Equivalent accuracy at a fraction of the cost: Overcoming temporal dispersion

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ABSTRACT

Numerical dispersion in finite difference modeling produces coherent artifacts, severely constraining the resolution of advanced imaging and inversion schemes. Conventionally, we deal with this by increasing the order of accuracy of the finite difference operators and resign ourselves to paying the high computational cost that incurs. But is there a way to reduce such dispersion without increasing cost or, conversely, decrease the cost without increasing numerical dispersion? To tackle this, we separate the finite difference numerical dispersion into pure time and pure space dispersion and address them independently. In this article, we focus on time dispersion. We show that finite difference time dispersion is virtually independent of the medium velocity and the spatial grid for propagation, and only depends on the time stepping scheme and the propagation time. Based on this, we devise post-propagation filtering to collapse the time dispersion effect of the finite difference modeling. Our dispersion correction filters are designed by comparing the input waveform with dispersive waveforms obtained by 1D propagation of that waveform forward in time. These filters are then applied on multi-dimensional shot records to eliminate the time dispersion by two schemes: (1) stationary filtering plus interpolation and (2) non-stationary filtering. We show with both 1D and 2D examples that the time dispersion is effectively removed by our post-propagation filtering at nearly no additional cost.

INTRODUCTION

Finite difference (FD) modeling for wave propagation has been widely used for advanced imaging techniques such as least squares reverse time migration (Lambare et al., 1992; Nemeth et al., 1999; Dai et al., 2010; Wong et al., 2011) and inversion schemes such as waveform impedance inversion (Kelly et al., 2010; Plessix and Li, 2013) and full waveform inversion (Tarantola, 1987; Virieux and Operto, 2009). In these methods, wavefields modeled by finite difference are compared to recorded data which, of course, contain no computational numerical dispersion. As a result, a great deal of the cost of these methods arises from the effort to make these comparisons meaningful by reducing finite difference dispersion.

Much effort has been put into suppressing the numerical dispersion of the FD methods. Kosloff and Baysal (1982) used the spatial Fourier transform to eliminate

all errors from FD approximation of the spatial derivatives, and chose a small enough time step to limit the numerical time dispersion. However, this method requires many Fourier transforms at each time step and also a rather small time step, and hence has been considered prohibitively expensive in practice. Therefore, many authors (Holberg, 1987; Fornberg, 1998; Etgen, 2007) set up an optimization problem to generate FD coefficients that minimize the misfit between the numerical phase velocity and the theoretical phase velocity for some range of frequency and velocity. Nonetheless, with computational cost controlled by the number of coefficients that are optimized, achieving both accuracy and efficiency at the same time is still challenging.

Recently, Stork (2013) proposed to separate the temporal and the spatial FD dispersion. Here we follow this approach, albeit only tackling the temporal dispersion in this article. For discussion of the spatial dispersion component, please refer to a companion paper Le et al. (2013). In this paper, we first analyze the source of the time dispersion and show that the time dispersion is independent of the spatial sampling and the velocity of the medium. To eliminate the space error associated with propagation we use a Fourier modeling method and propagate a wavelet through an arbitrary medium. Then we design the filters using 1D Fourier modeling results at discrete propagation times. These filters are applied to both 1D and 2D shot records to remove the time dispersion. We test two different filtration schemes: (1) stationary filtering plus interpolation and (2) non-stationary filtering. The results show that both filtration schemes can eliminate the time dispersion on a shot record with nearly no additional cost.

THEORY

Assuming constant density and a source free medium, the acoustic wave equation may be written

$$c^2(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})P = \frac{\partial^2}{\partial t^2}P, \tag{1}$$

where P is the pressure field and $c(x, y)$ is the velocity.

Both sides of equation 1 are approximated by numerical discretization: the left-hand side in space and the right-hand side in time. Using notation similar to Kosloff and Baysal (1982), the finite difference equation we are solving is

$$c^2 L P^n(i, j) = \frac{1}{\Delta t^2}\big[P^{n+1}(i, j) - 2P^n(i, j) + P^{n-1}(i, j)\big], \tag{2}$$

where $P^n(i, j)$ represents the value of the pressure field at time $t = n\Delta t$ at spatial location $x = x_0 + (i-1)\Delta x$, $y = y_0 + (j-1)\Delta y$. The term $c^2 L P^n(i, j)$ represents the numerical approximation of the left-hand side.

