Principles of migration using wavefield-continuation methods

Chapter 3
Challenging subsalt problem

Velocity Model

Figure 3.1
Challenging subsalt problem

Images

Wave-equation migration  CMPX=7500
Cross-line location (m)

Kirchhoff migration  CMPX=7500
Cross-line location (m)
Wavefield at $t=0$ s
Wavefield at $t=0.5 \text{ s}$
Wavefield at t=1 s
Wavefield recorded at the surface

Surface location (m)

7000  8000  9000  10000  11000

1.6  1.2  0.8

Eikonal solution

Figure 3.4
Wavefield-continuation migration

- Wavefield-continuation vs. Kirchhoff
- Reverse-time migration (RTM)
- Downward-continuation migration
  - Shot-profile migration
  - Source-receiver (S-R) migration a.k.a survey-sinking migration
- Reverse-time vs. downward-continuation migration
- Common-Image Gathers (CIG) by wavefield-continuation
  - CIG before imaging
  - CIG after imaging
Wavefield-continuation migration

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Seismic imaging experiment

Figure 3.5
Seismic imaging experiment

Figure 3.5

Reflectors

Wave propagation

Data
Seismic imaging experiment

Reflectors → Wave propagation → Data

Figure 3.5
Seismic imaging experiment

Reflectors

Wave propagation

Data

Figure 3.5
Seismic imaging experiment

Reflectors

Wave propagation

Data

Figure 3.5
Seismic imaging experiment

Reflectors

Wave propagation

Data

Slide #15  Figure 3.5  biondo@stanford.edu
Seismic imaging experiment

Reflectors

Wave propagation

Data

Figure 3.5
Seismic imaging experiment

Reflectors

Wave propagation

Data

Figure 3.5
Seismic imaging experiment

Reflectors

Wave propagation

Data
Seismic imaging experiment

Reflectors

Wave propagation

Data

Figure 3.5
Seismic imaging experiment

Reflectors

Data

Wave propagation

Slide #20  Figure 3.5  biondo@stanford.edu
Reverse-time migration (RTM)
- Wave propagation by numerical solution of full-wave (usually acoustic) wave equation
  - Source function ➔ Forward propagation
  - Recorded data ➔ Backward propagation
- Imaging condition

\[ I(z_\xi, x_\xi, y_\xi) = \sum_i \sum_t P^g(t, z = z_\xi, x = x_\xi, y = y_\xi; s_i)P^s(t, z = z_\xi, x = x_\xi, y = y_\xi; s_i) \]

where:  
- \( P^g \) and \( P^s \) are the propagated wavefields
- \( z_\xi, x_\xi, \) and \( y_\xi \) are the image space coordinates
- \( s_i \) are the individual shots in the survey
Imaging by back-propagation

Image progression at \( t = 1.65 \) s

Wave back-propagation

Data
Imaging by back-propagation

Image progression at \( t=1.50 \text{ s} \)

Data

Wave back-propagation
Imaging by back-propagation

Figure 3.6-3.8

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Imaging by back-propagation

Image progression at $t=1.20$ s

Data

Wave back-propagation

Source wavefield at $t=1.20$

Receiver wavefield at $t=1.20$

Figure 3.6-3.8

Slide #25
Imaging by back-propagation

Image progression at $t=1.05$ s

Wave back-propagation

Data

Source wavefield at $t=1.05$

Receiver wavefield at $t=1.05$

Slide #26

Figure 3.6-3.8

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Imaging by back-propagation

Image progression at $t=0.90$ s

Data

Wave back-propagation

Source wavefield at $t=0.90$

Receiver wavefield at $t=0.90$
Imaging by back-propagation

Image progression at t=0.75

Data

Wave back-propagation

Source wavefield at t=0.75

Receiver wavefield at t=0.75

t=0.75 s

Figure 3.6-3.8
Imaging by back-propagation

Image progression at t = .60

Data

Wave back-propagation

Source wavefield at t = .60

Receiver wavefield at t = .60

Figure 3.6-3.8
Imaging by back-propagation

Image progression at \( t = 0.45 \) s

Wave back-propagation

Data

Source wavefield at \( t = 0.45 \)

