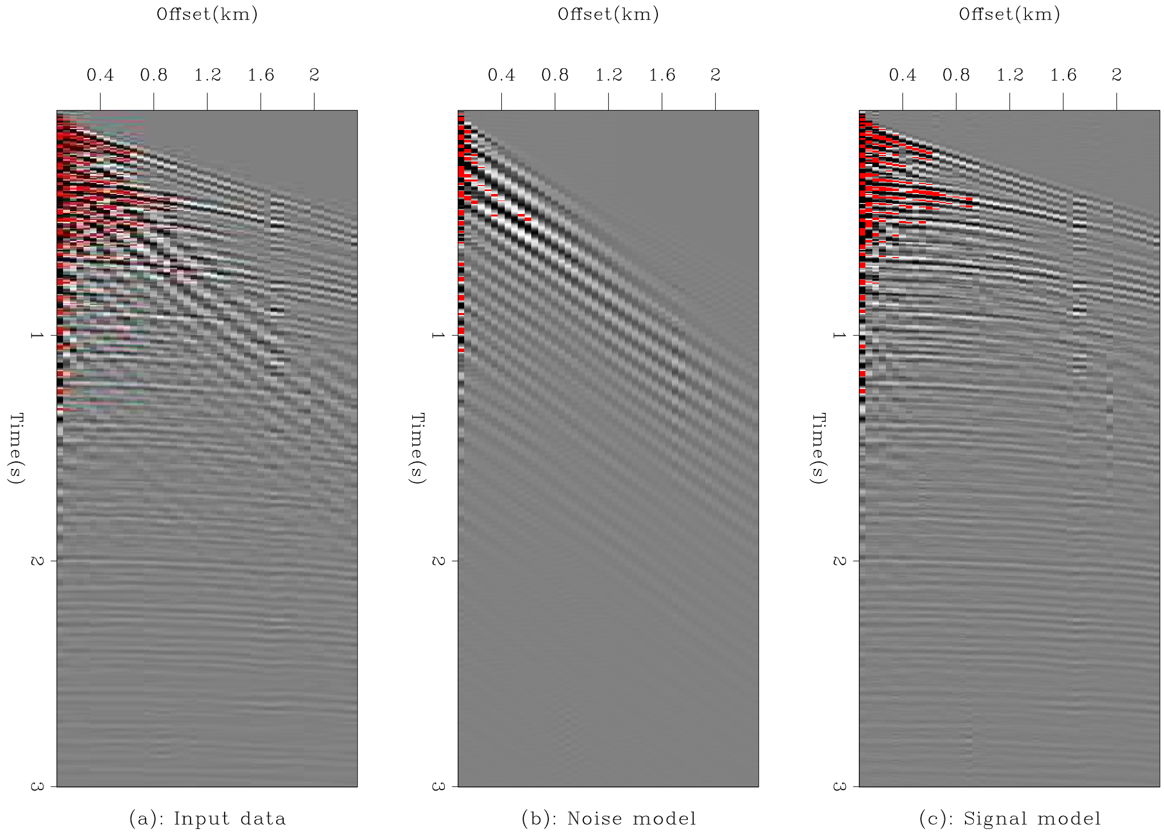

Figure 1: One shot gather from a 2-D land acquisition survey. The clipped values are shown in red. The input data on the left show high amplitudes at short offset. The middle and right panels display the noise and signal model, respectively. antoine2-input-data [ER,M]

distribution of the data. I then derive an appropriate weighting function from the AGC and estimated the noise and signal PEFs. To finish, I perform the signal separation on the raw data. Figure 4 displays the final noise attenuation result with the weighted PEF estimation. Figure 4a shows the estimated signal and Figure 4c the estimated noise. The noise has been successfully attenuated. Note that some signal has leaked in the noise in Figure 4c around 0.5 second. This is because I estimate only one noise and signal filter for the whole gather. But still, the noise is correctly attenuated. Figures 4b and 4d show a spike divided by the signal and noise PEFs respectively. Figure 4b demonstrates that the signal PEF correctly represents the quasi-flat signal. Figure 4d looks very similar to the noise with the correct amplitude behavior, as expected.

A last necessary comparison is by estimating the noise and signal PEFs from the weighted data [equation (5)] and look at the noise attenuation result. Figure 5 displays the noise attenuation result with the weighted data for the PEF estimation. The estimated signal in Figure 5a is satisfying with artifacts above 0.5 second (also visible in the estimated noise in Figure 5c). Figure 6 shows the difference between Figures 4a and 5a. In addition to the artifacts above 0.5 second, the noise is better removed at short offset in Figure 4a. The inverse PEF for the signal in Figure 5b is very similar to Figure 4b, but the amplitude decay is less important. The