Equation 2 represents an explicit, second-order time differencing scheme that is widely used in finite difference codes. Both the left-hand and the right-hand side of

equation 2 contain errors with respect to equation 1. We call the error from the left-hand side approximation spatial dispersion, and from the right-hand side temporal dispersion.

Conventionally, temporal dispersion can be reduced either by decreasing Δt or increasing the order of the finite difference (Figure 1). Both options significantly increase the computational cost. Stork (2013) suggests that temporal dispersion is independent of the medium velocity and the spatial grid for propagation. We verify this idea by numerical tests. Figure 2(a) shows the 1-D modeling results at different propagation times when the second-order time stepping is used. The spatial derivative is computed in the Fourier space to avoid any spatial dispersions. In this example, $\Delta t = 2$ms, $c = 2000$m/s, $\Delta x = 10$m. It is clear that the dispersion effects get stronger with the longer propagation time. Figure 2(b) shows the same modeling results as in figure 2(b), only with $c = 1000$m/s, $\Delta x = 6$m. Despite the differences in the spatial sampling and the medium velocity, the temporal dispersion remains the same as long as the Δt and the propagation time are the same. Therefore, inverse filters can be designed to remove the velocity-independent dispersions after propagation.
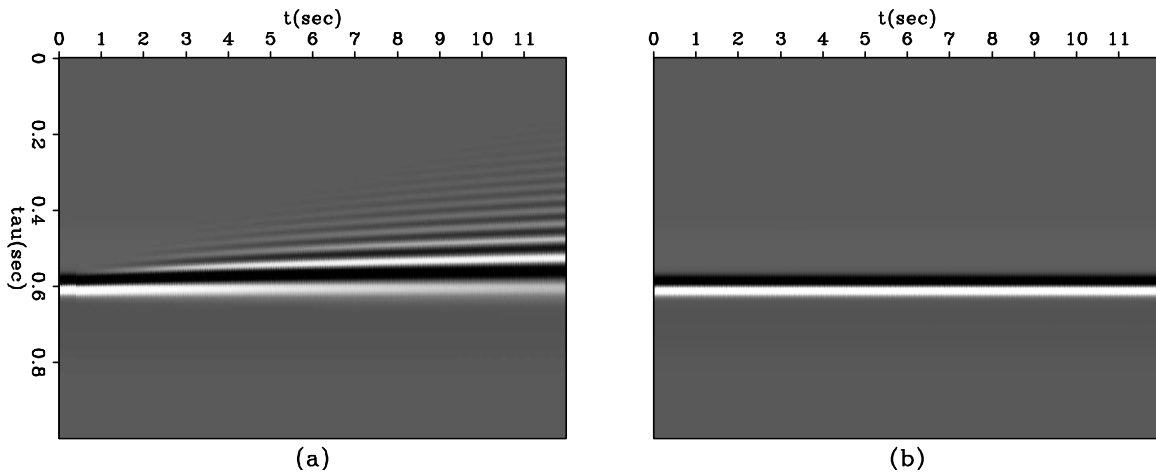

Figure 1: Wavefield from 1-D modeling using the Fourier method with (a) second-order time stepping and (b) fourth-order stepping. Time step is 2ms in both cases. Severe time dispersions are removed at twice the computation cost. [**ER**]

We estimate the inverse filters $f(\tau, t)$ by comparing the waveform $s(\tau, t)$ with the original waveform $s(\tau, 0)$ in the Fourier space:

$$F(\omega, t) = \frac{S(\omega, 0)}{S(\omega, t) + \epsilon},\qquad(3)$$

where $F(\omega, t)$ and $S(\omega, t)$ are the Fourier representation of $f(\tau, t)$ and $s(\tau, t)$, respectively. A small number ϵ is added to stabilize the division. The final inverse filters are band-limited within the frequency range of the original waveform. We only estimate the inverse filters at discrete propagation times. To apply these discrete filters on 2-D or 3-D continuous data record, we can choose from the following two schemes: stationary filtering plus interpolation, or non-stationary filtering.
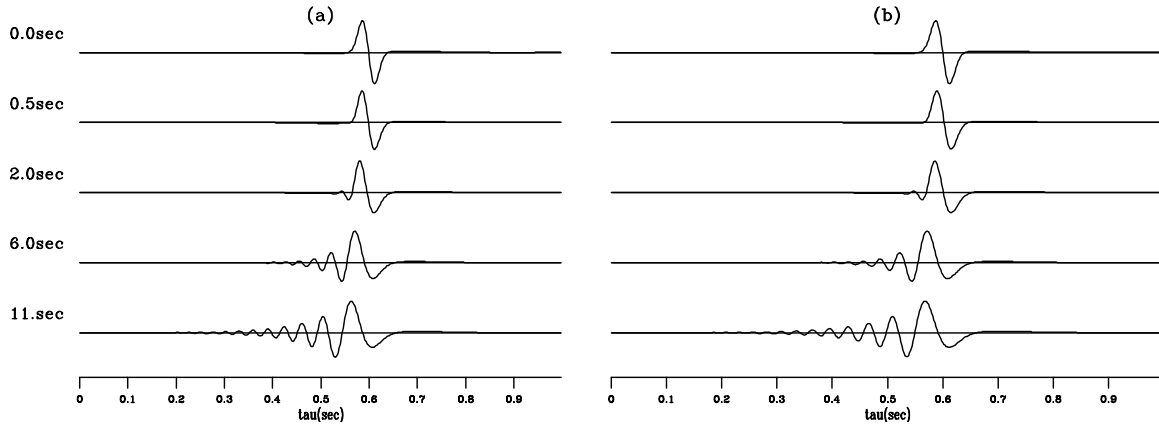
Figure 2: 1-D modeling results by Fourier method with different model parameters. In (a) $c = 2000$m/s, $\Delta x = 10$m. In (b) $c = 1000$m/s, $\Delta x = 6$m. The dispersion effects are the same. [**ER**]

Stationary filtering plus interpolation (SFPI)

Given a record $d(t, x)$ and inverse filters $f(\tau, t_i)$, we can first apply each of the inverse filters to the whole record:

$$d_i(t, x) = d(t, x) * f(\tau, t_i); \tag{4}$$

this is a trace-by-trace operation. On each filtered record $d_i(t, x)$, only the waveforms around $t = t_i$ are correctly filtered. The other parts of the record are either over or under compensated.

We then interpolate among the filtered records to obtain the temporal dispersion free record $\hat{d}(t, x)$:

$$\hat{d}(t, x) = \sum_i h_i d_i(t, x), \tag{5}$$

where $h(i)$ are the interpolation weights for each filtered records. In this paper, we use simple linear interpolation weights.

Non-stationary filtering (NSF)

In a non-stationary filtering scheme, we use a moving window, which has the same length as the filter, to select the data patch to convolve with the filter defined at the center of the moving window. We overlap the moving windows to ensure smooth transitions across the data patch. Mathematically, the filtering process can be formulated as follows:

$$\hat{d}(t, x) = \frac{1}{N} \sum_i (W_i d(t, x)) * f(\tau, t_i^w), \tag{6}$$

where W_i is the ith window acting on the data record, t_i^w is the center of this window, and N is the number of overlaps before moving to a new data patch.

Assuming the dispersion varies smoothly in time, we can build the filters at any propagation time by interpolating the estimated inverse filters as follows:

$$f(\tau, t_i^w) = \sum_i k(i) f(\tau, t_i), \qquad (7)$$

where k is the set of interpolation weights. The same linear interpolation scheme is used.

More advanced non-stationary filtering schemes (Margrave, 1997; Fomel, 2009) can be adapted in practice; however, in the examples we present in this paper, this simple patching method yields satisfactory results.

EXAMPLES

We test the proposed post-propagation filtering scheme on both 1-D and 2-D examples. All spatial derivatives in the numerical modeling are performed in the Fourier space to avoid the spatial dispersion. Figures 3(a) and 3(b) show the second-order and the fourth-order modeling results with $\Delta x = 10$m and $c = 2000$m/s. Dispersion is eliminated at twice the cost of second-order time stepping. Figure 3(c) and (d) show the second-order time stepping results after dispersion correction by SFPI and NSF, respectively. After the filtering, dispersion is eliminated at nearly no additional cost.

Figure 4 repeats the same modeling and filtering process with a different spatial sampling (6m) and a different velocity (1000m/s). Figure 4(a) shows the same dispersion effects as figure 3(a). We use the inversion filters estimated from 3(a) to process the data. Figures 4(c) and (d) show that the dispersion correction filters are effective as long as the time step Δt and the propagation time remain the same.

