Beam foucusing using double parameter fitting

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ABSTRACT

Beam-focusing using both moveout parameter (slope and curvature).

INTRODUCTION

The background is referred to (Biondi, 2010).

THEORY

First define the correlation panel after the beam(local window) decomposition:

$$C(h,\tau;x_s,x_g,v) = \int dt\, P_{cal}(h,t-\tau,x_s,x_g,v)\, P_{obs}(h,t;x_s,x_g). \tag{1}$$

Assume the move out function has the form

$$\theta_{x_s,x_g}(a,b,h) = ah^2 + bh = af(h) + bg(h), \tag{2}$$

in which h is the local offset, $f(h) = h^2, g(h) = h$.

The local maximization objective function that measure the flatness is

$$J_{FL}(x_s,x_g) = \int\int dh\, d\tau\, C(h,\tau+\theta(a,b);x_s,x_g,v_0)C(h,\tau;x_s,x_g,v(x)). \tag{3}$$

Keep in mind that a, b in the context of the paper is always a function of x_s, x_g, i.e. $a(x_s,x_g), b(x_s,x_g)$.

Because a, b maximize J_{FL}, then we have

$$\{ \begin{array}{l} \frac{\partial J_{FL}}{\partial a} = 0 \\ \\ \frac{\partial J_{FL}}{\partial b} = 0 \end{array} \tag{4}$$

To find out the relation between a, b and $v(x)$, differentiate the equation (4) with a, b and v. we have

$$\begin{bmatrix} \frac{\partial^2 J_{FL}}{\partial a^2} & \frac{\partial^2 J_{FL}}{\partial a \partial b} \\ \\ \frac{\partial^2 J_{FL}}{\partial a \partial b} & \frac{\partial^2 J_{FL}}{\partial b^2} \end{bmatrix} \begin{bmatrix} \frac{\partial a}{\partial v} \\ \\ \frac{\partial b}{\partial v} \end{bmatrix} = - \begin{bmatrix} \frac{\partial J_{FL}}{\partial a \partial v} \\ \\ \frac{\partial J_{FL}}{\partial b \partial v} \end{bmatrix}, \tag{5}$$

in which we can find

$$\frac{\partial J_{FL}}{\partial a} = \int\int dh\, d\tau \dot{C}(h,\tau+\theta;v_0) f(h) C(h,\tau,v) = 0$$
$$\frac{\partial J_{FL}}{\partial b} = \int\int dh\, d\tau \dot{C}(h,\tau+\theta;v_0) g(h) C(h,\tau,v) = 0$$

($\dot{C}, \ddot{C}$ indicate the first and second derviate on time axis.) and let

$$\frac{\partial^2 J_{FL}}{\partial a^2} = \int\int dh\, d\tau\, \ddot{C}(h,\tau+\theta;v_0) f^2(h) C(h,\tau;v) = E_{11}$$
$$\frac{\partial^2 J_{FL}}{\partial a \partial b} = \int\int dh\, d\tau\, \ddot{C}(h,\tau+\theta;v_0) f(h) g(h) C(h,\tau;v) = E_{12}$$
$$\frac{\partial^2 J_{FL}}{\partial b^2} = \int\int dh\, d\tau\, \ddot{C}(h,\tau+\theta;v_0) g^2(h) C(h,\tau;v) = E_{22}.$$

Then we have

$$\begin{bmatrix} E_{11} & E_{12} \\ E_{12} & E_{22} \end{bmatrix} \begin{bmatrix} \frac{\partial a}{\partial v} \\ \frac{\partial b}{\partial v} \end{bmatrix} = - \begin{bmatrix} \frac{\partial J_{FL}}{\partial a \partial v} \\ \frac{\partial J_{FL}}{\partial b \partial v} \end{bmatrix}, \tag{6}$$

Then the right hand side

$$\frac{\partial J_{FL}}{\partial a \partial v} = \int\int dh\, d\tau\, \dot{C}(h,\tau+\theta;v_0) f(h) \frac{\partial C(h,\tau;v)}{\partial v}$$
$$\frac{\partial J_{FL}}{\partial b \partial v} = \int\int dh\, d\tau\, \dot{C}(h,\tau+\theta;v_0) g(h) \frac{\partial C(h,\tau;v)}{\partial v}$$

and from Tarantola (1984) the waveform inversion theory, we have

$$\frac{\partial C(h,\tau;v)}{\partial v} = \frac{1}{v^3} \int dt\, P_{obs}(h,t+\tau;x_s,x_g) \dot{G}(x,t;x_g,h,0) * \dot{P}_{cal}(x,t;x_s), \tag{7}$$

in which G stands for Green's function for wavefield propagation. $\dot{G}, \dot{P}$ stands for first derivative over time as well, $*$ stands for convolution along time axis.

On the other side, the objective function we want to maximize is

$$\mathbf{J} = \frac{1}{2} \sum_{x_s} \sum_{x_g} \int d\tau \Big[\int dh\, C(h,\tau+\theta(a,b,h);x_s,x_g,v_0) \Big]^2$$

To calculate the gradient, (To give a concise notation, $C(h,\tau+\theta(a,b,h);x_s,x_g,v_0)$ is simply denoted as $C(h,\tau+\theta;v_0)$)

$$\frac{\partial J}{\partial v} = \sum_{x_s}\sum_{x_g} \int d\tau \int dh\, C(h,\tau+\theta;v_0) \Big[\int dh\, \dot{C}(h,\tau+\theta;v_0)(f(h)\frac{\partial a}{\partial v} + g(h)\frac{\partial b}{\partial v}) \Big]$$

notice that $\frac{\partial a}{\partial v}$, $\frac{\partial b}{\partial v}$ is independent of τ and h, so they can be taken out of the integral, denote

$$A(\tau, a, b; x_s, x_g, v_0) = \int dh\, C(h, \tau + \theta; x_s, x_g, v_0) \tag{8}$$

$$A_1(\tau, a, b; x_s, x_g, v) = \int dh\, \dot{C}(h, \tau + \theta; x_s, x_g, v) f(h) \tag{9}$$

$$B_1(\tau, a, b; x_s, x_g, v_0) = \int dh\, \dot{C}(h, \tau + \theta; x_s, x_g, v) g(h) \tag{10}$$

Then

$$\begin{aligned}
\frac{\partial J}{\partial v(x)} &= \sum_{x_s} \sum_{x_g} \int d\tau\, A(\tau; v_0) \left(A_1(\tau; v) \frac{\partial a}{\partial v} + B_1(\tau; v) \frac{\partial b}{\partial v} \right) \\
&= \sum_{x_s} \sum_{x_g} \int d\tau\, (A(\tau; v_0) A_1(\tau; v)) \frac{\partial a}{\partial v} + \int d\tau\, (A(\tau; v_0) B_1(\tau; v)) \frac{\partial b}{\partial v}
\end{aligned}$$