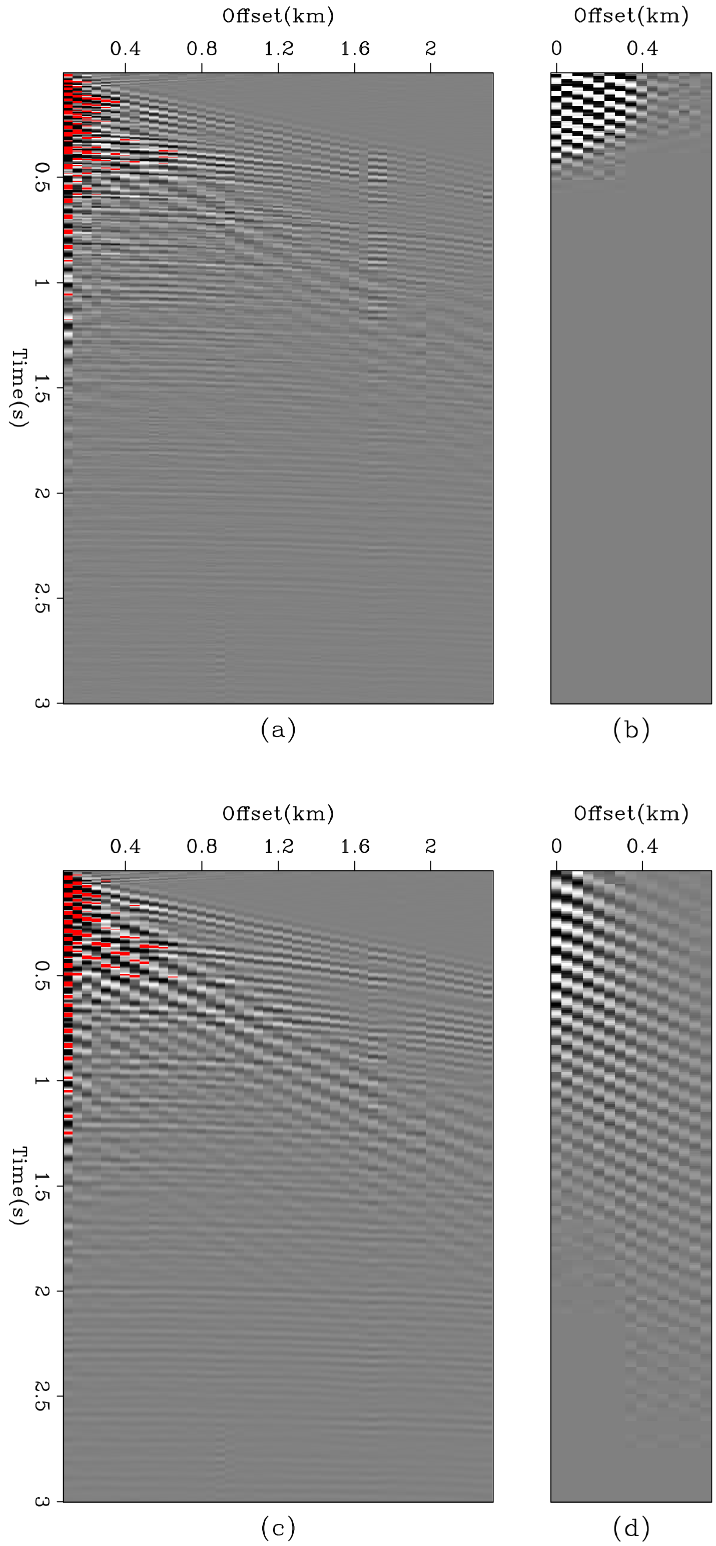

Figure 2: (a) Signal estimated with no weights for the PEF estimation (equation (1)). (b) Spike divided by the signal PEF. (c) Estimated noise. (d) Spike divided by the noise PEF. antoine2-separ [ER]

