Imaging overturned waves by plane-wave migration in tilted coordinates

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SUMMARY

We image overturned waves by decomposing the source and receiver wavefields into plane-waves. For each plane-wave, we extrapolate the source and receiver wavefields in a tilted coordinate system and cross-correlate them to obtain Common Image Gathers (CIGs). The tilting angle for the coordinate system is determined by the propagation direction of the plane wave. In tilted coordinates, the propagation direction is close to the extrapolation direction, so we can image steeply dipping reflectors and overturned waves with the one-way wave equation. We can also obtain robust, dip-dependent angle-domain CIGs (ADCIGs) by the same method as the one employed in reverse-time migration. These gathers provide moveout information and thus they are very important for velocity analysis on steeply dipping reflectors. Since our method is based on the one-way wave equation and it needs no padding for imaging overturned waves as needed by reverse-time migration, and so it is very efficient. We demonstrate our method on the Marmousi model by computing the Green's function for a point source on the surface. We also apply our method on a North Sea real dataset with overturned events.

INTRODUCTION

Downward continuation migration (Claerbout, 1985) gains more and more popularity with the continuing development of computer power. It is accurate, and can handle lateral velocity change and multi-pathing naturally. It is used to image complex geological structure, especially sub-salt reflectors. However, the dip angle of reflectors is limited in downward continuation, which makes it difficult to image steeply dipping reflectors and overturned waves. Downward continuation is based on the one-way wave equation and extrapolates the wavefields in the downward direction, although in reality waves propagate in all directions. When the propagation direction of waves is too far from the extrapolation direction, some of the energy will be lost.

Kirchhoff methods can handle steeply dipping reflectors and overturned waves, but they are based on a high frequency approximation and are less reliable to imaging complex geological structures, where multi-path events are present. Reverse-time migration (Whitmore, 1983; Baysal et al., 1983; Biondi and Shan, 2002), which is based on the two-way wave equation, can handle waves propagating in all directions, but it is still too expensive for today's computer power.

Many methods have been developed to handle waves that propagate at large angles to extrapolation direction. They solve the problem either by developing more accurate extrapolation operator to image high-angle waves, such as FFD (Ristow and Ruhl, 1994; Biondi, 2002) and GSP (de Hoop, 1996; Huang and Wu, 1996), or by making the extrapolation direction closer to the propagation direction, such as beam migration (Hill, 2001; Gray et al., 2002; Albertin et al., 2001; Brandsberg-Dahl and Etgen, 2003), beamlet migration (Chen et al., 2002) and coordinate transformation based methods (Zhang and McMechan, 1997; Etgen, 2002; Sava and Fomel, 2004). Our method is the latter one. We decompose both source and receiver wavefields into plane-waves and run plane-wave migration (Rietveld, 1995; Duquet et al., 2001; Liu et al., 2002; Zhang et al., 2003) on each of them within a tilted coordinate system.

Offset-domain CIGs for shot-profile migration and reverse-time migration are generated by cross-correlating the source and receiver wavefields with a horizontal shift (Rickett and Sava, 2002), and can be transformed into ADCIGs by slant stack (Sava and Fomel, 2003). However, CIGs obtained from downward continuation are contaminated by smearing noise at steeply dipping reflectors (Biondi and Shan, 2002). Reverse-time migration can provide both horizontal and vertical CIGs. Both are then merged into dip-dependent CIGs, which are robust and immune to the smearing noise (Biondi and Symes, 2003). By performing plane-wave migration in a tilted coordinates, we can obtain similar dip-dependent ADCIGs. These CIGs are robust and are useful for velocity analysis in presence of steeply dipping reflectors in the subsurface.

TILTED CARTESIAN COORDINATES

In Cartesian coordinates (x,z), the one-way wave equation is

$$\frac{\partial P}{\partial z} = \frac{i\omega}{v}\sqrt{1+\frac{v^2}{w^2}\frac{\partial^2}{\partial x^2}}P. \qquad (1)$$

It can be obtained by factoring the two-way acoustic wave equation. If the coordinates are rotated by an angle θ, the two-way acoustic wave equation doesn't change in the new coordinates (x',z'). Factoring the two-way wave equation in the tilted coordinates, we can obtain the same one-way wave equation. In the tilted coordinates, wavefields are now extrapolated in the z' direction, than in downward direction in downward continuation. And the source and receiver data are not on the line $z'=0$, but on $x'\sin\theta - z'\cos\theta = 0$ (Figure 1a).

