Attenuation of diffracted multiples with an apex-shifted tangent-squared radon transform in image space

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ABSTRACT

We propose to attenuate diffracted multiples with an apex-shifted tangent-squared Radon transform in angle domain common image gathers (ADCIG). Usually, where diffracted multiples are a problem, the wavefield propagation is complex and the moveout of primaries and multiples in data space is irregular. In our method, the complexity of the wavefield is handled by the migration provided reasonably accurate migration velocities are used. As a result, the moveout of the multiples is well behaved in the ADCIGs. For 2D data, our apex-shifted tangent-squared Radon transform maps the 2D image space into a 3D model space cube whose dimensions are depth, curvature and apex-shift distance. Well-corrected primaries map to or near the zero curvature plane and specularly-reflected multiples map to or near the zero apex-shift plane. Diffracted multiples map elsewhere in the cube according to their curvature and apex-shift distance. Thus, specularly reflected as well as diffracted multiples can be attenuated simultaneously. We illustrate our approach with a segment of a 2D seismic line over a large salt body in the Gulf of Mexico. We show that ignoring the apex shift compromises the attenuation of the diffracted multiples, whereas our approach attenuates both the specularly-reflected and the diffracted multiples without compromising the primaries.

INTRODUCTION

Surface-related multiple elimination (SRME) uses the recorded seismic data to predict and iteratively subtract the multiple series (Verschuur et al., 1992). 2D SRME can deal with all kinds of 2D multiples, provided enough data are recorded given the offset limitations of the survey line. Diffracted multiples from scatterers with a cross-line component cannot be predicted by 2D SRME but in principle can be predicted by 3D SRME as long as the acquisition is dense enough in both in-line and cross-line directions. With standard marine streamer acquisition, the sampling in the cross-line direction is too coarse and diffracted multiples need to be removed by other methods (Hargreaves et al., 2003) or the data need to be interpolated and extrapolated to a dense, large aperture grid (van Dedem and Verschuur, 1998; Nekut, 1998; Biersteker, 2001). In general, multiples may not have their moveout apex at zero offset on a CMP gather. Peg-leg multiples "split" into independent events when reflectors dip. These events look similar to diffracted multiples and may similarly hamper standard Radon demul-

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tiple and velocity analysis. Hargreaves et al. (2003) proposed a shifted hyperbola approach to attenuate split or diffracted multiples in CMP gathers. This approach, however, relies on the moveout of the multiples to be well approximated by hyperbolas in data space, which is problematic in complex media. A similar apex-shifted Radon transform was proposed by Trad (2002) for data interpolation.

In most situations in which diffracted multiples are a serious problem, the wave propagation is rather complex, for example for multiples diffracted off the edge of salt bodies. Thus, the moveout of primaries and multiples tend to be very complex, making the application of data-space moveout-based methods for the removal of multiples difficult. In ADCIGs, however, since the complexity of the wavefield has already been taken into account by prestack migration (to the extent that the presence of the multiples allows an accurate enough estimation of the migration velocity field), the residual moveout of multiples is generally smoother and better behaved (Sava and Guitton, 2003).

In this paper we focus on attenuating diffracted multiples in ADCIGs by redefining the tangent-squared Radon transform of Biondi and Symes (2003) to add an extra dimension to account for the shift in the apexes of the moveout curves of the diffracted multiples. We show with a 2D seismic line from the Gulf of Mexico that our approach is effective in attenuating both, the specularly-reflected and the diffracted multiples. In contrast, ignoring the apex shift results in poor attenuation of the diffracted multiples.

The real impact of our method for attenuating diffracted multiples is likely to be in 3D rather than in 2D, though the results that we show in this paper are limited to 2D. Biondi and Tisserant (2003) have presented a method for computing 3D ADCIGs from full 3D prestack migration. These 3D ADCIGs are functions of both the aperture angle and the reflection azimuth. Simple ray tracing modeling shows that out-of-plane multiples map into events with shifted apexes (like the 2D diffracted multiples) and different reflection azimuth than the primaries. Attenuation of these multiples from 3D ADCIGs can be accomplished with a methodology similar to one we present in this paper.

DIFFRACTED MULTIPLES ON ADCIGS

Figure **??** shows a stack over aperture-angle of a wave-equation migrated 2D line from the Gulf of Mexico over a large salt body. The presence of the salt creates a host of multiples that obscure any genuine subsalt reflection. Most multiples are surface-related peg-legs with a leg related to the water bottom, shallow reflectors or the top of salt. Below the edges of the salt we also encounter diffracted multiples (*e.g.*, CMP position 6000 m below 4000 m depth in Figure **??**). Figure **??** shows four ADCIGs obtained with wave-equation migration as described by Sava and Fomel (2003). The top two ADCIGs correspond to lateral positions directly below the left edge of the salt body (CMP positions 6744 m and 22056 m in Figure **??**). Notice how the apexes of the diffracted multiples are shifted away from the zero aperture angle (*e.g.*, the "seagull"-looking event at about 4600 m in panel (a)). For comparison, the bottom panels in Figure **??** show two ADCIGs that do not have diffracted multiples. Figure **??**c corresponds to an ADCIG below the sedimentary section (CMP 3040 m in Figure **??**) and