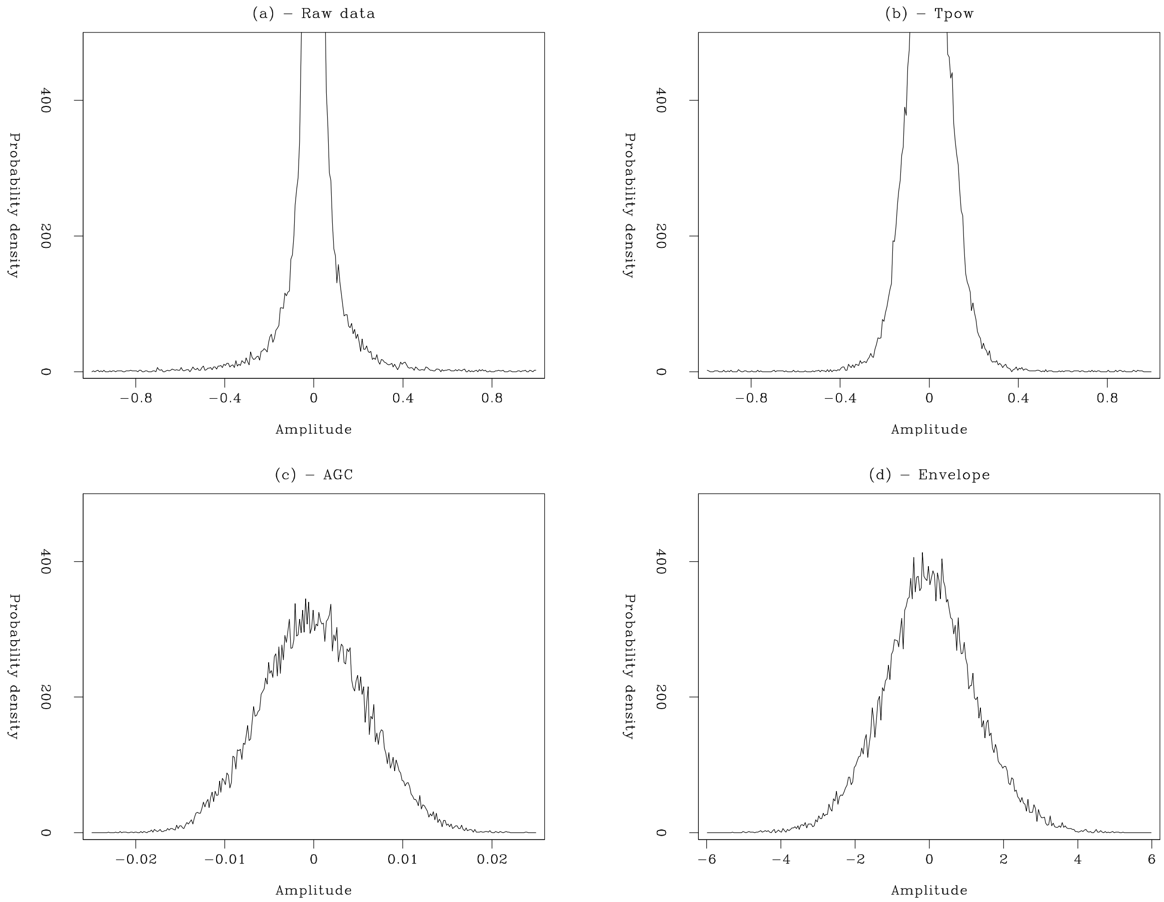

Figure 3: (a) Histogram of the input data. The amplitude of the data is not very well distributed. (b) Histogram of the data after geometrical spreading correction. The distribution is still not satisfying. (c) Histogram of the data after AGC. The data are very well balanced as indicated by the bell-shaped function. (d) Histogram of the data after envelope scaling. The distribution of the amplitudes is not as good as the AGC result. antoine2-comphisto [ER,M]

inverse PEF for the noise in Figure 5d is very different from Figure 4d. With the weighted residual, the filter is minimum phase whereas with the weighted data, it is not because the amplitude increases constantly with offset and time in Figure 5d.

Therefore, weighting the data or the residual for the PEF estimation produces very different filters affecting the final noise attenuation result. The best results are obtained when the residual is weighted to balance the relative importance of the regression equations. In the next section, I exemplify the weighted PEF estimation with a 3-D example and non-stationary filters.

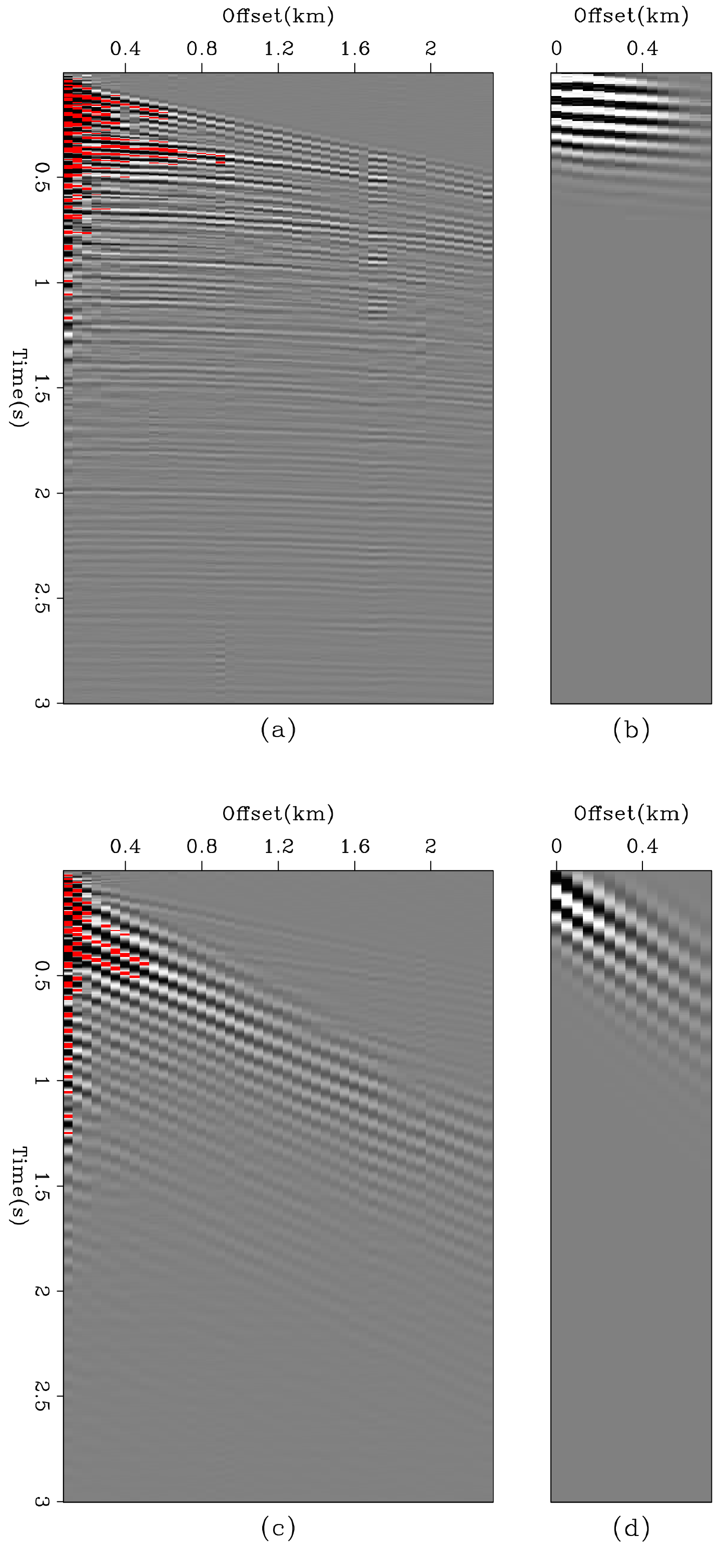

Figure 4: (a) Signal estimated with weighted residual for the PEF estimation (equation (6)). (b) Spike divided by the signal PEF. (c) Estimated noise. (d) Spike divided by the noise PEF. antoine2-separ-weight-AGC [ER]