Let

$$\int d\tau\, A(\tau; v_0) A_1(\tau; v) = A_2(v_0, v),$$

$$\int d\tau\, A(\tau; v_0) B_1(\tau; v) = B_2(v_0, v)$$

, then

$$\begin{aligned}
\frac{\partial J}{\partial v(x)} &= \sum_{x_s} \sum_{x_g} A_2(v_0; v) \frac{\partial a}{\partial v} + B_2(v_0; v) \frac{\partial b}{\partial v} \\
&= \sum_{x_s} \sum_{x_g} \left[\begin{array}{cc} A_2(v_0, v) & B_2(v_0, v) \end{array}\right] \begin{pmatrix} \frac{\partial a}{\partial v} \\ \frac{\partial b}{\partial v} \end{pmatrix}
\end{aligned} \tag{11}$$

from (6) and (7), we have

$$\begin{pmatrix} \frac{\partial a}{\partial v} \\ \frac{\partial b}{\partial v} \end{pmatrix} = - \begin{pmatrix} E_{11} & E_{12} \\ E_{12} & E_{22} \end{pmatrix}^{-1} \begin{pmatrix} \int \int dh\, d\tau\, \dot{C}(h, \tau + \theta; v_0) f(h) \frac{\partial C}{\partial v} \\ \int \int dh\, d\tau\, \dot{C}(h, \tau + \theta; v_0) g(h) \frac{\partial C}{\partial v} \end{pmatrix}, \tag{12}$$

Let

$$\left[\begin{array}{cc} E_{11} & E_{12} \\ E_{12} & E_{22} \end{array}\right]^{-1} = \left[\begin{array}{cc} F_{11} & F_{12} \\ F_{12} & F_{22} \end{array}\right].$$

and further

$$[A_2\ B_2] \left[\begin{array}{cc} F_{11} & F_{12} \\ F_{12} & F_{22} \end{array}\right] = [H_1\ H_2].$$

Then plug eq (12) into (11), we have

$$\frac{\partial J}{\partial v} = [A_2\ B_2]\begin{bmatrix} F_{11} & F_{12} \\ F_{12} & F_{22} \end{bmatrix}\begin{pmatrix} \int\int dh\, d\tau\, \dot{C}(h,\tau+\theta;v_0)f(h)\frac{\partial C}{\partial v} \\ \\ \int\int dh\, d\tau\, \dot{C}(h,\tau+\theta;v_0)g(h)\frac{\partial C}{\partial v} \end{pmatrix}$$
$$= [H_1\ H_2]\begin{pmatrix} \int\int dh\, d\tau\, \dot{C}(h,\tau+\theta;v_0)f(h)\frac{\partial C}{\partial v} \\ \\ \int\int dh\, d\tau\, \dot{C}(h,\tau+\theta;v_0)g(h)\frac{\partial C}{\partial v} \end{pmatrix}$$
$$= \int\int dh\, d\tau\, \dot{C}(h,\tau+\theta;v_0)(H_1 f(h) + H_2 g(h))\frac{\partial C}{\partial v} \quad (13)$$

Plug eq (7) in, we have

$$\frac{\partial J}{\partial v} = -\frac{2}{v^3}\sum_{x_s}\sum_{x_g}\{\int\int dh\, d\tau\, \dot{C}(h,\tau+\theta;v_0)(H_1 f(h)+H_2 g(h))\int dt\, P_{obs}(h,t+\tau;x_s,x_g)\dot{G}(x,t;x_g,h,0)*\dot{P}_{cal}(x,t;x_s)\} \quad (14)$$

Manipulate the internal triple integration in simlary way as in Tarantola (1984), we have

$$\frac{\partial J}{\partial v} = -\frac{2}{v^3}\sum_{x_s}\sum_{x_g}\int dt\, \dot{P}_{cal}(x,t;x_s)\int dh\, \dot{G}(x,-t;x_g,h,0)*S(t,h,\theta;x_s,x_g,v_0), \quad (15)$$

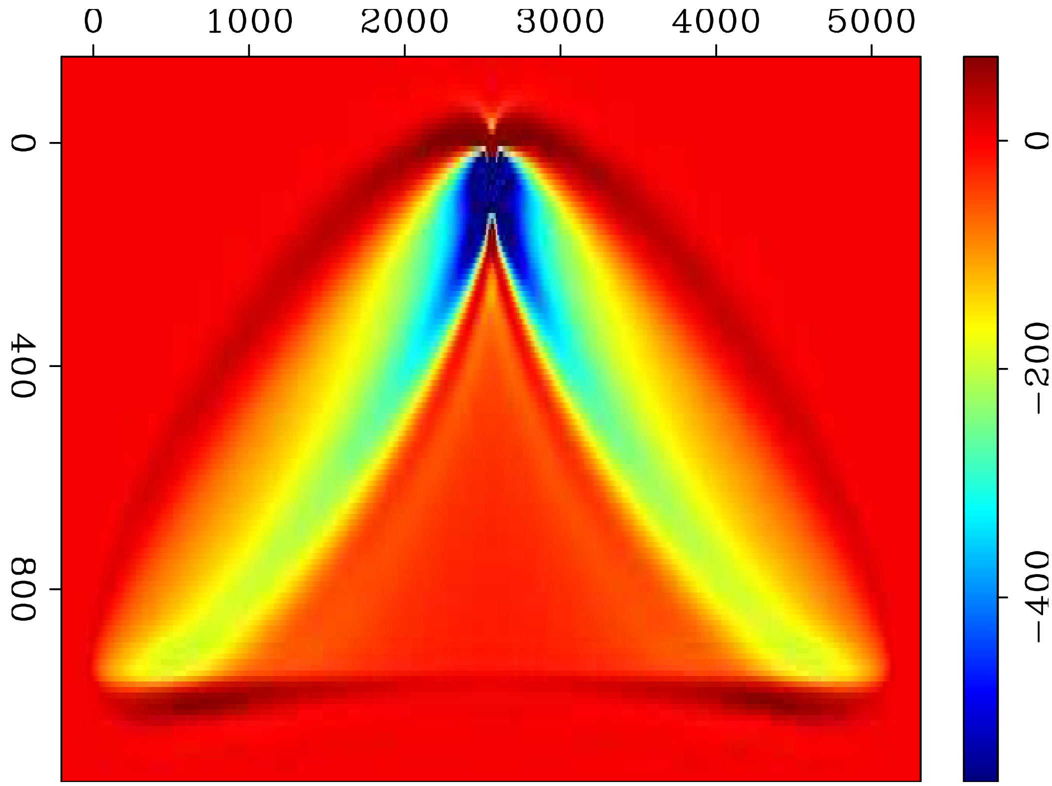
in which

$$S(t,h,\theta;x_s,x_g,v_0) = [H_1(x_g,x_s)f(h)+H_2(x_g,x_s)g(h)]\int d\tau[\dot{C}(h,\tau+\theta;v_0,x_g,x_s)P_{obs}(h,t+\tau,x_s,x_g)] \quad (16)$$