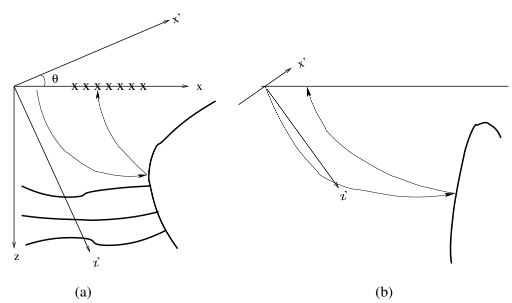

Figure 1: Tilted Cartesian coordinates.

For a point source, the waves propagate in all directions, making it impossible to cover all propagation directions with only one Cartesian coordinate system. We decompose the point source into plane waves and extrapolate each plane wave in a tilted coordinates, whose extrapolation direction z' is determined by the propagation direction of the plane wave. The propagation direction of the plane wave at the surface can be used as the extrapolation direction z' of the tilted coordinates, In this coordinates, the source and receiver wavefields of overturned waves can usually be caught, since the opening angle between them are usually small and the propagation directions of the source and receiver wavefields are close to each other (Figure 1b).

Another advantage of using a plane-wave rather than a point source is that no padding is needed to image steeply dipping reflectors. To catch the energy reflected by steeply dipping reflectors, the locations of sources and receivers are usually far from the reflector point (Figure 1b). Large extrapolation aperture is required to image these reflectors using a point source. In contrast, plane waves inherently have large apertures, which can easily cover the locations of source, receiver and reflector points. This greatly reduces the cost needed to image the steeply dipping reflectors.

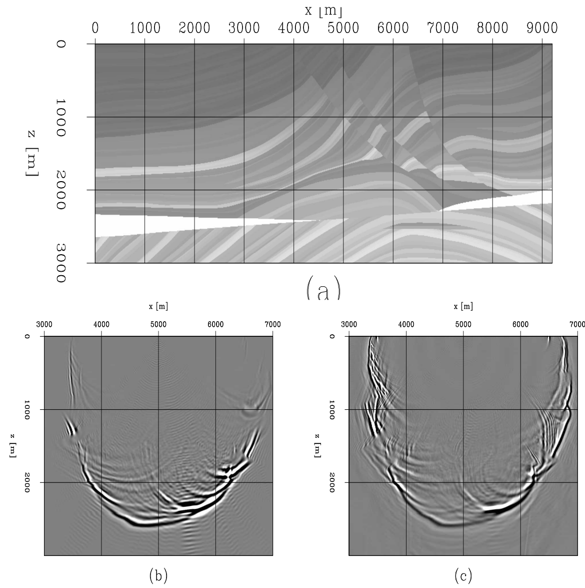

Figure 2: The Marmousi model: (a) velocity model; (b) wavefields obtained by downward continuation with FFD; (c) wavefields obtained by plane wave decomposition and extrapolation in tilted coordinates.

ANGLE-DOMAIN CIGS IN TILTED CARTESIAN COORDINATES

In shot-profile migration, offset-domain CIGs can be obtained by cross-correlating the source and receiver wavefields with a horizontal shift (Rickett and Sava, 2002). The offset-domain CIGs can be transformed into ADCIGs by slant-stack (Sava and Fomel, 2003). In reverse-time migration, in addition to horizontal offset-domain CIGs, we have vertical offset-domain CIGs, which are obtained by cross-correlating the source and receiver wavefields with a vertical shift (Biondi and Shan, 2002). Both horizontal and vertical offset-domain CIGs can be transformed into ADCIGs and merged into dip-dependent CIGs by transforming the horizontal and vertical offsets into apparent geological offset as follows (Biondi and Symes, 2003):

$$h_x = \frac{h_0}{\cos\alpha}, \quad (2)$$

$$h_z = \frac{h_0}{\sin\alpha}, \quad (3)$$

where α is the dip angle of the subsurface reflector, h_x is the horizontal offset, h_z is the vertical offset and h_0 is the apparent geological offset.