Receiver wavefield at \( t = 0.45 \)

Figure 3.6-3.8

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Imaging by back-propagation

Image progression at $t=0.30$ s

Wave back-propagation

Data

Source wavefield at $t=.30$

Receiver wavefield at $t=.30$

Figure 3.6-3.8

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Reverse-time migration (RTM)

- Wave propagation by numerical solution of full-wave (usually acoustic) wave equation

- Source function $\rightarrow$ Forward propagation

- Recorded data $\rightarrow$ Backward propagation

- Imaging condition

$$I(z_\xi,x_\xi,y_\xi) = \sum_i \sum_t P^g(t,z = z_\xi,x = x_\xi,y = y_\xi;s_i) P^s(t,z = z_\xi,x = x_\xi,y = y_\xi;s_i)$$

where: $P^g$ and $P^s$ are the propagated wavefields

$z_\xi, x_\xi,$ and $y_\xi$ are the image space coordinates

$s_i$ are the individual shots in the survey

Slide #32  Eq. 3.1  biondo@stanford.edu
Prestack imaging condition

“Poststack” imaging condition

\[ I(z_{\xi}, x_{\xi}, y_{\xi}) = \sum_{i} \sum_{t} P^g(t, z = z_{\xi}, x = x_{\xi}, y = y_{\xi}; s_i) P^s(t, z = z_{\xi}, x = x_{\xi}, y = y_{\xi}; s_i) \]

“Prestack” imaging condition

\[ I(z_{\xi}, x_{\xi}, y_{\xi}, x_{\xi_h}, y_{\xi_h}) = \sum_{i} \sum_{t} P^g(t, z_{\xi}, x_{\xi} + x_{\xi_h}, y_{\xi} + y_{\xi_h}; s_i) P^s(t, z_{\xi}, x_{\xi} - x_{\xi_h}, y_{\xi} - y_{\xi_h}; s_i) \]

where: \( x_{\xi_h} \) and \( y_{\xi_h} \) are the subsurface offsets
Example of prestack image

Zero subsurface offset section

Subsurface offset common image gather (CIG)
Wavefield-continuation migration

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  - CIG after imaging
Downward-continuation

Recursive downward propagation

\[ P_{z+\Delta z}(\omega,k_x,k_y) = P_z(\omega,k_x,k_y)e^{ik_z\Delta z} \]

where: \( \omega \) is the temporal frequency 
\( k_x, k_y \) and \( k_z \) are the spatial wavenumbers

Solving the one-way wave equation with the Single Square-Root operator:

\[ k_z = SSR(\omega,k_x,k_y) = -\sqrt{\frac{\omega^2}{v(z,x,y)^2} - (k_x^2 + k_y^2)} \]

where: \( v(z,x,y) \) is the velocity function
Shot-profile migration

- Downward continuation
  - Source function: \( P^s_z = P^s_{z=0}e^{-ik_zz} \)
  - Recorded data: \( P^g_z = P^g_{z=0}e^{ik_zz} \)

- Imaging condition

\[
I\left(z_\xi, x_\xi, y_\xi, x_{\xi_h}, y_{\xi_h}\right) = \sum_i \sum_\omega P^g_z \left( \omega, x_\xi + x_{\xi_h}, y_\xi + y_{\xi_h}; s_i \right) \left[ P^s_z \left( \omega, x_\xi - x_{\xi_h}, y_\xi - y_{\xi_h}; s_i \right) \right]
\]
Wavefield-continuation migration

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Source-receiver migration
a.k.a.
Survey-sinking migration

Sources

Receivers

z
Source-receiver migration
a.k.a.
Survey-sinking migration

\begin{align*}
    z &= -\frac{\omega^2}{\nu(s,z)^2} - k_s^2 \\
    z &= -\frac{\omega^2}{\nu(g,z)^2} - k_g^2
\end{align*}