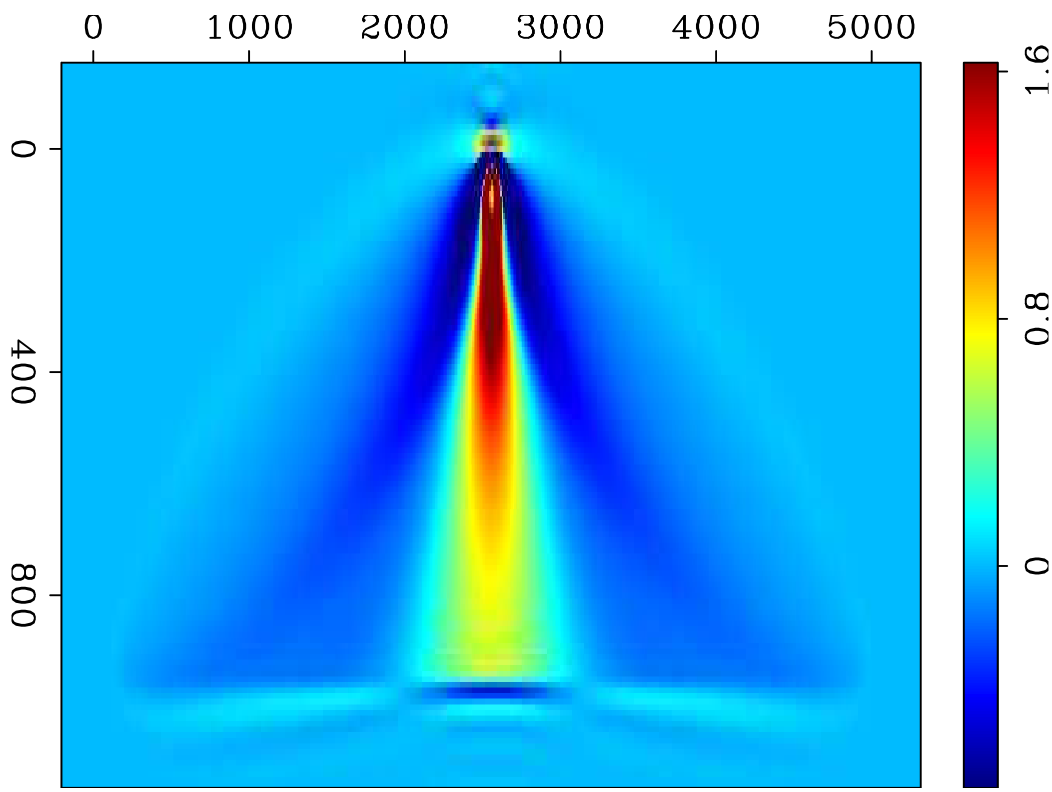
We can further simplify it by changing the integration varibles. Let $x_r = x_g + h$, replace the integration variable x_g, then we have

$$\sum_{x_g}\int dh\, \dot{G}(x,-t;x_g,h,0) *_t [H_1 f(h) + H_2 g(h)]\int d\tau[\dot{C}(h,\tau+\theta;v_0,x_g,x_s)P_{obs}(h,t+\tau,x_s,x_g)] =$$
$$\sum_{x_r}\dot{G}(x,-t;x_r,0) *_t \left(\int d\tau \dot{C}(\tau+\theta;v_0,x_r)P_{obs}(x_r,t+\tau)\int dh H_1(x_r-h)f(h) + H_2(x_r-h)g(h)\right) \quad (17)$$

Then if we evaluate $\frac{\partial J}{\partial v}$ at $v = v_0$, then $\theta = 0$. and the equation (15) & (16) illustrates us how to implement the gradient calculation on computers: The major procedure is the same as in FWI case. The only difference is now we have a more complex way of computing the residual for back propagation. First the correlation panel between observed data and calculated data is calculated, then divide the correlation panel into local beams(sub windows). Then for each subwindow, calculate matrix E and A_2, B_2 to get weight H_1, H_2, do the cross-correlation between the first derivative of correlation panel($\dot{C}$) and the observed data, scale each trace by $H_1 f(h) + H_2 g(h)$, finally put the result back to the data space aligned to the position of the original beam position.



(a)



(b)

Figure 1: (a) Velocity Gradient obtained by YiLuo's Traveltime Tomography method; (b) Velocity Gradient obtained by this method.

PRELIMINARY RESULTS

Here I implemented a simple scenario where we have constant velocity error. The velocity grid has a grid size of 10m by 10m, 130 grid points in depth and 552 grid points in width. One shot is placed on the center top of the domain, receivers down the bottom, spacing is also 10m. The true velocity is 3000m/s, and the initial velocity is 3300m/s. The beam size is 15 grid pts, i.e 140m width.

Figure 1 shows the gradient calculated from two methods: the Traveltime Tomography method and the method presented in this report. In the following I am showing the calculation procedure provided by the **T**heory section step by step.

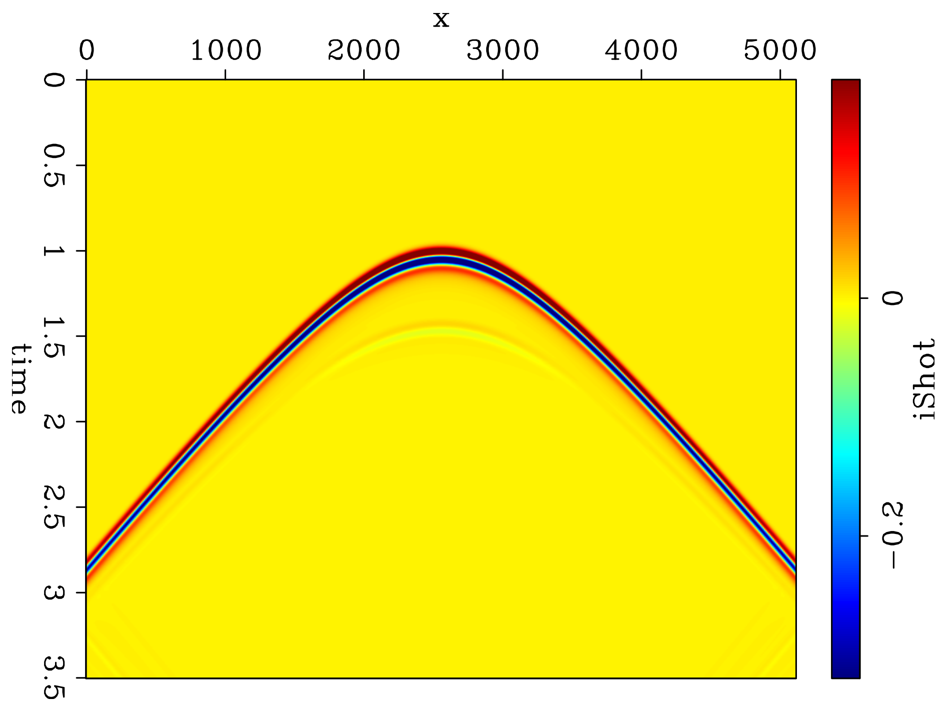


Figure 2: The observed data.