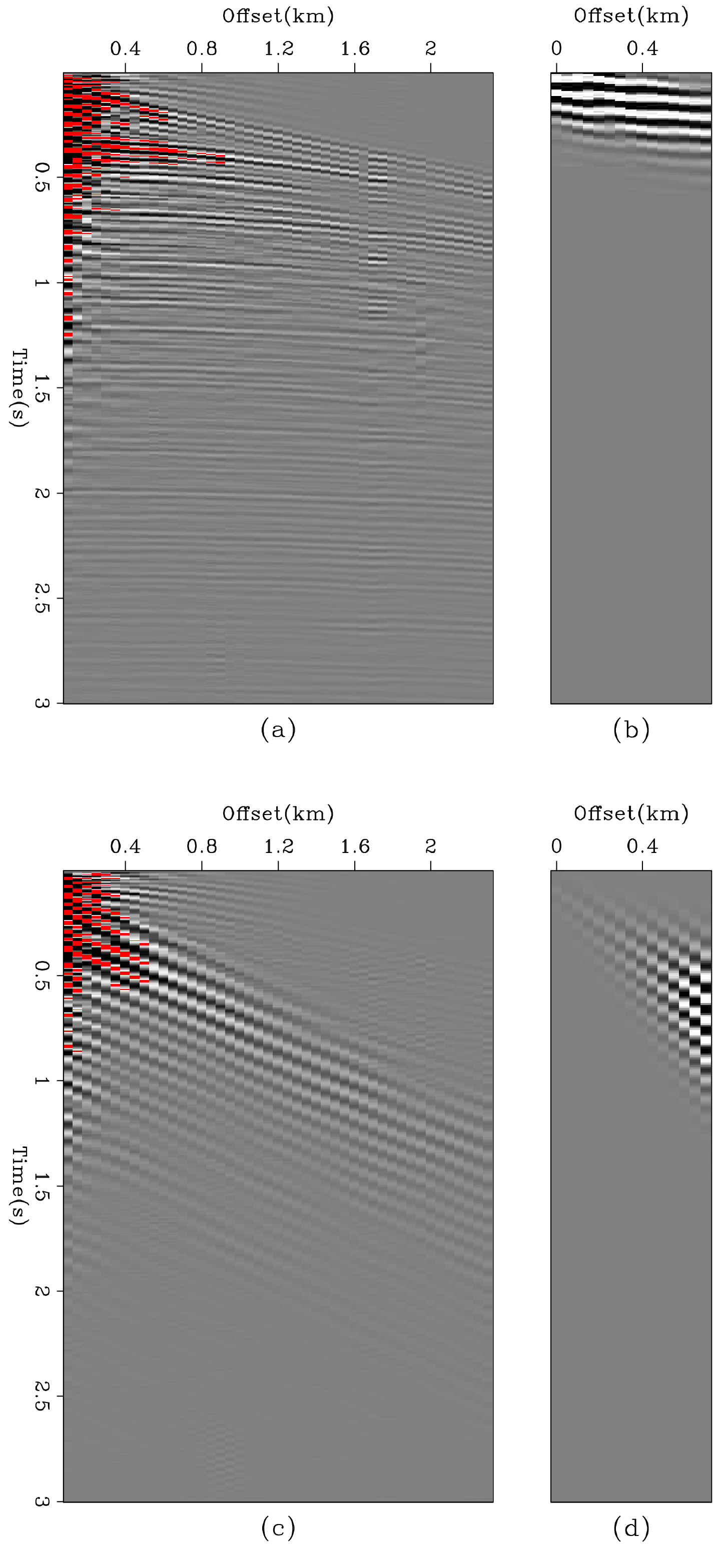

Figure 5: (a) Signal estimated with weighted data for the PEF estimation (equation (5)). (b) Spike divided by the signal PEF. (c) Estimated noise. (d) Spike divided by the noise PEF. antoine2-separ-AGC2 [ER]

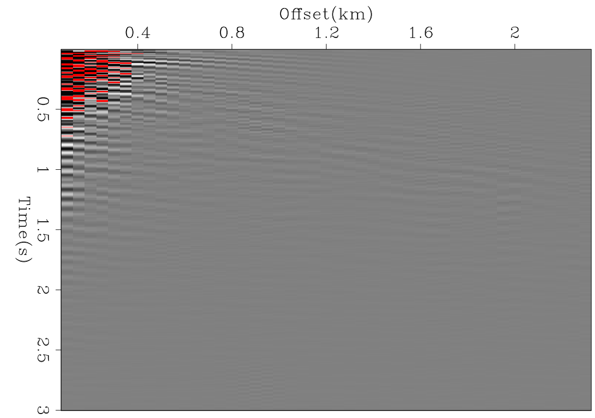

Figure 6: Difference plot between Figures 4 and 5. Both results are very similar but the weighted data strategy leads to a less efficient noise attenuation at short offset and leaves artifacts on top of the gather. antoine2-comp-2d [ER]

A 3-D land data example

I illustrate the weighted PEF estimation on a near-offset section from a 3-D land survey dataset. Figure 7 displays the 3-D data. The noise comes in different flavors. First, we have missing traces, second, we have large amplitude differences from trace to trace (common with land data), and then we have surface waves with strong amplitude variations along the time axis. I first design a weighting function that takes into account the missing traces and correct for the amplitude problems (Figure 8). This weighting function is going to be utilized for the PEF estimation. With the time domain formulation, there is no limit in the complexity of the weighting operator since it can mix zero traces with weights of any kind. This would be much more difficult, if not impossible, in the Fourier domain.