Figure 5 shows the modeling and filtering results on a 2D record. Panels (c) and (d) show that both filtering schemes can remove the dispersion effects on the shot record without increasing the FD order in time. Figure 6 shows a zoom-in view of the shot record in figure 5. All the parallel events leading the main lobe are temporal dispersions. The post-propagation filterings remove them all and perfectly match the fourth-order modeling results.

Figure 7 shows the later waveform at receiver $x = 2000$m. The wave packet between 9s and 9.5s is severely dispersed so that any waveform-based inversion scheme would fail. Post-propagation filterings restore the phase of the waveform at no additional cost.
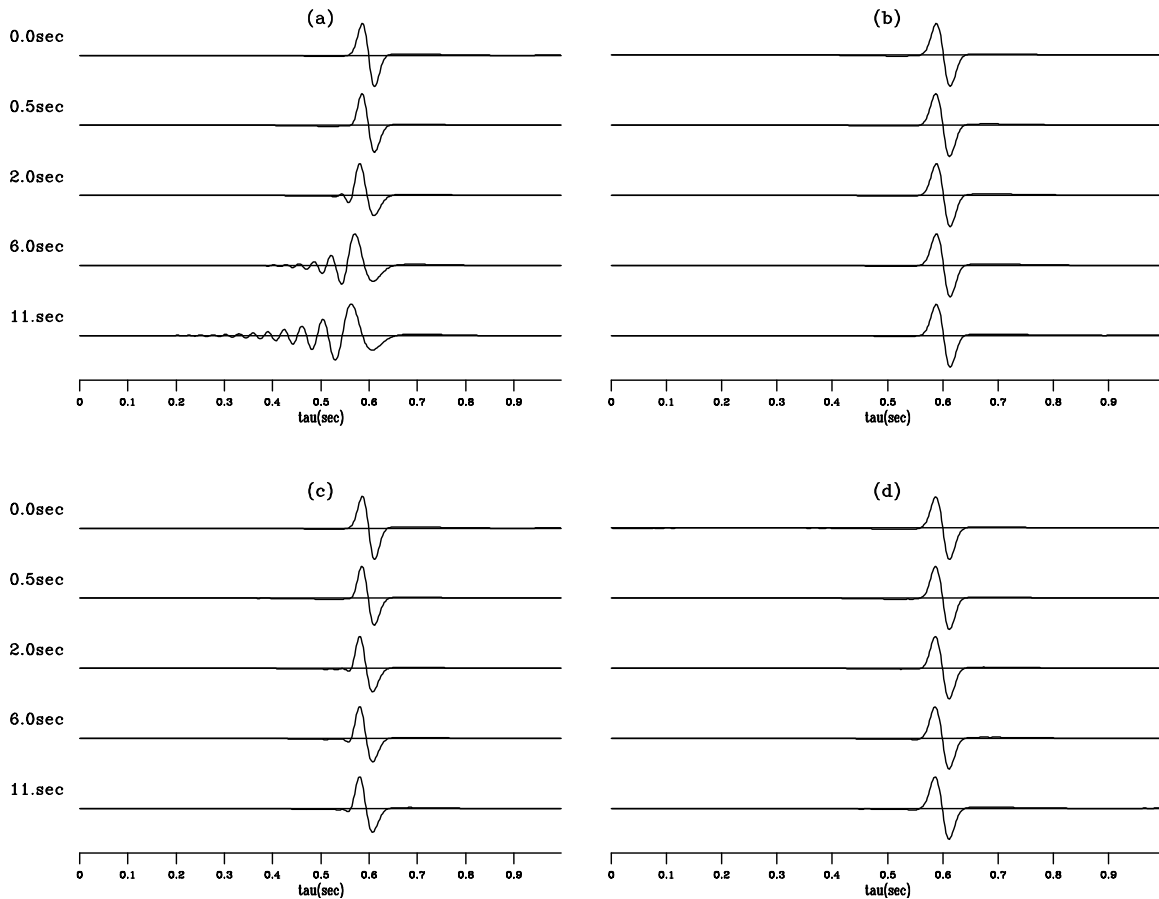
Figure 3: One-D modeling and filtering results in TK domain with 10m spacing and 2000m/s velocity. Waveforms are shown after being propagated for 0sec, 0.5sec, 2sec, 6sec