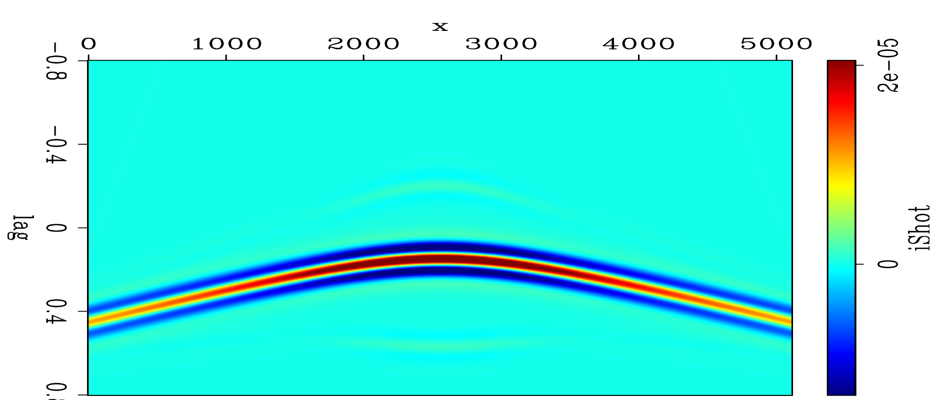
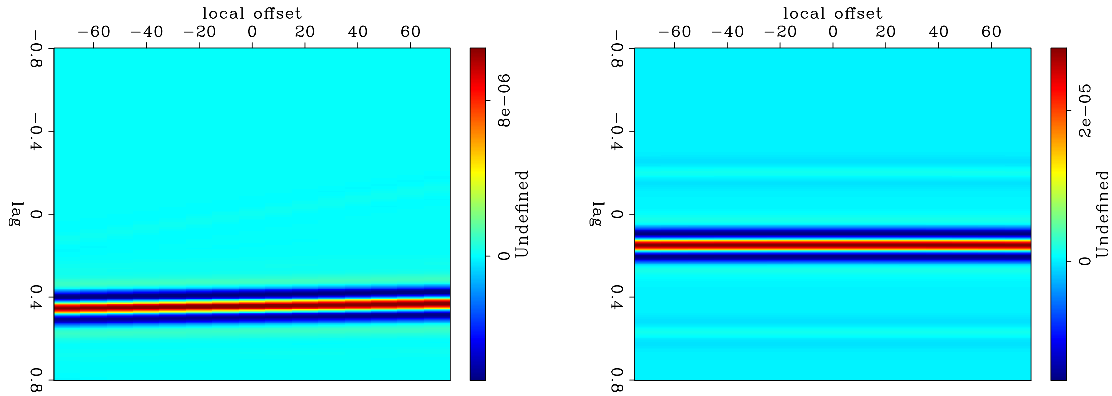
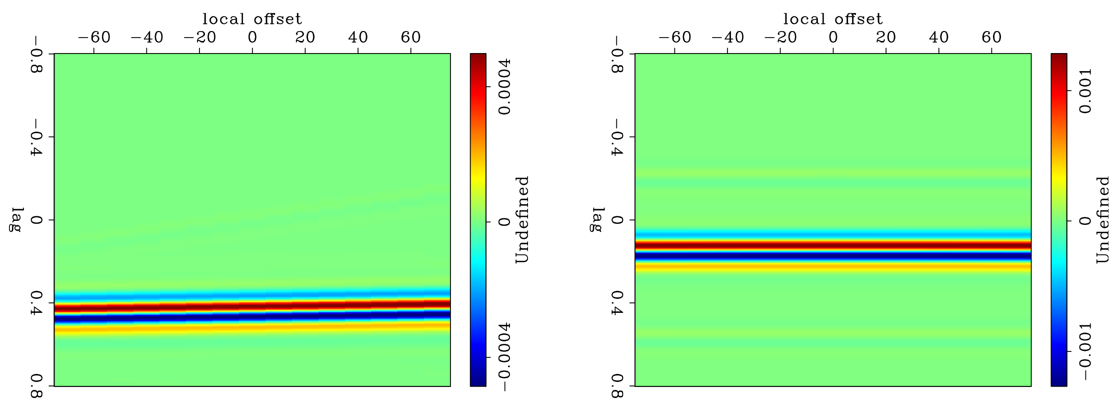


Figure 3: The correlation between d_{obs} and d_{cal}.



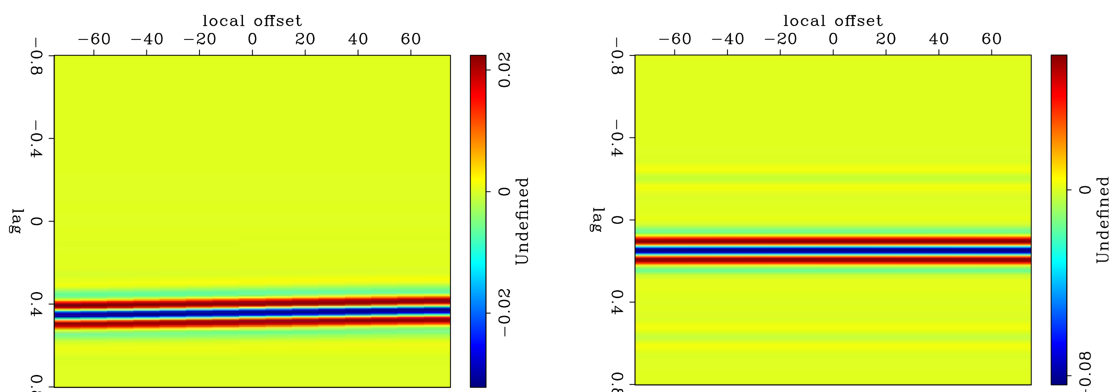
(a) (b)

Figure 4: the correlation $C(h, \tau)$ for the local beam at $x_g = 100m$ and $x_g = 2560m$ respectively; In plot (a) the event is slightly tilted.