For this 3-D example, I estimate non-stationary 3-D filters. These filters are constant within a patch. For this dataset, I have a total of 90 patches with ten patches in the time axis and three in the crossline and inline directions. More details on the non-stationary filters can be found in Crawley (2000) and Claerbout and Fomel (2002) . The first step consists in estimating the noise and signal non-stationary PEFs. The noise in Figure 7 is mainly ground-roll. A good noise model can be then obtained by low-passing the data and a signal model by high-passing the data.

Figures 11 and 12 show the noise attenuation results with and without residual weighting, respectively. However, a mask has been applied for the PEF estimation in Figure 11 to not include the missing traces. Figure 12 shows better results in the lower part (below 0.42 s.) of the section where some low-frequency noise is still visible. The difference is particularly obvious in the time section where a channel is clearly visible in Figure 12. One problem with the input data is that the amplitude decreases with time, especially for the noise (Figure 7). Therefore, during the signal and noise PEFs estimation steps, most of the solver efforts are directed toward the PEFs where the noise and signal is the strongest, i.e, the upper part. Thus, we end-up with “good” PEFs in the top, and “bad” PEFs in the bottom where the noise separation is the less efficient. Figure 9 displays the estimated PEFs for the noise and signal with and without residual weight. Note that in Figures 9a and 9c the coefficients (especially the first two) vary a lot with the patch number. They almost go to zero for the PEFs at the bottom. On the contrary, in Figures 9b and 9d, the PEFs coefficients are much much uniform across time and offset, as expected with the weighted PEF estimation technique.

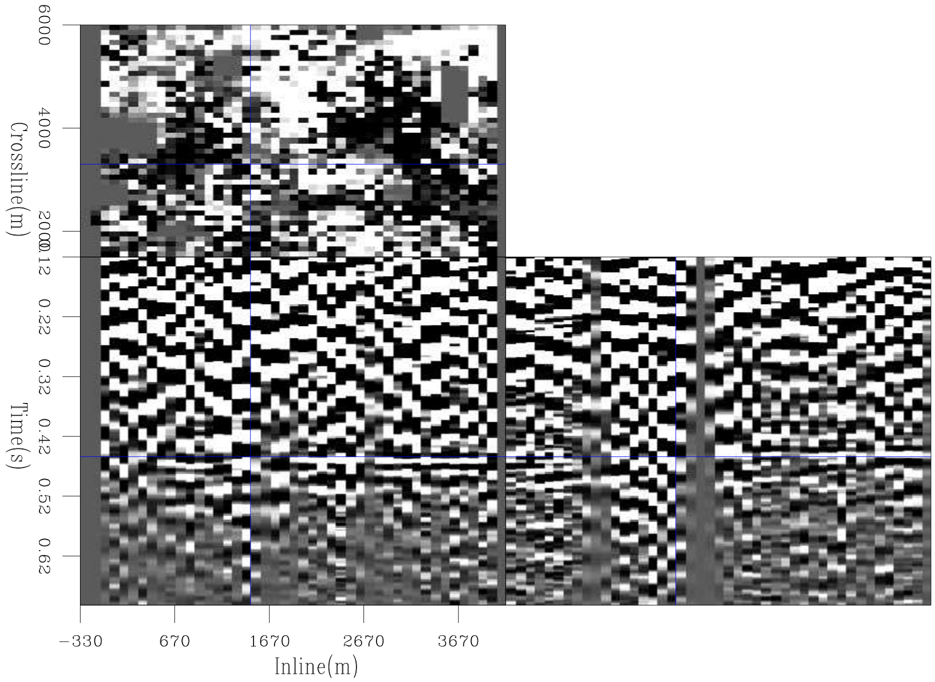

Figure 7: A near-offset section of a 3-D land survey. Some signal is visible near 0.42 s. This section is contaminated with ground-roll. The amplitude varies across time and offset with missing traces as well. antoine2-data3d [ER,M]

To finish with the 3-D data example, I show in Figure 13 the noise attenuation result when the noise and signal models are weighted prior to the PEF estimation. In that case, the residual is not weighted, but only the data are, i.e., equation (5). Note that the same inversion parameters (ϵ in equation (8), number of iterations, patch geometry, PEF sizes) are used for both cases. This result is also satisfying but the signal is not as well preserved as it is in Figure 12. I show a difference plot between the two results in Figure 14. Looking closely at the time slices (upper panel) of Figures 12 and 13, we see at large black area between 6000 and 4000 meters in the crossline direction and between 2670 and 3670 meters in the inline direction. We can see the same black shape in the difference plot of Figure 14, thus demonstrating that more noise has been subtracted in Figure 12. The same conclusion holds for the white area above the black shape in Figures 12 and 13. These differences prove that weighting the residual is the correct way of handling amplitude problems with seismic data.