and 11sec. (a): second-order time stepping results. Leading time dispersion gets greater with propagation time. (b): fourth-order time stepping results. Dispersion is eliminated at twice the cost of second-order time stepping. (c): second-order time stepping results after dispersion correction by SFPI. (d): second-order time stepping results after dispersion correction by NSF. In (c) and (d), dispersion is eliminated at nearly no additional cost. **[ER]**
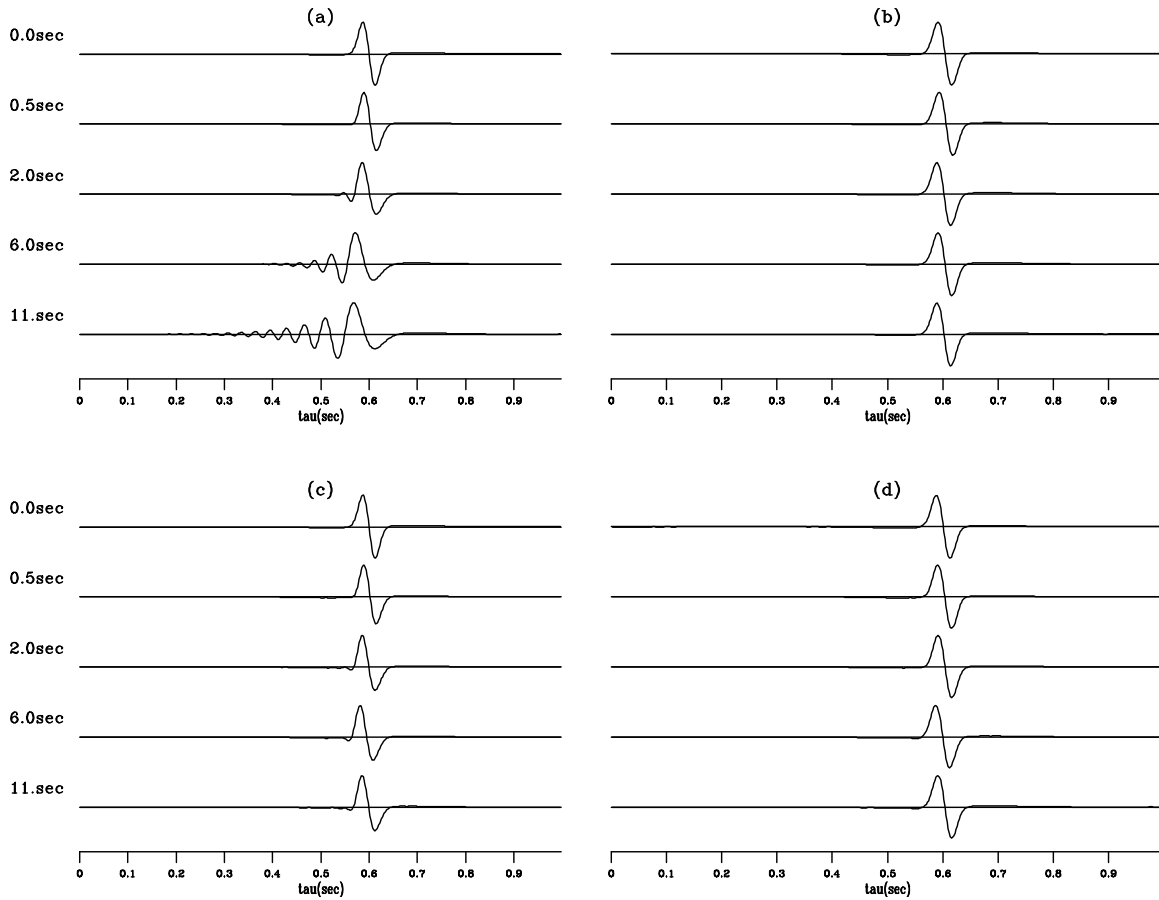
Figure 4: One-D modeling and filtering results in TK domain with 6m spacing and 1000m/s velocity. Waveforms are shown after being propagated for 0sec, 0.5sec, 2sec, 6sec and 11sec. (a): second-order time stepping results. Dispersion gets greater with propagation time. The dispersion effect is the same as in figure 3(a). (b): fourth-order time stepping results. Dispersion is eliminated at twice the cost of second-order