In the tilted coordinates, wavefields are extrapolated in the z' direction. The offset-domain CIGs are generated by cross-correlating the source and receiver wavefields with an x' direction shift. So the subsurface offset is in the x' direction. As with the apparent geological offset h_0, the x' direction offset $h_{x'}$ can be transformed to horizontal and vertical offsets as follows

$$h_x = \frac{h_{x'}}{\cos\theta}, \quad (4)$$

$$h_z = \frac{h_{x'}}{\sin\theta}, \quad (5)$$

where θ is the tilting angle in Figure 1. As for reverse-time migration, horizontal and vertical offset-domain CIGs can be transformed into ADCIGs and merged into dip-dependent ADCIGs. A simple way is to merge them with the following weights:

$$w_{h_x} = \cos^2\alpha, \quad (6)$$

$$w_{h_z} = \sin^2\alpha, \quad (7)$$

where α is the apparent dip angle of the reflector. Dip-dependent residual moveout (Shan and Biondi, 2003) can be used to analyze dip-dependent ADCIGs to provide useful moveout information for velocity analysis.

In tilted coordinates, the direction of subsurface offsets is close to that of apparent geological offset, since the extrapolation direction of the wavefields is close to the propagation direction of the waves. Within a limited length of subsurface offsets, we can obtain much more accurate CIGs at steeply dipping reflectors than with standard downward continuation.

EXAMPLES

Our first example concerns waves propagating in the Marmousi model. Figure 2 presents (a) the velocity of the Marmousi model, (b) the wavefields obtained by downward continuation with Fourier finite difference (FFD), and (c) the wavefields obtained by plane-wave decomposition and extrapolating each plane wave with FFD in a tilted coordinates. The source location is at $x = 5000$m. In Figure 2b, although we extrapolate the wavefield with a wide-angle extrapolation operator, the wavefield is not accurate for large incident angles. In contrast, Figure 2c does not have this angle limitation and the wavefield is extrapolated better for high-angle waves.

Our second example images the overturned waves of a 2-D line from the North Sea dataset. This dataset presents challenges due to the interaction between the salt edge and a chalk layer. The migration velocity is a smooth version of a tomographic result obtained by inverting the moveouts measured on ADCIGs computed from downward-continuation prestack migration (Clapp, 2001). The image from Reverse-time migration (Biondi and Shan, 2002) shows the presence of overturned energy. Figure 3 shows (a) the image obtained by downward continuation, (b) the image obtained by reverse-time migration, (c) the image obtained by plane-wave migration in tilted coordinates and (d) the rays shot at the steeply dipping reflector. There are two steeply dipping reflectors at the edge of the salt, and one of them is almost vertical. Rays from these two reflectors (Figure 3d) show that the waves are overturned. Downward continuation (Figure 3a) doesn't image these two steeply dipping reflectors, failing to handle overturned energy. In contrast, both reverse-time prestack migration (Figure 3b) and plane-wave migration in tilted coordinates (Figure 3c) preserve the overturned energy and image the steeply dipping reflectors. The images of the steep reflectors by these two methods are comparable, but the cost is different.

Figure 4 compares the horizontal CIGs at the surface location $x = 4000$m from plane-wave migration in tilted coordinates with those from reverse-time migration. Figure 4a and Figure 4c show offset and angle domain CIGs from reverse-time migration, respectively. Figure 4b and Figure 4d show offset and angle domain CIGs from plane-wave migration in tilted coordinates, respectively. The ADCIGs from plane-wave migration in tilted coordinates look similar to those from reverse-time migration.