CONCLUSION

Estimating filters is an important step of many processing techniques like signal/noise separation, data interpolation, missing data restoration or regularized tomography. The stationarity of the data is at the heart of many filter estimation techniques. In the case where the data have strong amplitudes variations, I show that a weighting function that leverages the fitting

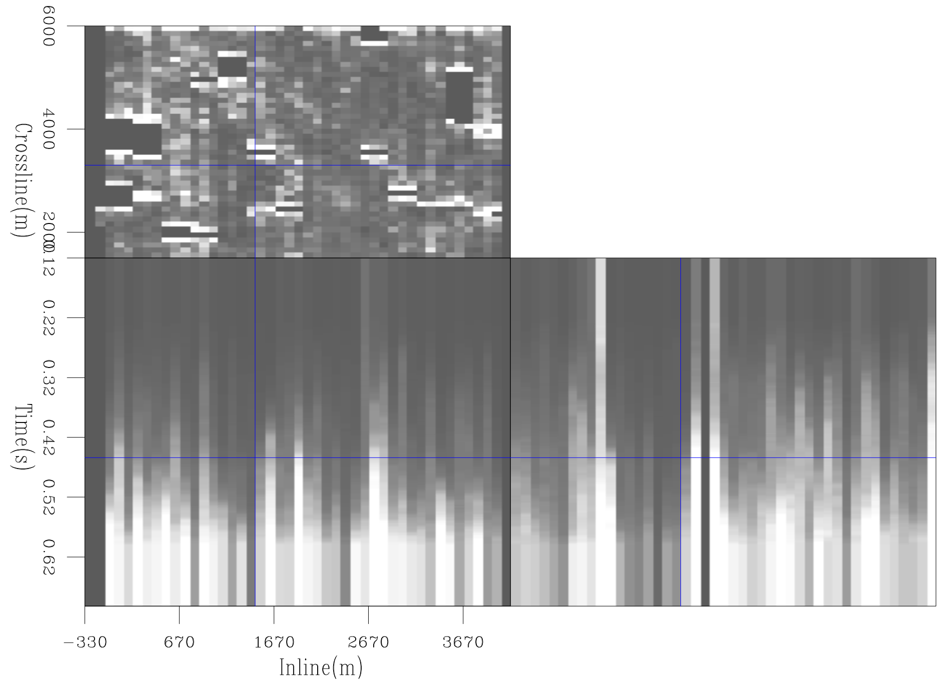

Figure 8: Weighting function for the non-stationary PEFs estimation. Whiter means larger weight. The weight is zero where traces are missing. antoine2-data3dweight [ER,M]

equations during the filter estimation step must be used for improved results. In other words, the residual should be weighted and not the data points. Test with 2-D and 3-D field data with stationary and non-stationary filters for two signal-noise separation problems prove that a weighted residual PEF estimation scheme give the best estimated noise and signal while preserving the amplitudes.

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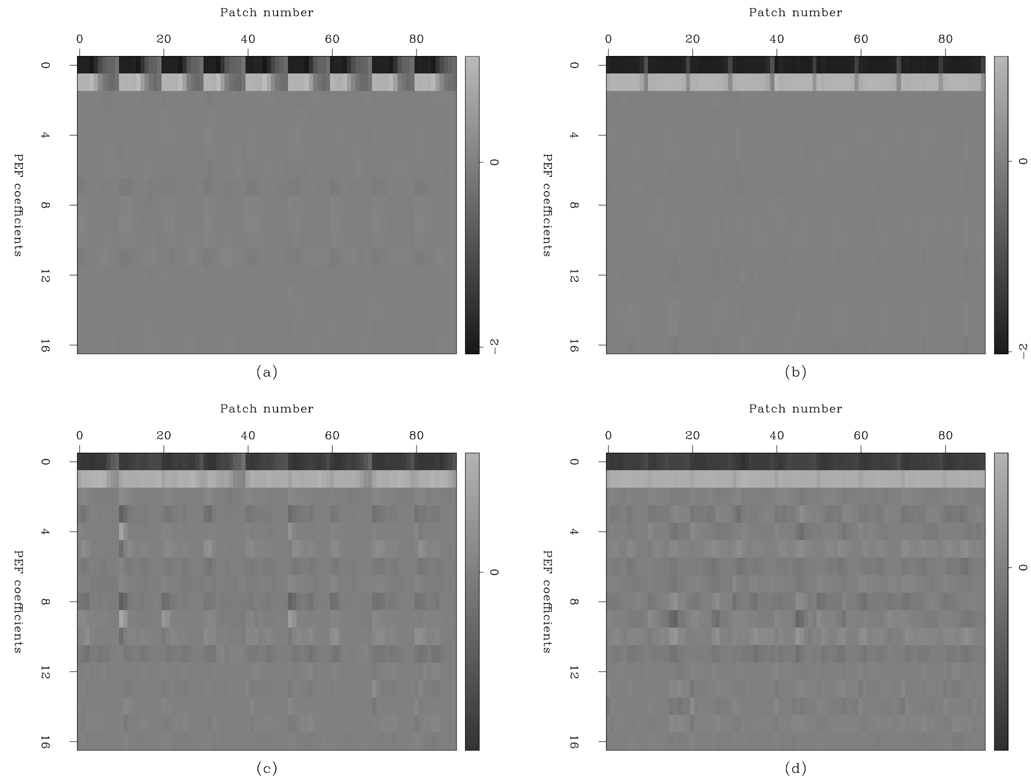