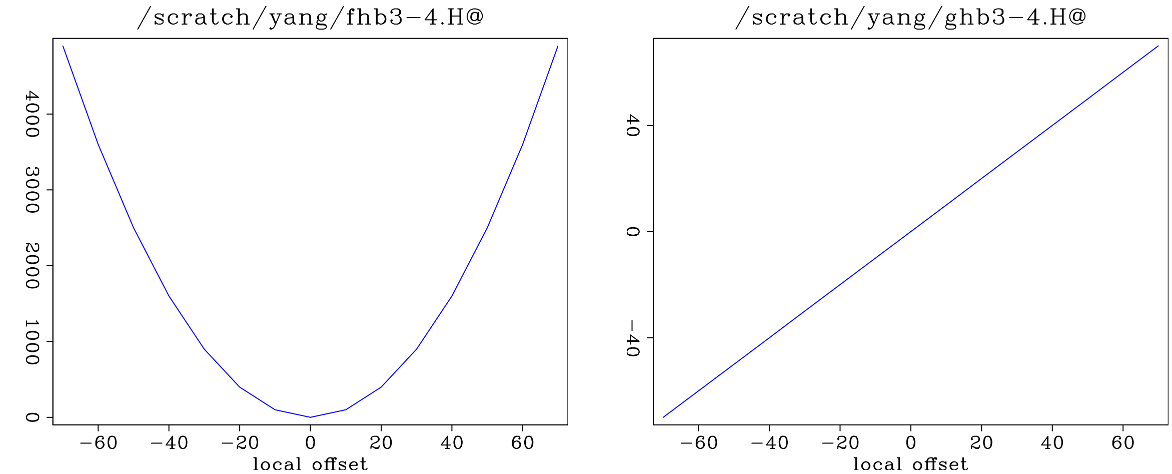
(a) (b)

Figure 5: the first derivative of correlation $\dot{C}(h, \tau)$ for the local beam at $x_g = 100m$ and $x_g = 2560m$ respectively.



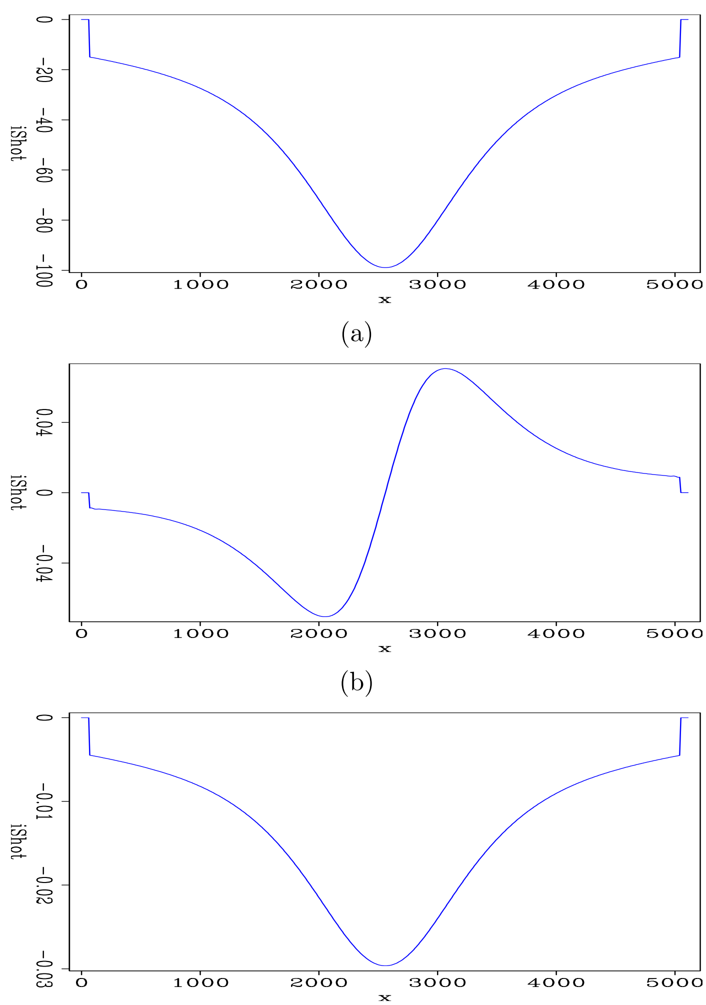
(a) (b)

Figure 6: the first derivative of correlation $\ddot{C}(h, \tau)$ for the local beam at $x_g = 100m$ and $x_g = 2560m$ respectively.



(a) (b)

Figure 7: the local moveout function $f(h) = h^2$ and $g(h) = h$.

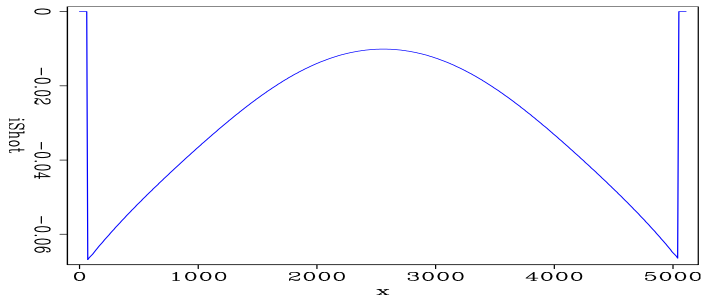


(a)

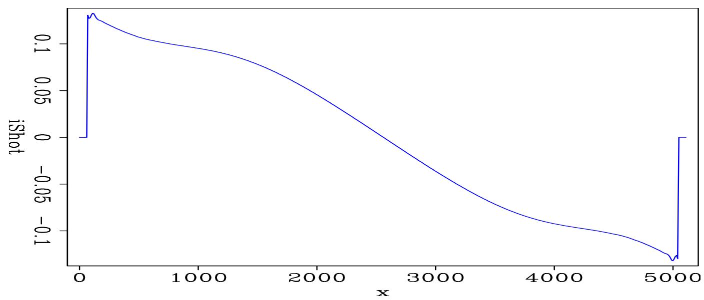
(b)

(c)

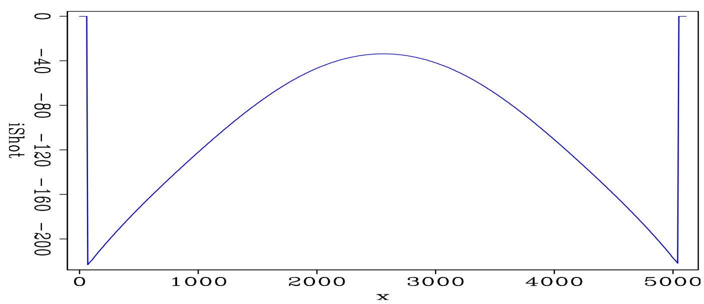
Figure 8: the E matrix (E11,E12,E22) for each beam location x_g.