time stepping. (c): second-order time stepping results after dispersion correction by SFPI. (d): second-order time stepping results after dispersion correction by NSF. In (c) and (d), the same set of filters applied in figures 3(c) and (d) are applied here. [**ER**]
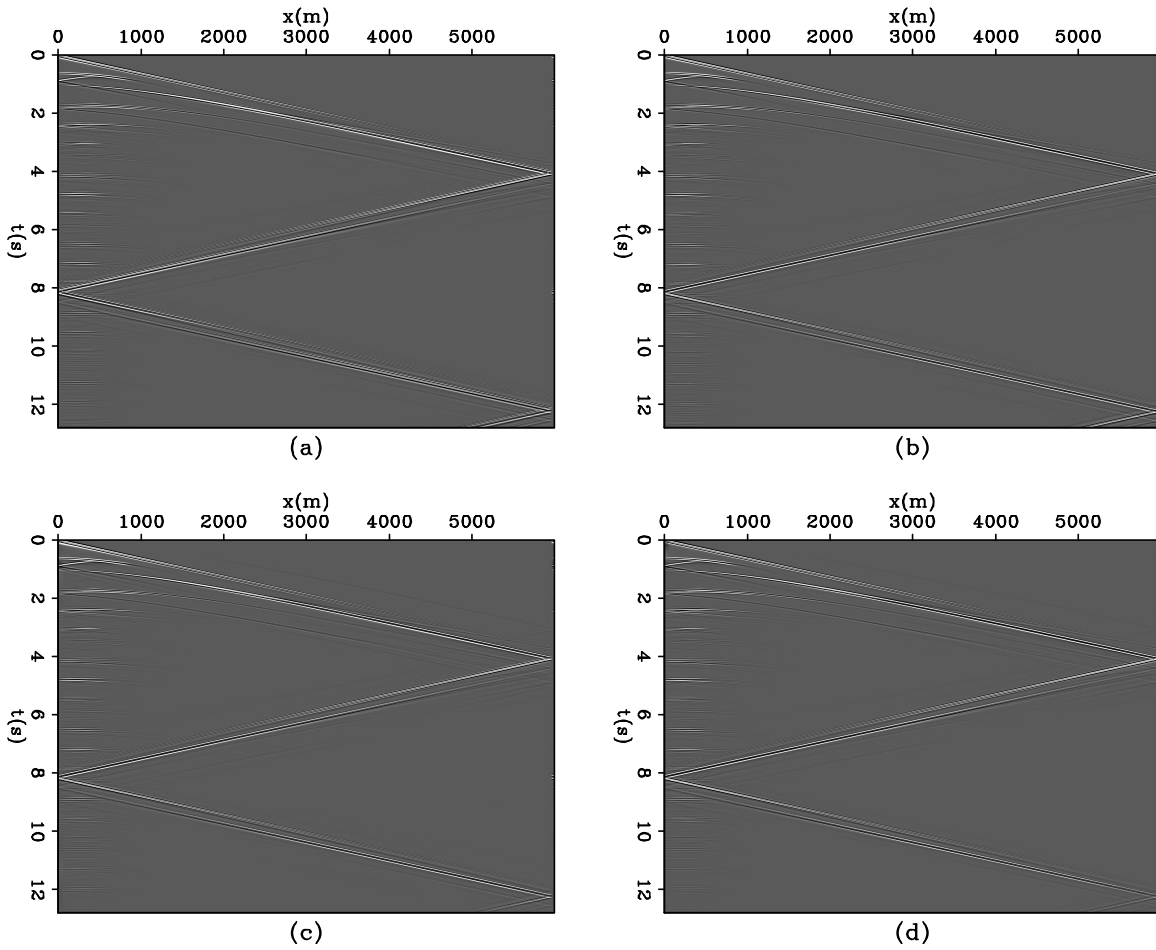
Figure 5: Shot record by two-dimensional TK domain modeling. (a) Shot record with 2nd-order modeling. Time dispersion gets greater with time. (b) Shot record with 4th-order modeling. Dispersion was eliminated by twice the cost of 2nd-order time stepping. (c) Shot record (a) after dispersion correction by SFPI. (d) Shot record (a) after dispersion correction by NSF. [**ER**]
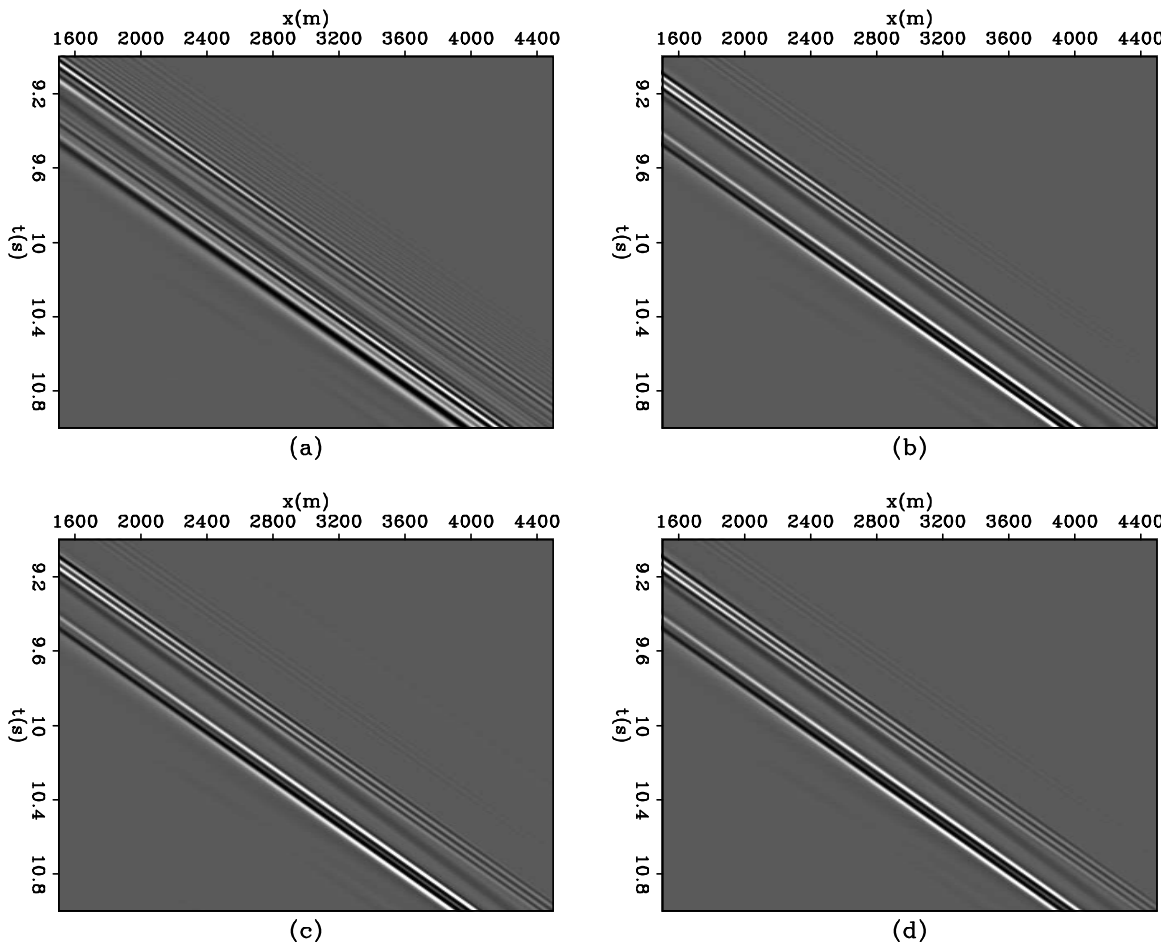
Figure 6: Zoom-in view of a shot record by two-dimensional TK domain modeling. (a) Shot record with 2nd-order modeling. Time dispersion gets greater with time. (b) Shot record with 4th-order