Figure 9: PEFs estimated for each patch for the noise and signal. The vertical axis represents the PEF coefficients (the leading one coefficient is omitted) and the horizontal axis represents a patch number (Figure 10). (a) Noise PEFs with unweighted residual. (b) Noise PEFs with weighted residual. Note that the noise PEFs are very close to a 1-D second derivative (1,-2,1). (c) Signal PEFs with unweighted residual. (d) Signal PEFs with weighted residual. antoine2-pef.comp [CR]

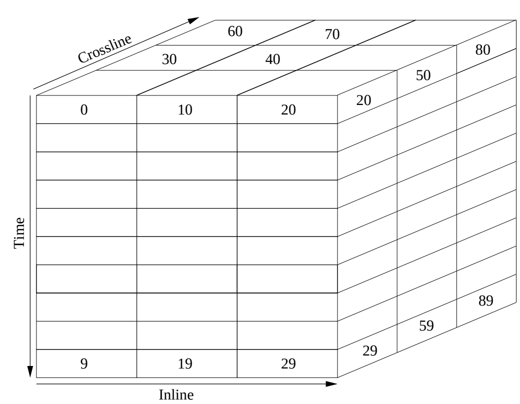

Figure 10: Geometry of the patches. The numbers correspond to the vertical axis of Figure 9. antoine2-patch-geom [NR]

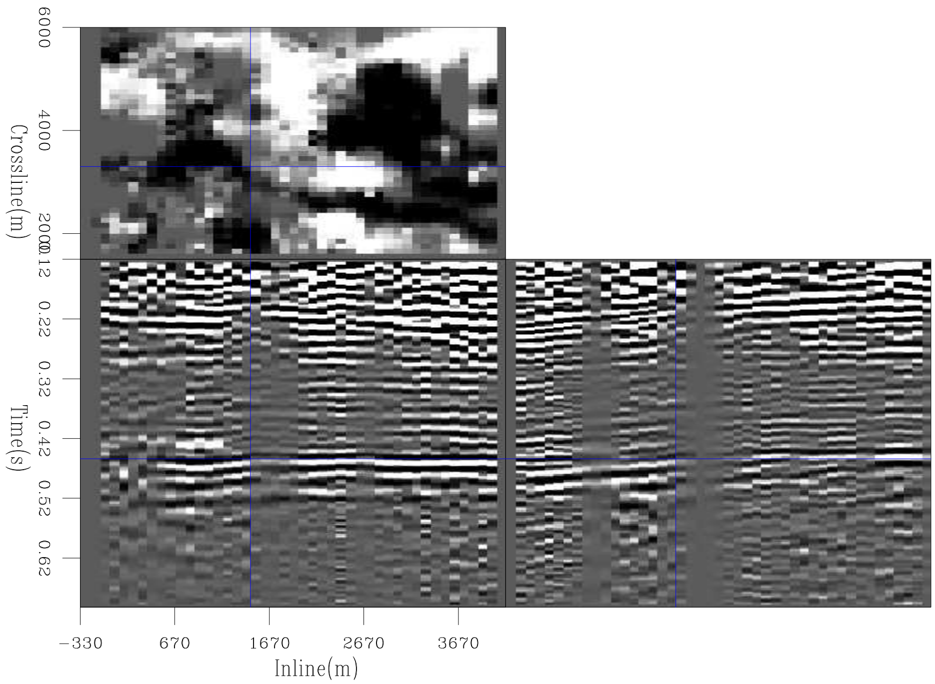

Figure 11: Signal estimated without weight for the signal and noise PEFs estimation. Some noise remains below 0.42 s. antoine2-separ-ns-3d [CR,M]