(a)

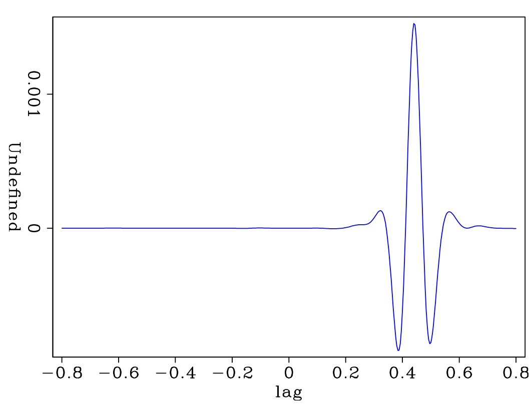


(b)

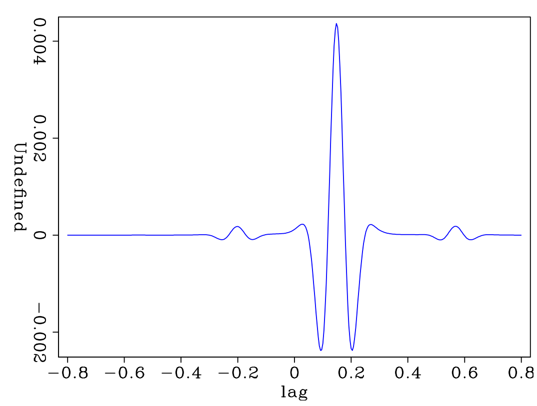


(c)

Figure 9: the F matrix (F11,F12,F22) for each beam location x_g. Notice $F22$ has much bigger amplitude than $F11$.

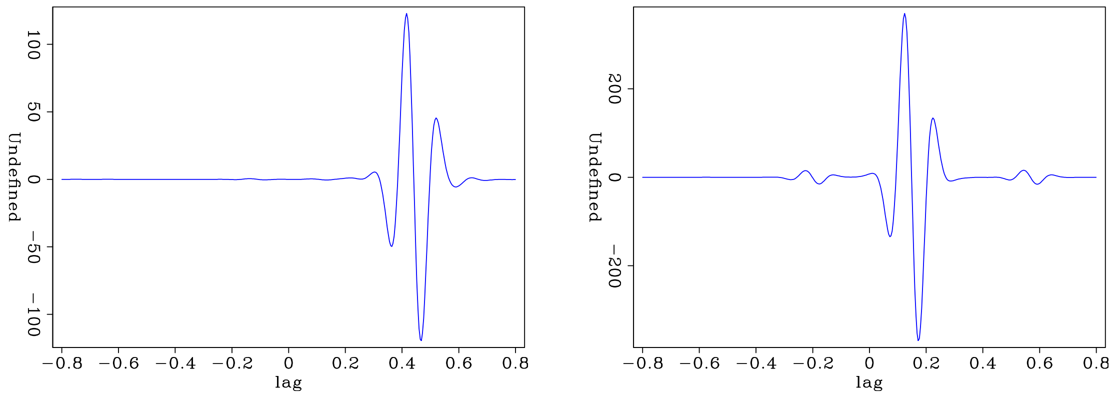


(a)



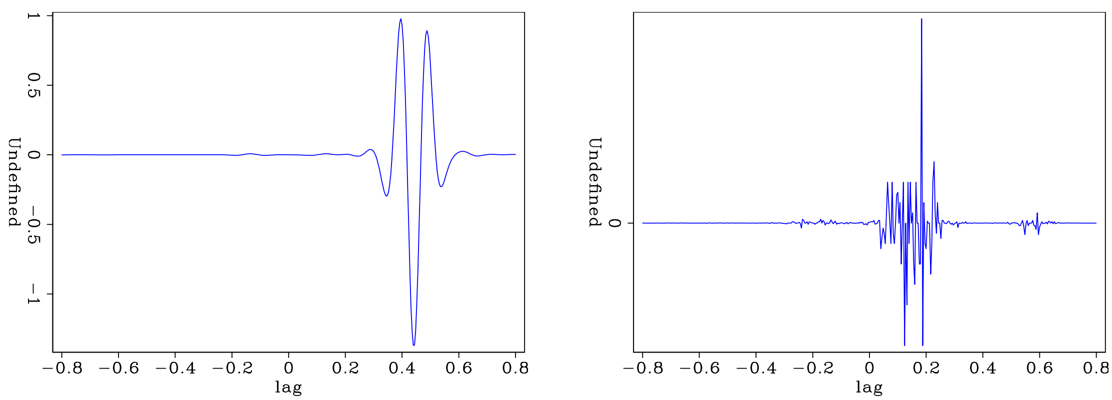
(b)

Figure 10: The trace $A(\tau)$ for the local beam at $x_g = 100m$ and $x_g = 2560m$ respectively.



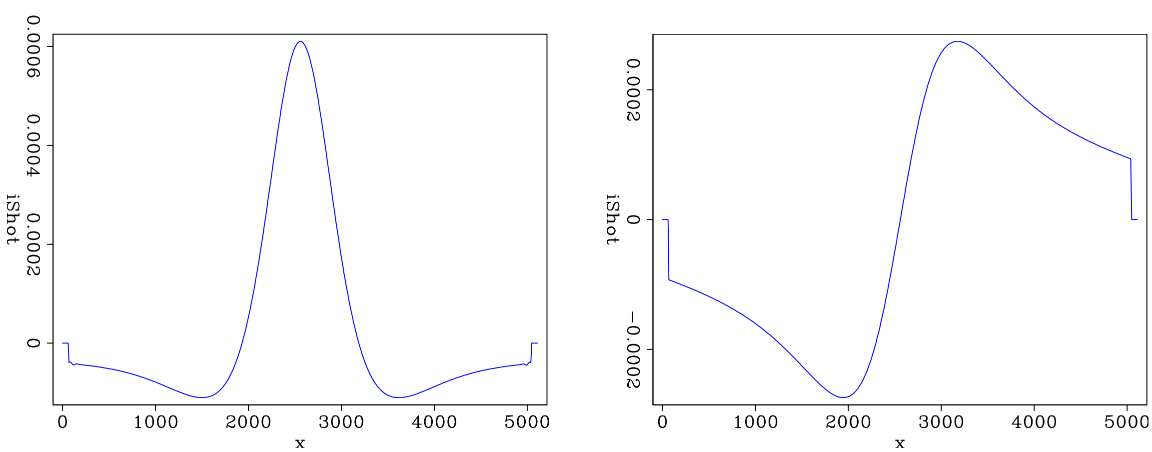
(a) (b)

Figure 11: The trace $A1(\tau)$ for the local beam at $x_g = 100m$ and $x_g = 2560m$ respectively.