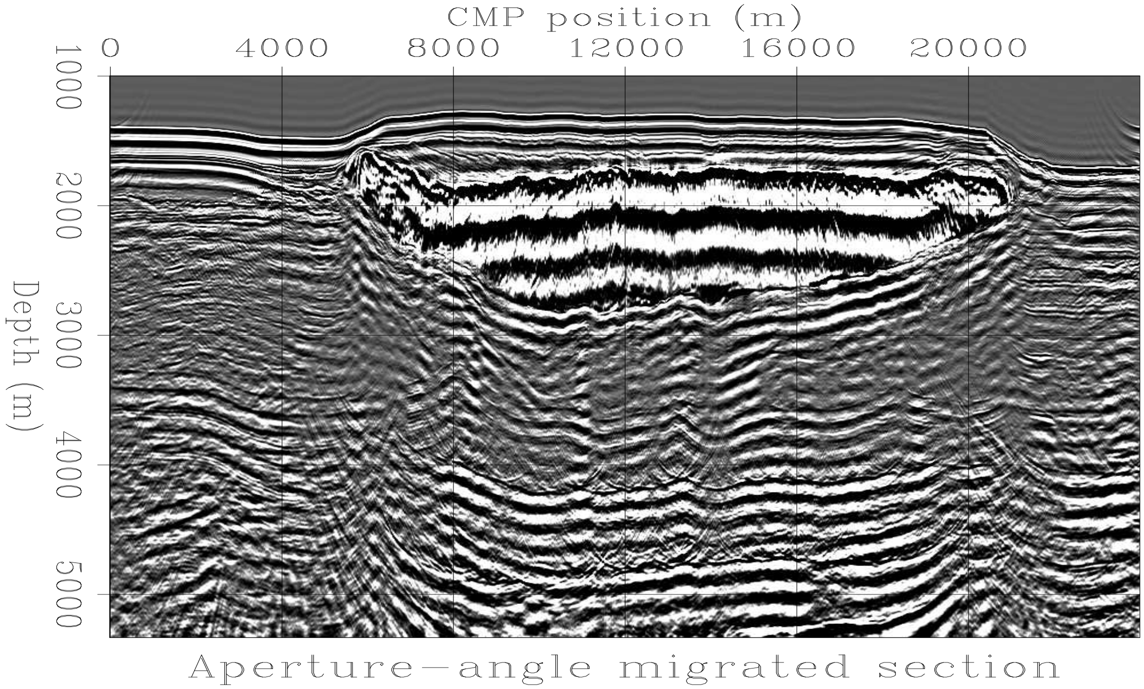


Figure 1: Angle stack of migrated ADCIGs of 2D seismic line in the Gulf of Mexico. Notice that multiples below the salt obscure any primary reflections. angle_stack [CR]

Figure **??**d to and ADCIG below the salt body (CMP position 12000 m in Figure **??**). In these ADCIGs all the multiples are specularly-reflected and thus have their apexes at zero aperture angle. Notice also that although the data is marine, the ADCIGs show positive and negative aperture angles. We used reciprocity to simulate negative offsets and interpolation to compute the two shortest-offset traces not present in the original data. The offset gathers were then converted to angle gathers. The purpose of having both positive and negative aperture angles is to see more clearly the position of the apexes of the diffracted multiples.

APEX-SHIFTED RADON TRANSFORM

In order to account for the apex-shift of the diffracted multiples (h), we define the forward and adjoint Radon transforms as a modified version of the "tangent-squared" Radon transform introduced by Biondi and Symes (2003). We define the transformation from data space (ADCIGs) to model space (Radon-transformed domain) as:

$$m(h,q,z') = \sum_{\gamma} d(\gamma, z = z' + q\tan^2(\gamma - h)),$$

and from model space to data space as

$$d(\gamma, z) = \sum_{q}\sum_{h} m(h,q,z' = z - q\tan^2(\gamma - h)),$$

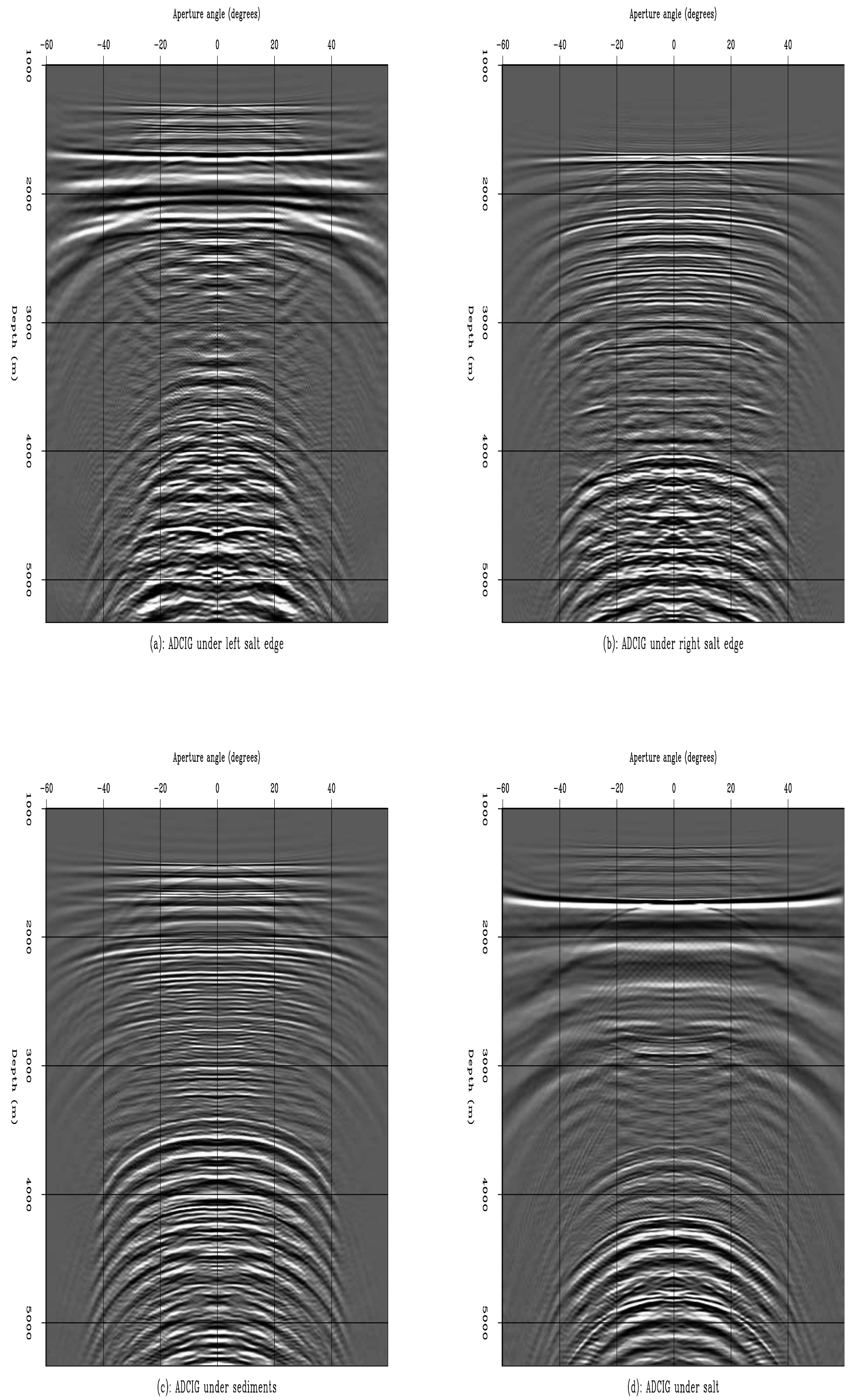


Figure 2: Angle domain common image gathers. (a) under the left edge of the salt, CMP at 6744 m; (b) under the right edge of the salt, CMP at 22056 m; (c) below the sedimentary section, CMP at 3040; (d) below the salt body, CMP at 12000 m. cags [CR]

where z is depth in the data space, γ is the aperture angle, z' is the depth in the model space, q is the moveout curvature and h is the lateral apex shift. In this way, we transform the two-dimensional data space of ADCIGs, $d(z,\gamma)$, into a three-dimensional model space, $m(z',q,h)$.

In the ideal case, primaries would be perfectly horizontal in the ADCIGs and would thus map in the model space to the zero-curvature ($q=0$) plane, *i.e.*, a plane of dimensions depth and apex-shift distance (h,z'). Specularly-reflected multiples would map to the zero apex-shift distance ($h=0$) plane, *i.e.*, a plane of dimensions depth and curvature (q,z'). Diffracted multiples would map elsewhere in the cube depending on their curvature and apex-shift distance.

SPARSITY CONSTRAINT

As a linear transformation, the apex-shifted Radon transform can be represented simply as

$$\mathbf{d} = \mathbf{L}\mathbf{m}, \tag{1}$$

where **d** is the image in the angle domain, **m** is the image in the Radon domain and **L** is the forward apex-shifted Radon transform operator. To find the model **m** that best fits the data in a least-squares sense, we minimize the objective function:

$$f(\mathbf{m}) = \|\mathbf{L}\mathbf{m}-\mathbf{d}\|^2 + \epsilon^2 b^2 \sum_{i=1}^{n} \ln\left(1+\frac{m_i^2}{b^2}\right), \tag{2}$$

where the second term is a Cauchy regularization that enforces sparseness in the model space. Here n is the size of the model space, and ϵ and b are two constants chosen a-priori: ϵ which controls the amount of sparseness in the model space and b related to the minimum value below which everything in the Radon domain should be zeroed (Sava and Guitton, 2003). The least-squares inverse of **m** is

$$\hat{\mathbf{m}} = \left[\mathbf{L}'\mathbf{L} + \epsilon^2 \mathbf{diag}\left(\frac{1}{1+\frac{m_i^2}{b^2}}\right)\right]^{-1} \mathbf{L}'\mathbf{d}, \tag{3}$$

where **diag** defines a diagonal operator. Because the model space can be large, we estimate **m** iteratively. Notice that the objective function in Equation (2) is non-linear because the model appears in the definition of the regularization term. Therefore, we use a limited-memory quasi-Newton method (Guitton and Symes, 2003) to find the minimum of $f(\mathbf{m})$.

A LOOK AT THE 3D RADON DOMAIN

In this section we will illustrate the mapping of the different events between the image space (z,γ) and the 3D Radon space (z',q,h) using the ADCIG at CMP 6744 m (Figure **??**a). This ADCIG has no discernible primaries below the salt, but nicely shows the apex-shifted moveout of the diffracted multiples.

Figure **??**a shows the $h = 0$ plane from the 3D volume. This plane corresponds to the zero apex-shift and is therefore similar to the standard 2D Radon transform. Primaries are mapped near the $q = 0$ line and specularly-reflected multiples are mapped to other q values. Diffracted multiples, since their moveout apex is not zero, are not mapped to this plane and so do not obscure the mapping of the specularly-reflected multiples. For comparison, Figure **??**b shows the 2D Radon transform, plotted at the same clip value as Figure **??**a. Notice how the diffracted-multiple energy is mapped as background noise, especially at the largest positive and negative q values. Notice also that the primary energy is higher than in Figure **??**a since with the 3D transform the primary energy is mapped not only to the $h = 0$ plane but to other h planes as well. This is further illustrated on Figure **??** which shows the zero-curvature ($q = 0$) plane for the same ADCIG. The nearly flat primaries have zero curvature for all values of the apex-shift distance h, so in this plane they appear as horizontal lines. Neither the specularly-reflected nor the diffracted multiples map to this plane.