Figure 5 compares the horizontal CIGs at the surface location $x = 5000$m from plane-wave migration in tilted coordinates with those from reverse-time migration. Panels are organized as in Figure 4. The vertical salt edge is located at surface location x=5000m, in the vertical interval $1500 < z < 2000$m. At the vertical salt edge, we show that the ADCIGs from reverse-time migration and from plane-wave migration in tilted coordinates are similarly smeared

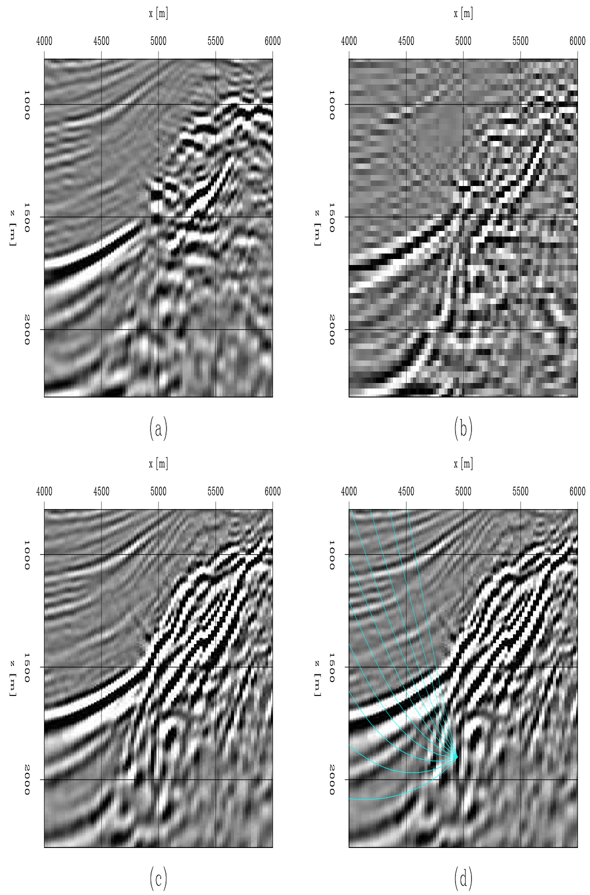

Figure 3: Migration of North Sea real dataset: (a) the image from downward continuation; (b) the image from reverse-time migration; (c) the image from plane-wave migration in tilted coordinates; (d) the rays shot at $x = 5000$m, $z = 1900$m.

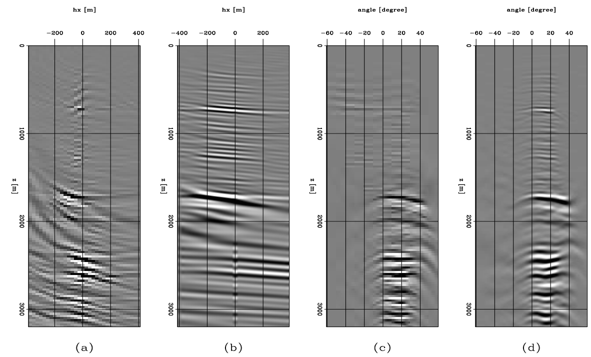

Figure 4: Horizontal CIGs at $x = 4000$m: (a) offset-domain CIGs obtained by reverse-time migration; (b) offset-domain CIGs obtained by plane-wave migration in tilted coordinates; (c) ADCIGs obtained by reverse-time migration; (d) ADCIGs obtained by plane-wave migration in tilted coordinates.

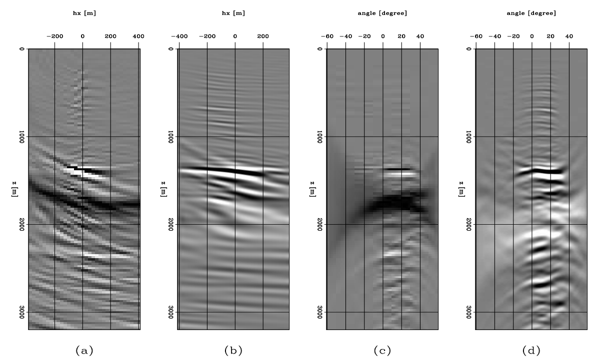

Figure 5: Horizontal CIGs at $x = 5000$m: (a) offset-domain CIGs obtained by reverse-time migration; (b) offset-domain CIGs obtained by plane-wave migration tilted coordinates; (c) ADCIGs obtained by reverse-time migration; (d) ADCIGs obtained by plane wave migration in tilted coordinates.

along the angle axis. This smearing noise makes horizontal CIGs a poor source of the information for velocity analysis at these steeply dipping reflectors.