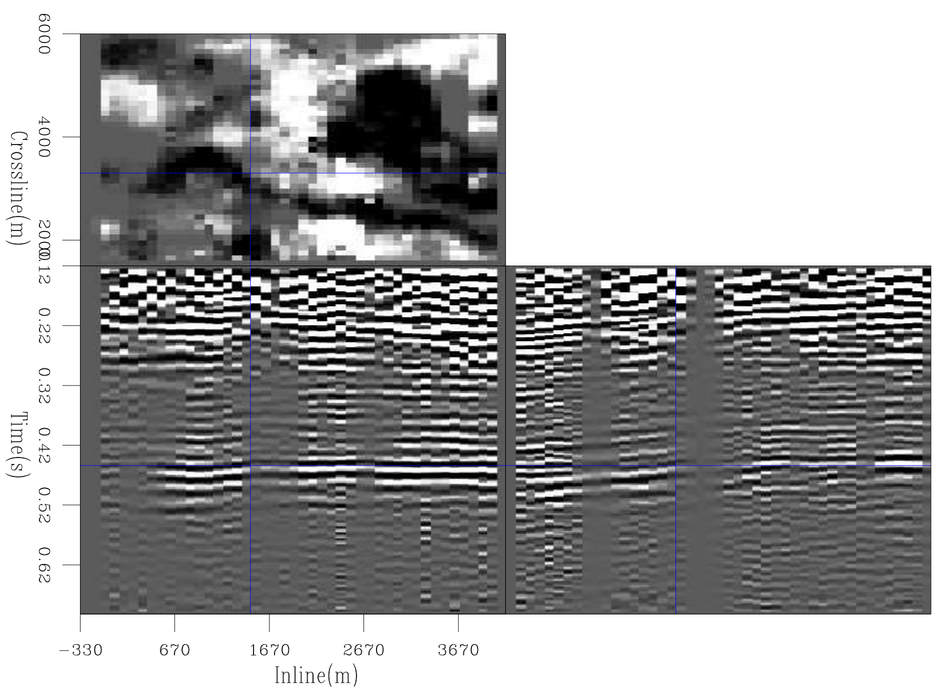

Figure 12: Signal estimated with weight for the signal and noise PEFs estimation. The noise is well attenuated. The time slice displays the channel more clearly than in Figure 11. antoine2-separ-ns-weight-AGC-3d [CR,M]

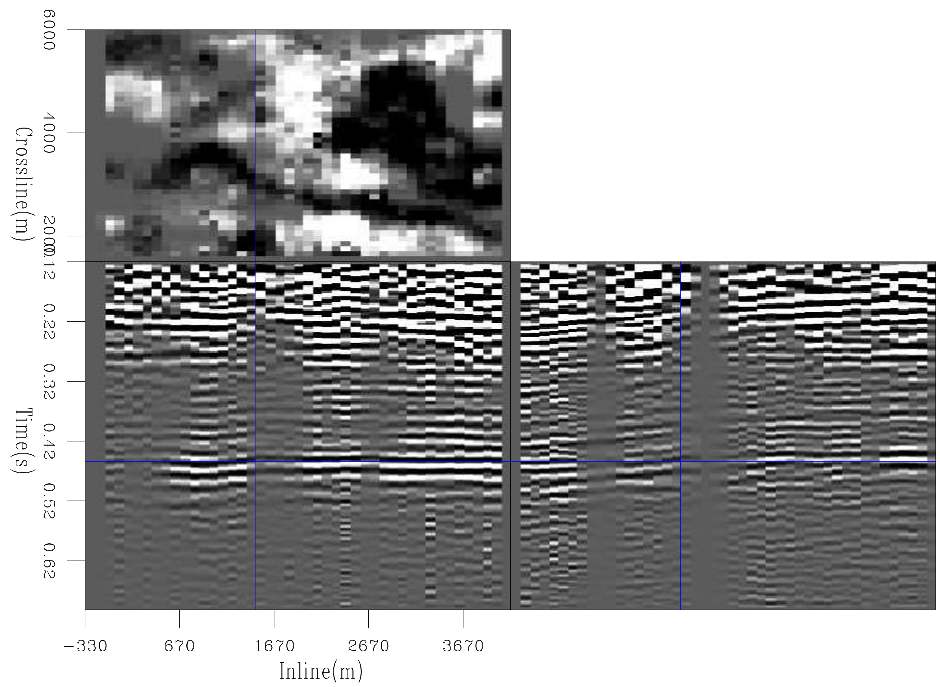

Figure 13: Signal estimated with a weighted noise and signal model for the PEFs estimation. antoine2-separ-ns-AGC-3d [CR,M]

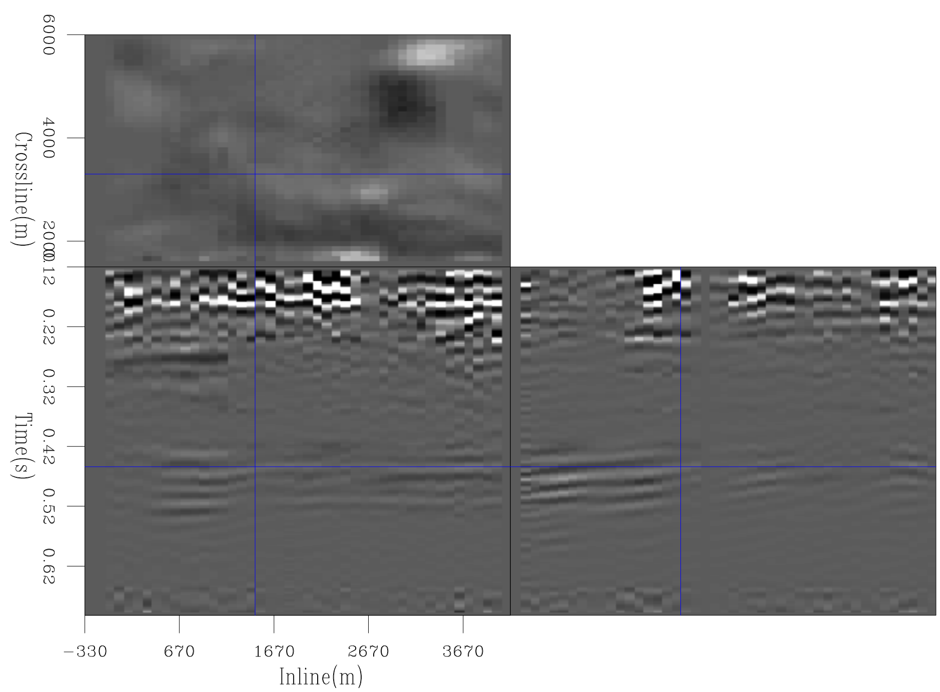

Figure 14: Difference between Figure 12 and Figure 13. The signal is better preserved with the weighted residual than with the weighted noise and signal models. antoine2-comp-3d [CR,M]

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