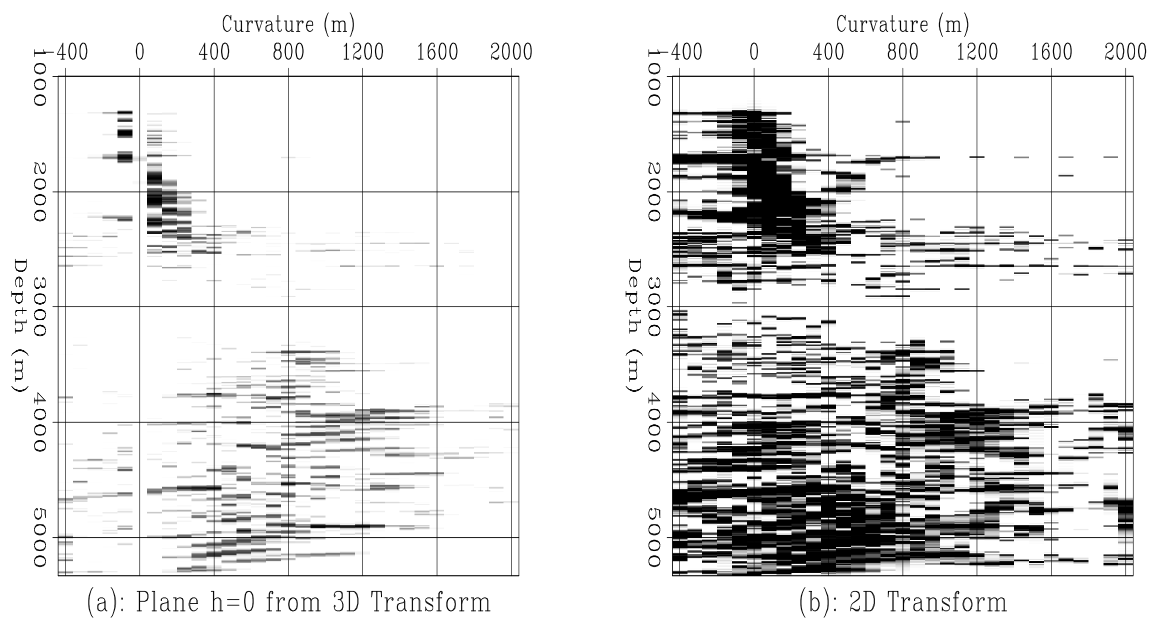


Figure 3: Radon transforms of the ADCIG in Figure **??**b. (a): 2D transform. (b): $h = 0$ plane of the apex-shifted 3D transform. env_comp [CR]

ATTENUATION OF DIFFRACTED AND SPECULARLY-REFLECTED MULTIPLES

With ideal data, attenuating both specularly-reflected and diffracted multiples could, in principle, be accomplished simply by zeroing out (with a suitable taper) all the q-planes except the one corresponding to $q = 0$ in the model cube $m(z', q, h)$ and taking the inverse apex-shifted Radon transform. In practice, however, the primaries may not be well-corrected and primary energy may map to a few other q-planes. Energy from the multiples may also map to those planes and so we have the usual trade-off of primary preservation versus multiple attenuation. The advantage now is that the diffracted multiples are well focused to their corresponding

