You can see the sunset twice in one day

Where the sun sets over the ocean, the horizon line casts a shadow moving up the sides of mountains, buildings, trees, and even people. As the sinking sun is increasingly occluded by the horizon it becomes more of a point source. Thus the rising shadow line should be quite sharp. Think of a man seated at the ocean's edge with his eyes at an altitude of one meter. I will calculate that it requires 8 seconds for the shadow to reach from the water's edge up to his eyes. Three seconds later at 11 seconds the shadow reaches two meters, the eyes of a woman standing there. When the air is clear, the wink-out of the top edge of the sun can be quick compared to three seconds. On a clear day the sun seems to blink off in a sort of a brief moment. We may think of this as the sharp shadow washing upward across their faces.

Taking sunset at $t = 0$, a beam of light skims the earth (a circle of radius $R = 6371$ km). After touching the earth at $x = 0$ the earth drops away from the beam to altitude $h(t)$ at distance $x(t)$. We are aiming to tabulate $h(t)$.

$$
\begin{aligned}
R^2 + x^2 &= (R+h)^2 \qquad \text{pythagoras} && (1)\\
x^2 &= 2Rh \qquad \text{ignoring } h^2 && (2)\\
x &= \sqrt{2R}\sqrt{h} \;=\; 3550\sqrt{h} \qquad \text{in meters} && (3)\\
\alpha &= \frac{\text{earth circumference}}{\text{duration of day}} && (4)\\
x &= \alpha t \;=\; \frac{\pi(2R)}{24\cdot 60\cdot 60}\, t \qquad \alpha \text{ is the speed of the earth surface} && (5)\\
h &= x^2/(2R) \;=\; (\alpha t)^2/(2R) \;=\; (\,(\alpha/\sqrt{2R})\; t)^2 && (6)\\
h &= \left(\frac{\pi\sqrt{2R}}{24\cdot 60\cdot 60}\, t\right)^2 && (7)\\
h &= (0.13\; t)^2 \qquad \text{and} \qquad t = 8\sqrt{h} && (8)
\end{aligned}
$$

In Python: import math, then: 3.14 * math.sqrt(2*6371000)/(24*60*60) = 0.13

To make an appealing video we need to chose an altitude, an altitude for a video of a face or of a whole body. The upward velocity $v = dh/dt$ is

$$
\begin{aligned}
v &= dh/dt = .13^2\cdot 2t \;=\; .13^2\cdot 2\cdot 8\sqrt{h} \;=\; .27\sqrt{h} && (9)\\
h &= (v/.27)^2 \;=\; (4v)^2 \qquad \text{or alternately} \qquad v \;=\; \sqrt{h}/4 && (10)
\end{aligned}
$$

Try making a selfie during sunset. If you get a successful Youtube, I can link it here[1].

I read that taking the elevator up the Burj Khalifa[2], you can see the sunset twice, first at the ground and later at the top. That's what led us here.

—Jon Claerbout 9/11/2019

[1] http://sep.stanford.edu/sep/jon/sunset.pdf

[2] https://gizmodo.com/did-you-know-that-the-burj-khalifa-is-so-tall-that-you-5917230

Observations

Bill Symes wrote me (11/3/19) saying:

I was out rowing around in West Sound this evening, with a line of hills to the SW, far enough away (maybe 2 miles) that the horizon was fairly sharp. I watched the sun until its top limb just grazed the hill line and "winked out", then immediately stood up. The top limb was visible again for perhaps 2 s. Estimating the height-of-eye difference at 1 m, and allowing for my reaction time and the time taken to stand up (carefully—it's just a rowboat), the experience was consistent with the 3 s/m estimate.

You could certainly say that I saw two sunsets.

Shadow of sunset cast by a mountain

My memory of seeing the sun "wink out" at sunset includes the presence of a cloud. This is a little like Bill seeing the wink-out over some hills. In this case the atmospheric physics is quite different because the ray does not graze the ocean.

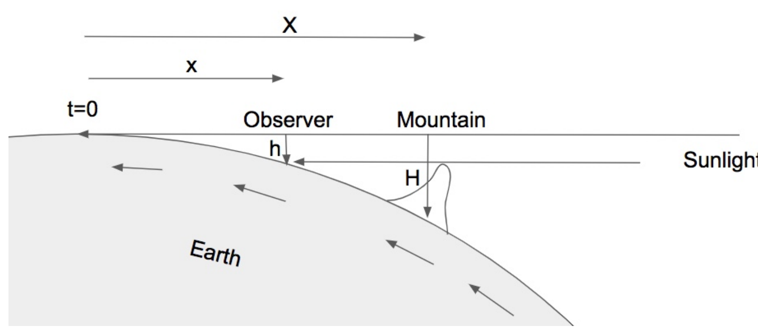

Figure 1: Sunset is earlier because of the mountain.