Eq. 3.9
**Source-receiver migration**

*a.k.a.*

**Survey-sinking migration**

**Imaging:**

\[
I(z_{\xi}, x_{\xi}, y_{\xi}) = \sum_{\omega} P_z(\omega, x_{\xi}, y_{\xi}, x_h = 0, y_h = 0)
\]

\[
- \frac{\omega^2}{\sqrt{v(s,z)^2 - k_s^2}} - \frac{\omega^2}{\sqrt{v(g,z)^2 - k_g^2}}
\]

Eq. 3.9 & 3.12

*Slide #41*

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Source-receiver migration 
a.k.a.
Survey-sinking migration

Imaging: 
\[ I(z_{\xi}, x_{\xi}, y_{\xi}, x_{\xi h}, y_{\xi h}) = \sum_{\omega} P_z(\omega, x_{\xi}, y_{\xi}, x_h = x_{\xi h}, y_h = y_{\xi h}) \]
Zero-offset migration

Survey sinking in midpoint-offset space

\[ k_z = -\sqrt{\frac{\omega^2}{v(s,z)^2} - \frac{1}{4}(k_m - k_h) \cdot (k_m - k_h) - \sqrt{\frac{\omega^2}{v(g,z)^2} - \frac{1}{4}(k_m + k_h) \cdot (k_m + k_h)}} \]
Zero-offset migration

- Survey sinking in midpoint-offset space

\[ k_z = -\sqrt{\frac{\omega^2}{v(s,z)^2} - \frac{1}{4}(k_m - k_h) \cdot (k_m - k_h)} - \sqrt{\frac{\omega^2}{v(g,z)^2} - \frac{1}{4}(k_m + k_h) \cdot (k_m + k_h)} \]

- Zero-offset survey sinking and imaging

\[ k_z = -\sqrt{\frac{4\omega^2}{v(m,z)^2} - k_m \cdot k_m} \]

\[ I(z_\xi, x_\xi, y_\xi) = \sum_\omega P_z(\omega, x_m = x_\xi, y_m = y_\xi) \]
Troubles with exploding-reflectors assumption

Zero-offset section

CMP gather

Slide #45

Figure 3.16 & 3.17

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Wavefield-continuation migration

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  - CIG before imaging
  - CIG after imaging
Imaging of steep dips

Downward continuation migration
Imaging of steep dips with RTM

Reverse time migration
Imaging of steep dips with RTM

Reverse-time migration

Downward-cont. mig.

Figure 3.15
Migration artifacts in RTM - t=1.2 s

True velocity model ➡ Artifacts

Smooth velocity model ➡ No artifacts

Slide #50  Figure 3.6 & 3.9  biondo@stanford.edu
Migration artifacts in RTM - $t=0.75$ s

True velocity model $\rightarrow$ Artifacts

Smooth velocity model $\rightarrow$ No artifacts

Figure 3.7 & 3.10
Migration artifacts in RTM - t=.3 s

True velocity model $\Rightarrow$ Artifacts

Smooth velocity model $\Rightarrow$ No artifacts

Slide #52 Figure 3.8 & 3.11 biondo@stanford.edu
Migration artifacts in RTM
Practical problem with salt bodies

Cross-correlation image

Liu et al., SEG, 2007

Source wavefield at 2.4 s

Receiver wavefield at 2.4 s
Anisotropic downward-continuation.

- Isotropic downward-continuation

\[ k_z = \text{SSR}(\omega, k_x, k_y) = -\sqrt{\frac{\omega^2}{v(z,x,y)^2} - (k_x^2 + k_y^2)} \]

- Anisotropic (VTI) downward continuation

\[ k_z = -\frac{\omega}{V_V(z,x,y)} \sqrt{\frac{\omega^2 - V_H^2(z,x,y) [k_x^2 + k_y^2]}{\omega^2 + [V_N^2(z,x,y) - V_H^2(z,x,y)] [k_x^2 + k_y^2]}} \]

where:  \( V_V, V_H \) and \( V_N \) are the "VTI" velocities.