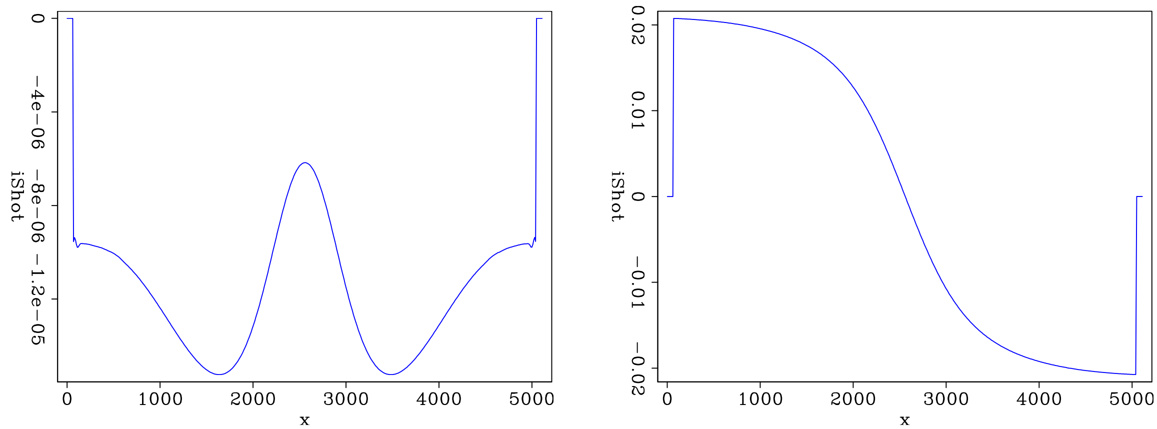
(a) (b)

Figure 12: The trace $B1(\tau)$ for the local beam at $x_g = 100m$ and $x_g = 2560m$ respectively.



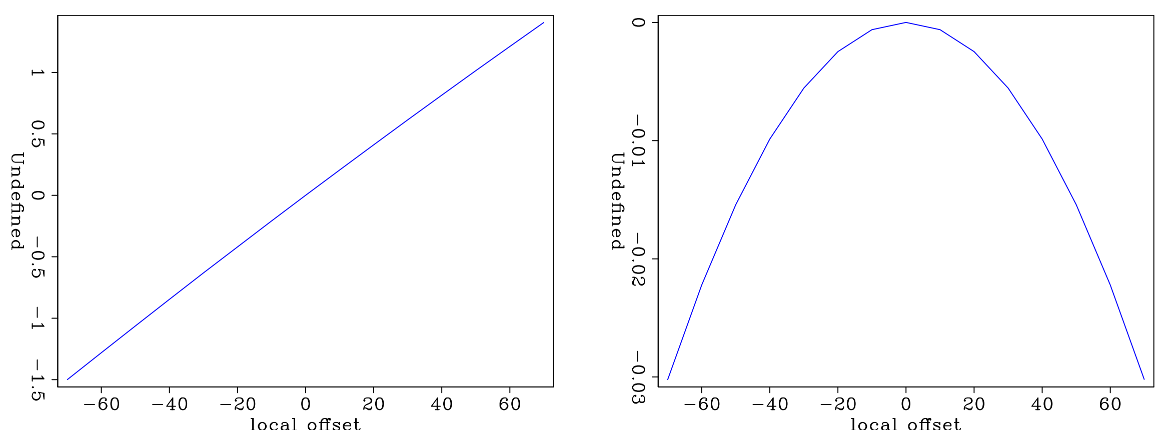
(a) (b)

Figure 13: The $A2, B2$ (i.e. $\frac{\partial J}{\partial a}$ and $\frac{\partial J}{\partial b}$) for each beam location. Notice that (a) plot has negative values on the edge.



(a) (b)

Figure 14: The H1 and H2 term for each beam location.



(a) (b)

Figure 15: $H_1 f(h) + H_2 g(h)$ term for the local beam at $x_g = 100m$ and $x_g = 2560m$.

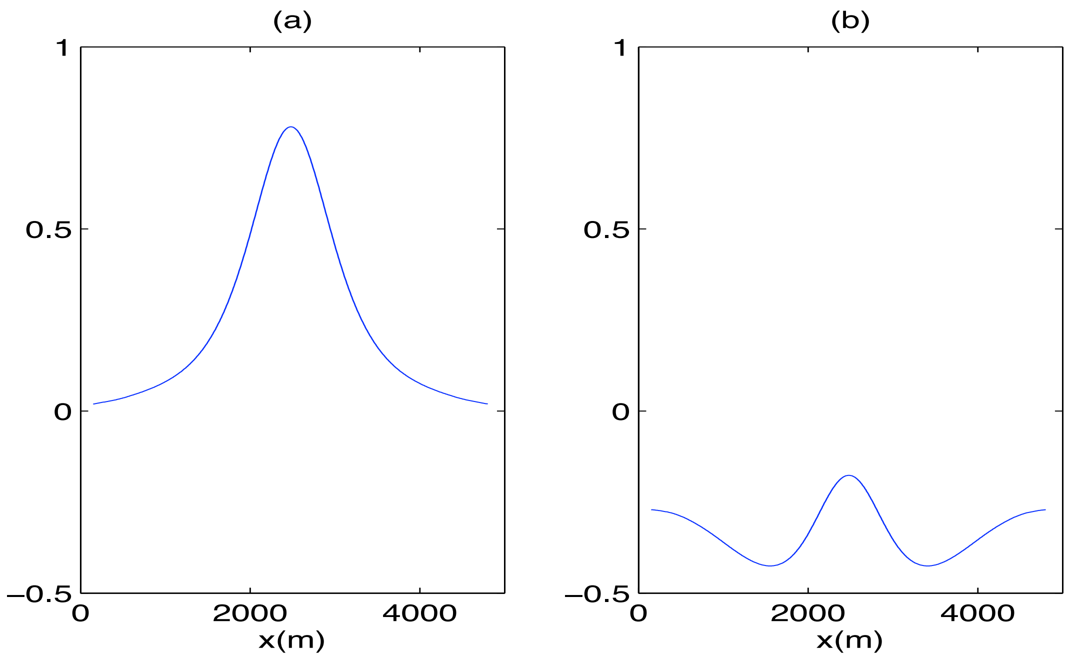


Figure 16: (a) Convolution of H1 and f; (b) Convolution of H2 and g; see eq (17).