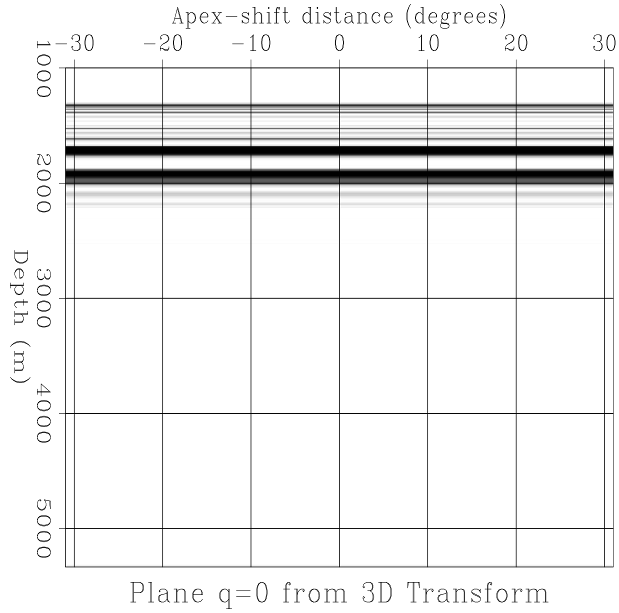


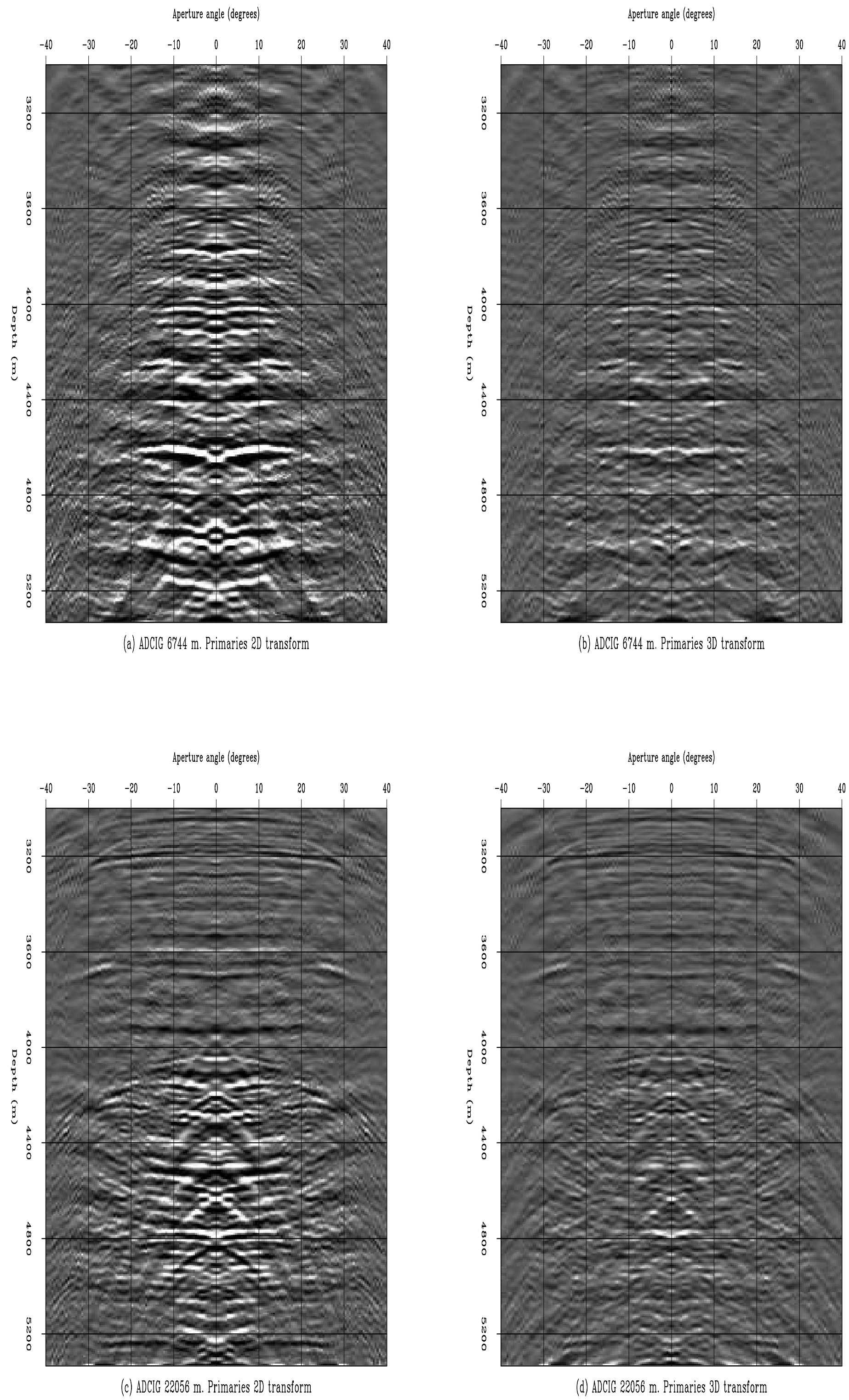
Figure 4: Zero curvature plane from 3D Radon transform of ADCIG in Figure **??**b. Flat primaries are mapped to the zero curvature line for all *h* values. env_qplane [CR]

h-planes and can therefore be easily attenuated. Rather than suppressing the multiples in the model domain, we chose to suppress the primaries and inverse transform the multiples to the data space. The primaries were then recovered by subtracting the multiples from the data.

Figure **??** shows a close-up comparison of the primaries extracted with the standard 2D transform (Sava and Guitton, 2003) and with the apex-shifted Radon transform for the two ADCIGs at the top in Figure **??**. The standard transform (Figures **??**a and **??**c) was effective in attenuating the specularly-reflected multiples, but failed at attenuating the diffracted multiples (below 4000 m), which are left as residual multiple energy in the primary data. Again, this is a consequence of the apex shift of these multiples. There appears not to be any subsalt primary in Figures **??**a and **??**b and only one clearly visible subsalt primary in Figures **??**c and **??**d (just above 4400 m). This primary was well preserved with both transformations.

Figure **??** shows a similar comparison for the extracted multiples. Notice how the diffracted multiples were correctly identified and extracted by the 3D Radon transform, in particular in Figure **??**b. In contrast, the standard 2D transform misrepresent the diffracted multiples as though they are specularly-reflected multiples as seen in Figure **??**a. We can take advantage of the 3D model representation to separate the diffracted multiples from the specularly-reflected ones. This is shown in Figure **??**. The diffracted multiples are clearly seen in Figure **??**c.

In order to assess the effect of better attenuating the diffracted multiples on the angle stack of the ADCIGs we processed a total of 310 ADCIGs corresponding to horizontal positions 3000 m to 11000 m in Figure **??**. Figure **??** shows a close-up view of the stack of the primaries extracted with the 2D Radon transform, the stack of the primaries extracted with the 3D Radon transform and their difference. Notice that the diffracted multiple energy below the edge of the salt (5000 m to 7000 m) that appears as highly dipping events with the 2D transform, has been attenuated with the 3D transform. This is shown in detail in Figure **??**c. It is very difficult to identify any primary reflections below the edge of the salt, so it is hard to assess if the



(a) ADCIG 6744 m. Primaries 2D transform

(b) ADCIG 6744 m. Primaries 3D transform

(c) ADCIG 22056 m. Primaries 2D transform

(d) ADCIG 22056 m. Primaries 3D transform

Figure 5: Comparison of primaries extracted with the 2D Radon transform (a) and (c) and with the apex-shifted Radon transform (b) and (d). Notice that some of the diffracted multiples remain in the result with the 2D transform. comp_prim1 [CR]