modeling. Dispersion was eliminated at twice the cost of 2nd-order time stepping. (c) Shot record (a) after dispersion correction by SFPI. (d) Shot record (a) after dispersion correction by NSF. [**ER**]
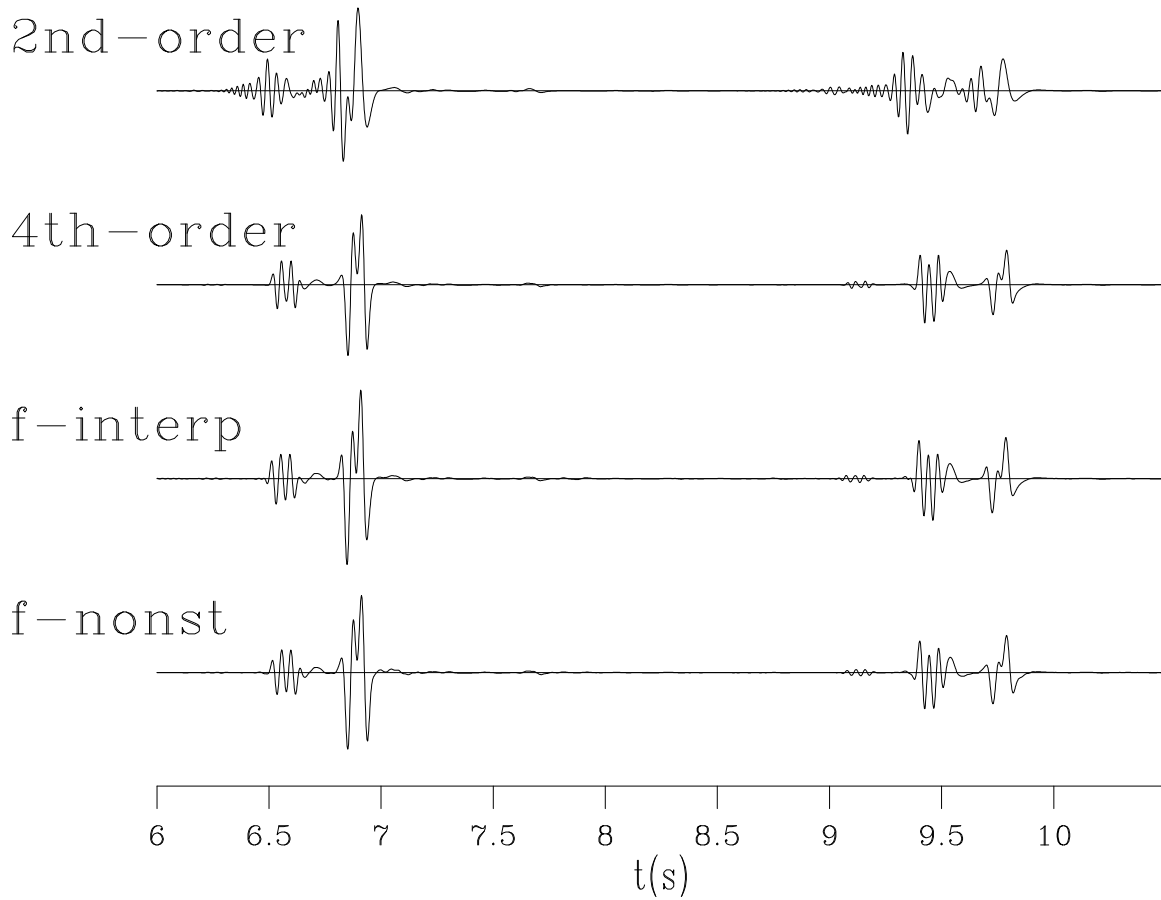
Figure 7: Full waveform at receiver $x = 2000$m by two-dimensional Fourier domain modeling. Dispersion gets greater in time in the 2nd-order time stepping. The energy packet between 9s and 9.5s is severely distorted by dispersion in the 2nd-order modeling result. Post-propagation filtering can remove the dispersion at nearly no additional cost. [**ER**]

CONCLUSIONS

We have developed two post-propagation filtering schemes to remove the temporal dispersion caused by the inaccuracy of the second-order FD approximation to the time derivatives when solving the wave equation. We show that the temporal dispersion is independent of the medium velocity and spatial sampling, which is the reason why post-propagation filtering is possible. We design two different filtering schemes: stationary filtering plus interpolation and non-stationary filtering. The filtering results on both 1-D and 2-D second-order modeling show that we can successfully remove the dispersion artifacts at nearly no additional cost.

Although the filtering results of both schemes are similar, the SFPI scheme involves many (the number of filters) passes of convolution across the data record, whereas the NSF scheme involves only one convolutional pass. Furthermore, the SFPI scheme interpolates among multiple copies of the data record, whereas the NSF scheme interpolates among the filter coefficients. Therefore, considering both the computation and memory requirements, the non-stationary filtering scheme is more suitable for large scale computations.

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