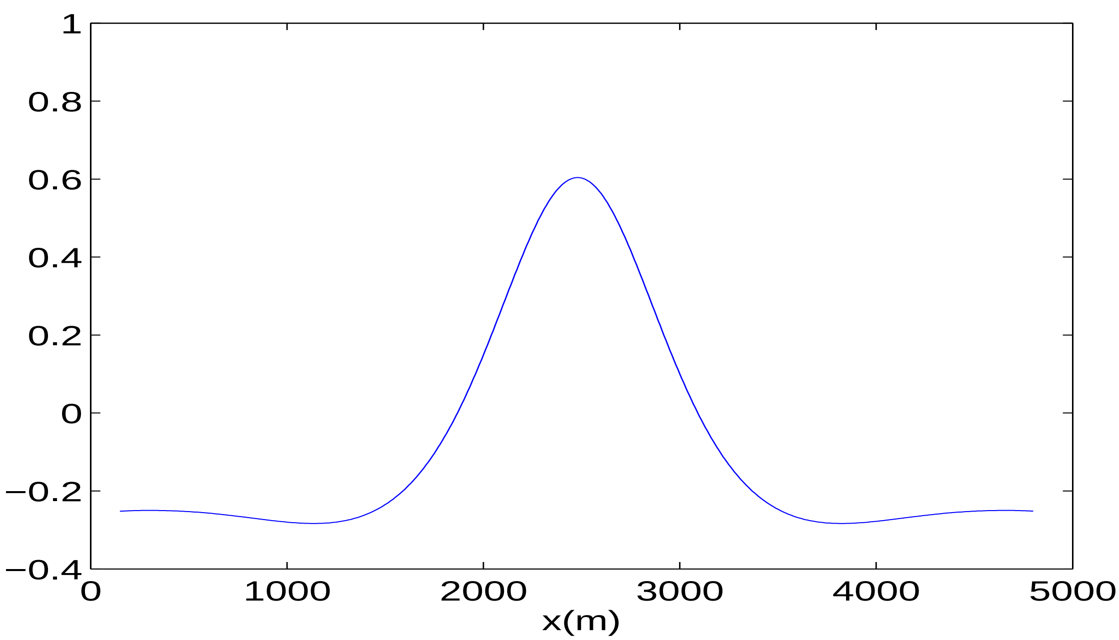
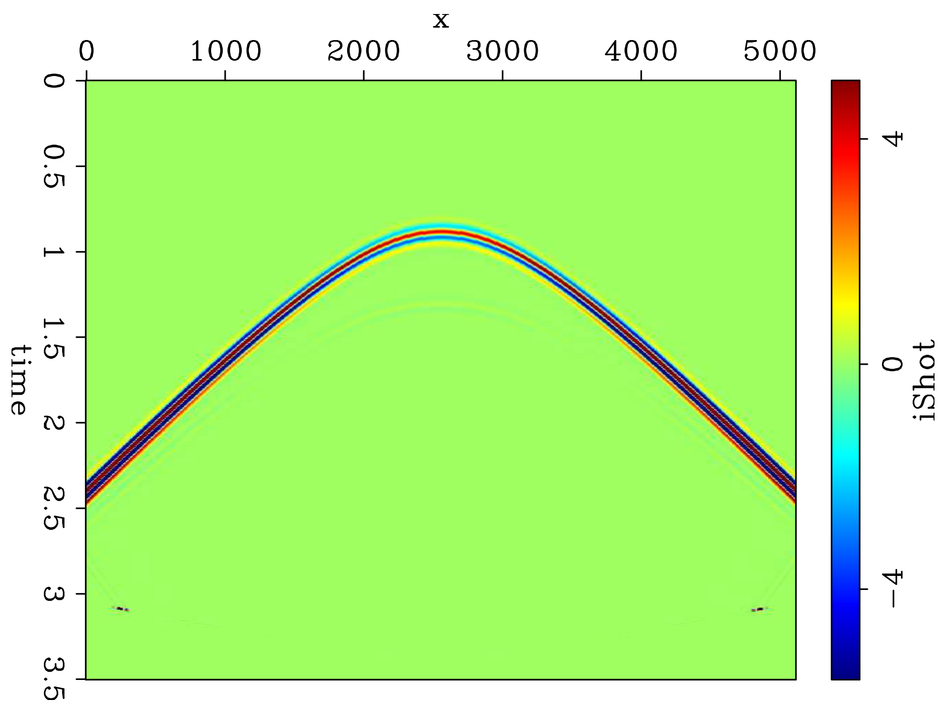
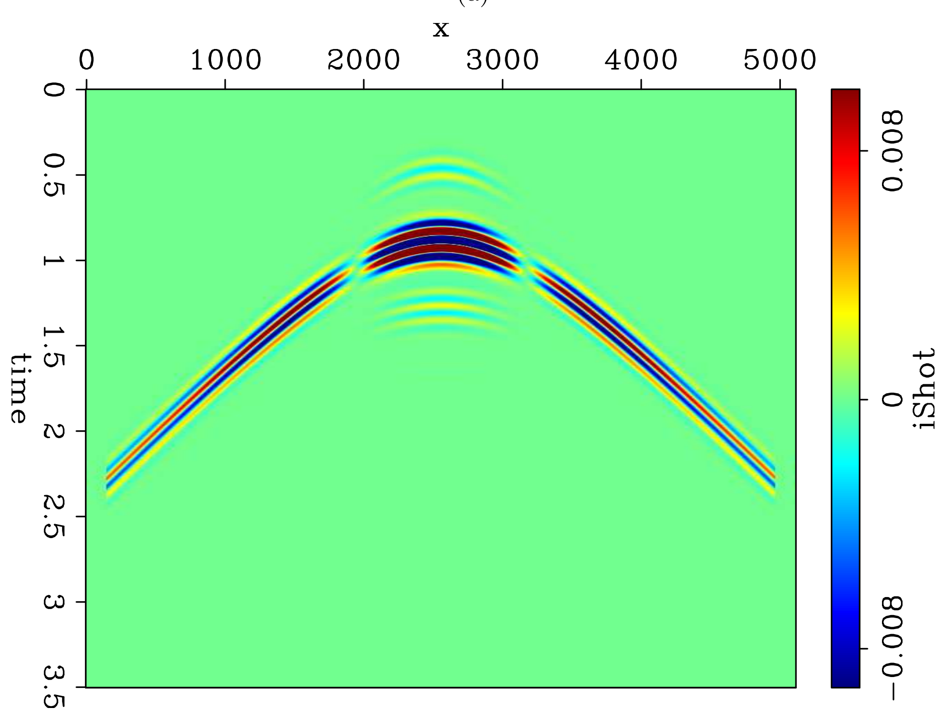


Figure 17: Sum of the two terms in Figure 16, see eq (17).



(a)



(b)

Figure 18: (a) Back projected data by YiLuo's Traveltime Tomography method; (b) Back projected data obtained by this method,notice the sign of the signal flipped when going from the center to the ends along x axis, this can also be seen from figure 17's polarity change along x axis. (The secondary weak events in (b) is the modeling artifact from the imperfect absorbing boundaries.

FUTURE WORK

ACKNOWLEDGMENTS

REFERENCES

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Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation: Geophysics, **49**, 1259–1266.