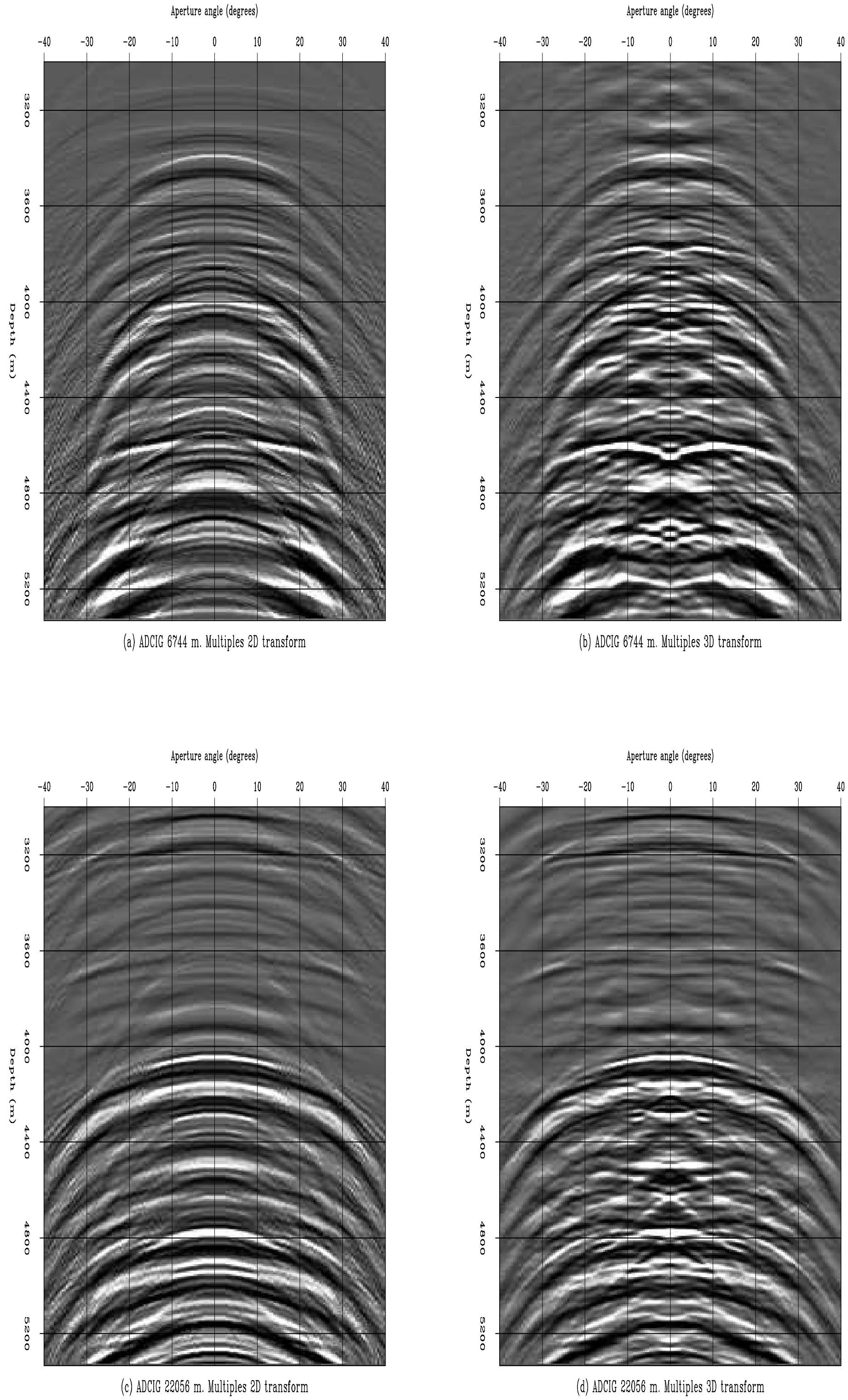


Figure 6: Comparison of multiples extracted with the 2D Radon transform (a) and (c) and with the apex-shifted Radon transform (b) and (d). comp_mult1 [CR]

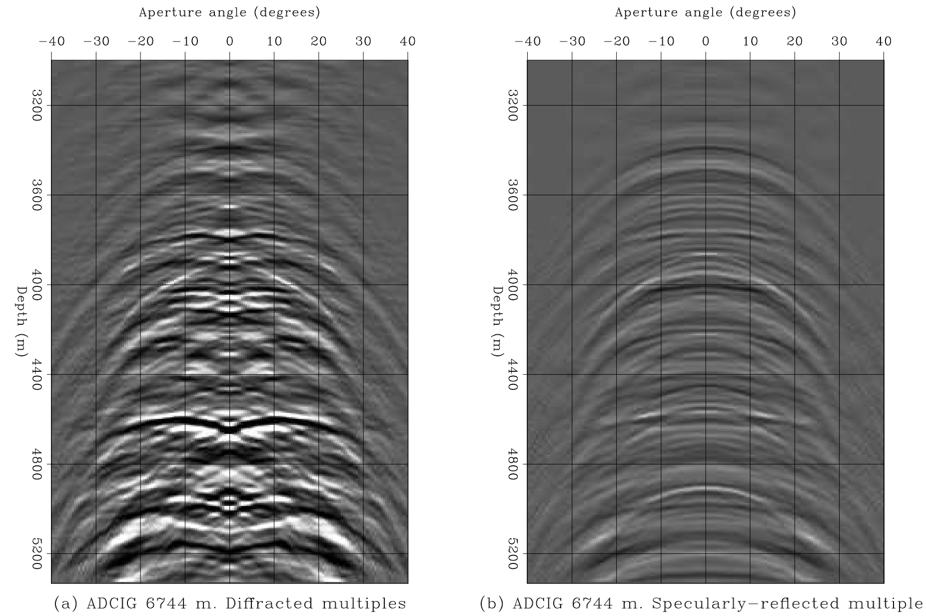


Figure 7: Comparison of (a) diffracted and (b) specularly-reflected multiples for the ADCIG in Figure **??**a. Notice the lateral shifts in the apexes of the diffracted multiples. comp_mult2 [CR]