Figure 6 compares the vertical CIGs at depth $z = 1900$m from plane-wave migration in tilted coordinates with those from reverse-time migration. Panels are organized as in Figure 4 but in the vertical direction. The vertical ADCIGs from both two methods are similar. In contrast to horizontal CIGs, vertical CIGs focus at the vertical salt edge and can provide useful information for velocity analysis at the salt edge.

Figure 7 shows the dip-dependent ADCIGs obtained by merging horizontal and vertical ADCIGs with the weights in equation (6) and (7). Comparing with horizontal and vertical ADCIGs, we find that dip-dependent ADCIGs are more robust. The smearing noise is avoided and both horizontal and vertical reflectors are well focused.

CONCLUSION

We image the overturned waves by decompose the source and receiver wavefields into plane waves. Each plane wave is extrapolated in tilted Cartesian coordinates. Offset-domain CIGs are generated by cross-correlating the source and receiver wavefields, with a shift in the direction normal to the extrapolation direction. The offset-domain CIGs are decomposed into horizontal and vertical CIGs, which are merged into robust, dip-dependent ADCIGs.

We apply our method to a North Sea dataset and compare the image with the ones obtained from downward continuation and reverse-time migration. The image obtained by plane-wave migration in tilted coordinates is better compared with the image obtained from standard downward continuation. The steeply dipping salt edge are imaged by one-way plane-wave migration in tilted coordinates, but it is missing in the image obtained by downward continuation. The plane-wave migration results are also comparable to those obtained by reverse-time migration and produces similar horizontal and vertical CIGs. The dip-dependent ADCIGs merged from horizontal and vertical CIGs are robust and provide useful moveout information for reflectors with a wide range of dips.

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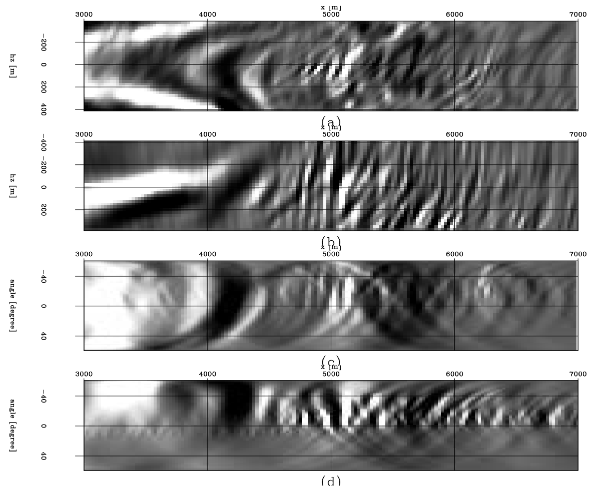

Figure 6: Vertical CIGs at $z = 1850$m: (a) offset domain CIGs obtained by reverse-time migration; (b) offset domain CIGs obtained by tilted coordinates plane wave migration; (c) ADCIGs obtained by reverse-time migration; (d) ADCIGs obtained by tilted coordinates plane wave migration.

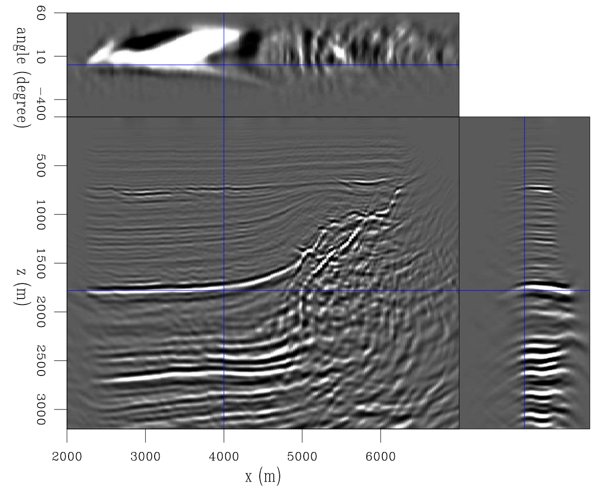

Figure 7: Dip-dependent ADCIGs: Notice that both horizontal and vertical reflectors are focused very well.

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