primaries have been equally preserved with both methods. It is known, however, that for this dataset, there are no multiples above a depth of about 3600 m, between CMP positions 3000 m to 5000 m. The fact that the difference panel appears nearly white in that zone shows that the attenuation of the diffracted multiples did not affect the primaries. Of course, this is only true for those primaries that were correctly imaged, so that their moveout in the ADCIGs was nearly flat. Weak subsalt primaries may not have been well-imaged due to inaccuracies in the migration velocity field and may therefore have been attenuated with both the 2D and the 3D Radon transform.

For the sake of completeness, Figure **??** shows the extracted multiples with the 2D and the 3D Radon transform and their difference. Again, the main difference is largely in the diffracted multiples.

DISCUSSION

The results shown in the previous section demonstrate that with the 3D Radon transform it is possible to attenuate, although not completely remove, the diffracted multiples. It should be noted, however, that in this seismic section it is very difficult to find a legitimate primary reflection below the salt and in particular below the edge of the salt, where the contamination by the diffracted multiples is stronger. It is somewhat disappointing that the attenuation of the diffracted multiples didn't help in uncovering any meaningful primary reflections in this case. We expect the situation to be different with other datasets.

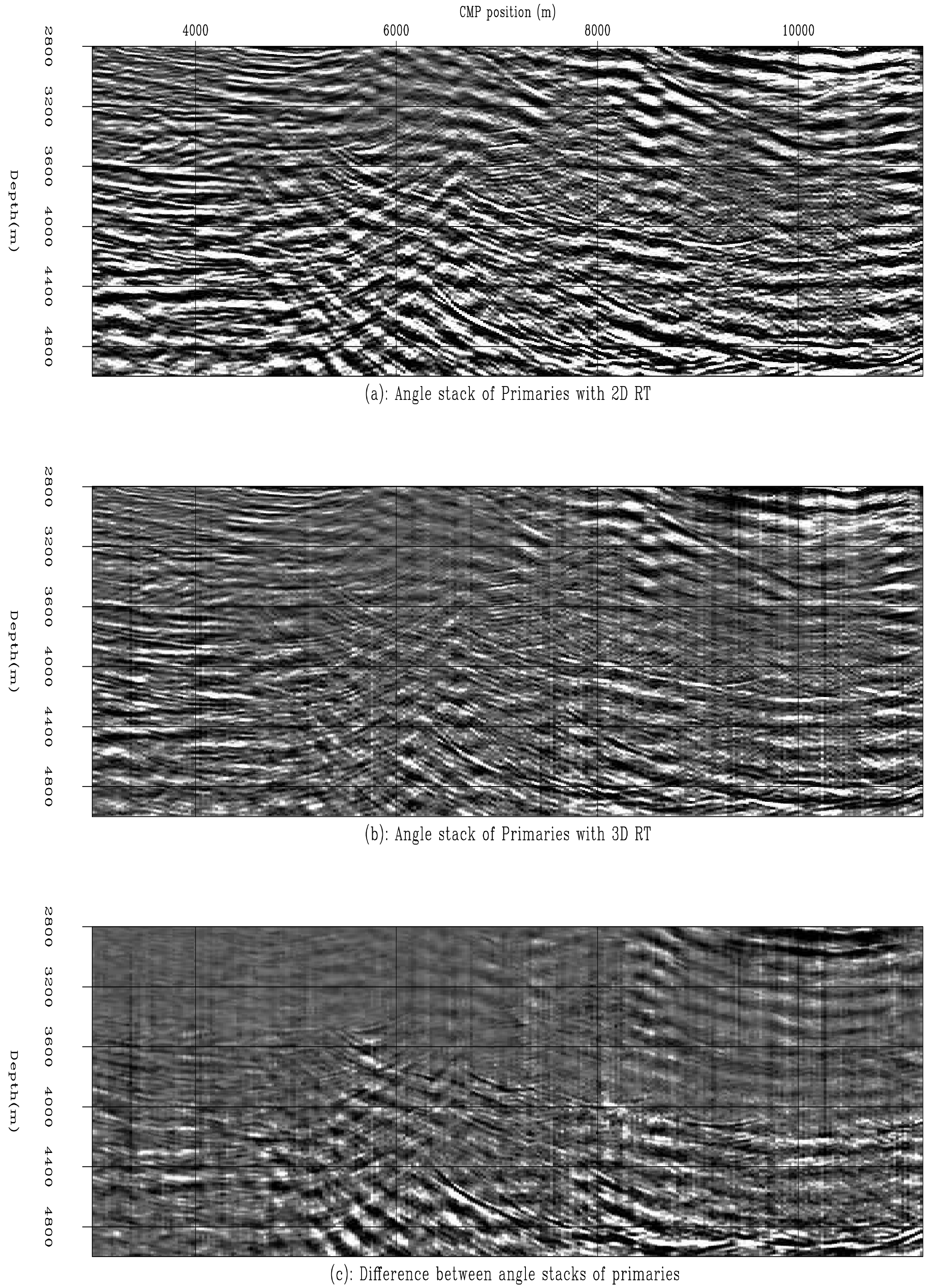


Figure 8: Comparison of angle stacks for primaries. comp_prim1_stack [CR]

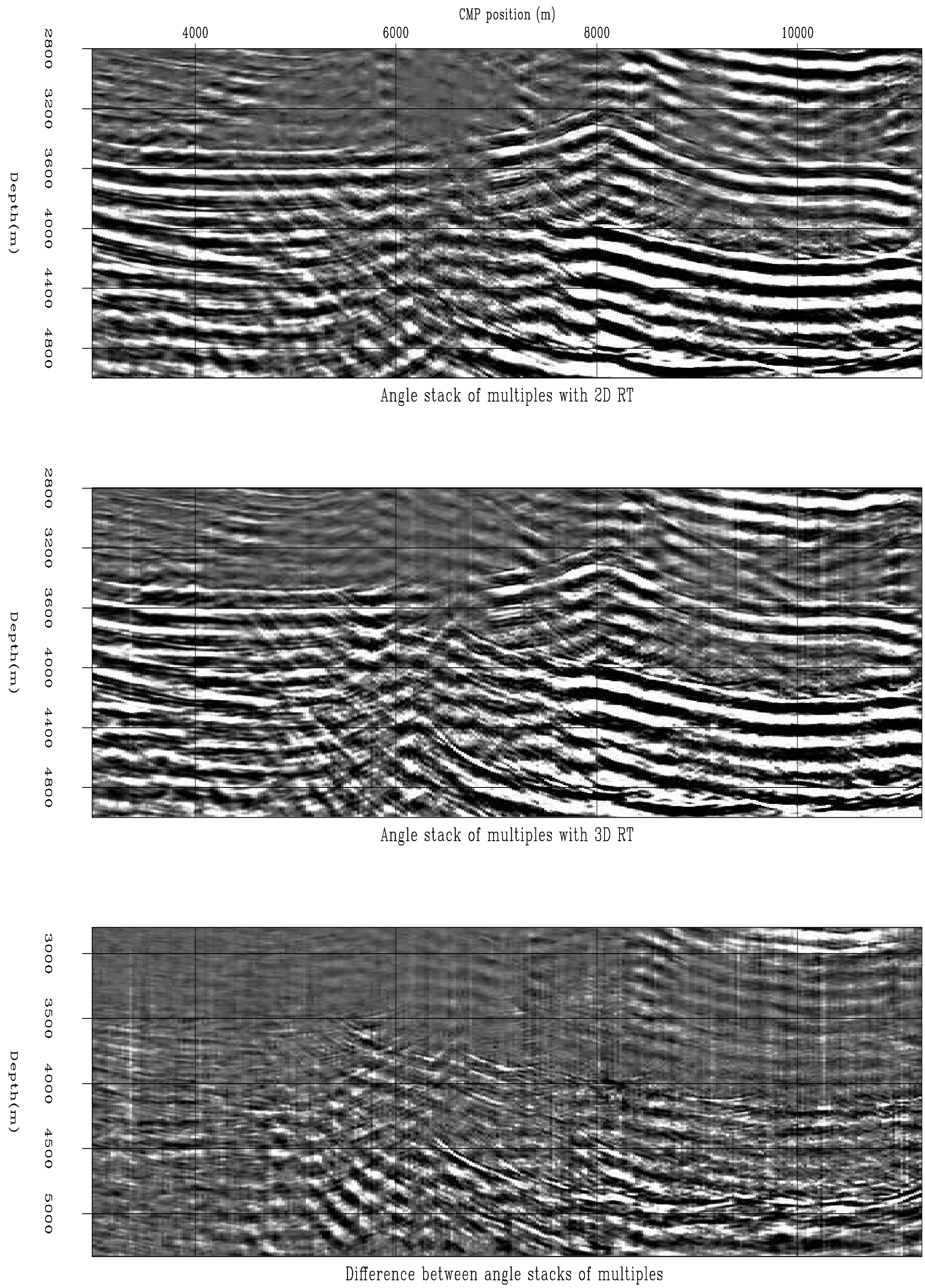


Figure 9: Comparison of angle stacks for multiples. comp_mult1_stack [CR]

We mentioned before that working in image space (ADCIGs in this case) is convenient because the migration takes care of the complexity of the wavefield propagation, but attenuating the multiples after migration does not come without a price. The estimation of the migration velocities may be more difficult and less accurate because of the presence of the multiples. There is therefore an inherent trade-off when choosing to work in image space. Good migration velocities for weak subsalt primaries may be particularly difficult to estimate in the presence of the multiples. On the other hand, the parabolic or hyperbolic assumption for the moveout of the multiples in data space may not be appropriate at all in complex media. An alternative could be to do a standard Radon demultiple before prestack migration to facilitate the choice of the migration velocities and 3D Radon demultiple on the ADCIGs to attenuate residual multiples, in particular diffracted multiples.

We should also emphasize that adding the extra dimension to deal with the diffracted multiples does not in itself resolve the usual problem that non-flat primaries may map to the multiple region and therefore we have to trade primary preservation for multiple attenuation. We saw this limitation in this case, which forced us to let some residual multiple energy leak into the extracted primaries. Obviously, the flatter the primaries in the ADCIGs, the better our chances to reduce the residual multiple energy.

CONCLUSION

The combination of choosing the image space in the form of ADCIGs and the apex-shifted tangent-squared transformation from (z,γ) to (z',q,h) has proven to be effective in attenuating specularly-reflected and diffracted multiples in 2D marine data. The residual moveout of both multiples in ADCIGs is well-behaved and the extra dimension provided by the apex-shift allows the attenuation of the multiples without compromising the integrity of the primaries.

ACKNOWLEDGMENTS

We thank the sponsors of the Stanford Exploration Project for their support to carry out this study and WesternGeco for providing the dataset.

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