## Wave Equation Imaging Comparisons: Survey Sinking vs. Shot Profile Methods

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### Summary

We examine the differences and similarities between the two common categories of wave-equation migration by looking at their mathematical formulations and examining their imaging results. We compare different implementations of shot profile and double square root downward continuation imaging in the context of the overall imaging challenge, including image accuracy, velocity model building, and the ability to generate prestack gathers. We examine the strengths and advantages of the methods by considering the approximations that go into them, the resulting images, and the relative costs of the methods.

## Introduction

The challenges of exploration in increasingly complex geological areas has led to increased industry interest in wave-equation depth migration methods (Bevc and Biondi, 2002; Ritchie, 2003). Commonly available wave-equation migration solutions available in the industry are those based on Survey Sinking or Double Square Root downward continuation (DSR) and Shot Profile Migration (SPM). It has been shown mathematically that properly implemented DSR and SPM have equivalent accuracy and produce equivalent results (Wapenaar and Berkhout, 1987; Biondi, 2002).

The advantage of DSR methods is that they have significant potential for speedup, and some methods such as Common Azimuth Migration (Biondi and Palacharla, 1996), are 60 times faster than competing Shot Profile implementations. In the Common Azimuth (ComAz) implementation, the speedup comes from exploiting the observation that marine streamer data are acquired over a narrow azimuth range, and that the five-dimensional downward continuation operator can be approximated to a high-degree of accuracy by analytically removing crossline offset wavenumber dependency to obtain a four-dimensional downward continuation operator.

While ComAz produces excellent results (Fliedner et al., 2002), the Common Azimuth approach may not correctly position steeply dipping reflections at high strike angles to the acquisition geometry; the worst case being at 45 degrees strike to streamer direction. To solve this potential shortcoming, Biondi developed the so-called Narrow Azimuth migration, which retains enough crossline offset wavenumbers to overcome the possible shortcomings of ComAz (Biondi, 2001). This allows the downward continuation of enough crossline offset wavenumbers, to adequately sample the full crossline azimuth wavenumber range in the data.

## Wave-equation migration methods

The resurgence in popularity of wave-equation methods in 3-D has been spurred by two factors: (1) Clever algorithms, and (2) fast and cheap computers. Wave equation methods can be generally grouped by the classification of their computational domain (shot-profile, source-receiver, survey sinking, plane wave) and by the numerical method used to extrapolate or downward continue the wavefield (finite difference, frequency domain, generalized screen propagator (GSP), Fourier finite difference (FFD), etc.). In addition, wave equation methods can be solutions of the two-way wave equation (reverse time) or the one-way wave equation. Two-way wave equation methods are computationally more expensive, although they do promise potential advantages for imaging overturned rays. In this discussion, we limit ourselves to more commonly available solutions based on one-way wave equation downward continuation, and we look at the differences in wave equation methods based on the characterization of their computational domain. In terms of classification by numerical extrapolation method, we simply assert that any choice of migration method must incorporate an extrapolator that uses a high-order efficient extrapolator capable of handling strong lateral velocity variations and steep dips - most commercial applications should incorporate these essential elements, and the technical literature is full of detailed analysis of the various methods.

# Shot profile compared to double square root downward continuation

One of the most common computational domain divisions between wave equation methods in the industry today is that between shot-profile migration (SPM) and sourcereceiver migration. Source-receiver is also commonly referred to as survey sinking or double-square root (DSR) method, and despite the name, is commonly applied in the midpoint-offset domain (Claerbout, 1985, Popovici, 1996).

To understand the two methods, we briefly outline how they work. In shot profile migration, each shot record is migrated individually into an image volume by:

- 1. downward continuing the receiver wavefield,
- 2.downward continuing the source (i.e. modeling the shot), and
- 3.imaging by cross correlation of the two wavefields and extracting the zero lag.

Source receiver downward continuation is performed by applying the DSR equation at each depth step to simultaneously:

1. downward continue the receiver wavefield, and

2. downward continue the source wavefield,

at each depth step, the imaging condition is applied by extracting the wavefield at zero time and zero offset.

The observant reader will note that steps one and two are similar. In fact, the downward continuation of receiver and source wavefield commutes, and the order can be rearranged. With some manipulation of equations, it can be shown that the two methods are mathematically equivalent (Wapenaar and Berkhout, 1987; Biondi, 2002). Therefore, shot-profile and source-receiver downward continuation are theoretically equivalent. This means, that properly implemented, the two methods should yield equivalent accuracy and comparable imaging results. The difference then becomes purely an engineering issue, and as we describe below, source-receiver methods offer significant opportunities for algorithmic efficiency based purely on the computational domain.

Two of the first economically feasible implementations of wave-equation migration were common azimuth migration (Biondi and Palacharla, 1996) and offset plane-wave migration (Mosher et al., 1997). Biondi's implementation exploits the fact that most marine data are acquired in streamer geometry that is very nearly zero azimuth, or can be easily corrected to zero azimuth using an azimuth moveout operator (Biondi et al., 1998). This results in a 4-D downward continuation that is extremely efficient, and is 60 times faster than the equivalent 5-D downward continuation that does not take into account the streamer geometry and the common azimuth approximation. For areas where the common azimuth approximation may be in question, this same approach can be used in a narrow or wide azimuth formulation by including some crossline offset wavenumbers in the downward continuation. The downward continuation propagator applied in common azimuth and plane wave migration is commonly some form of an extended split-step method or generalized screen propagator. Properly applied, these propagators are capable of imaging steep dips in the presence of strong lateral velocity variations.

Shot profile migration is commonly applied using a finite difference propagator and a cross-correlation imaging condition (Jacobs, 1982). The shot profile approach is a full 5-D downward continuation (shot x,y, receiver x,y, and z), and therefore requires much more cpu than common azimuth or narrow azimuth migration. Its obvious advantage is that it retains all data azimuths, so it is better suited to many land and ocean-bottom cable acquisition geometries. To get around the extreme computational cost of shot profile migration, many practitioners decimate the input data and/or reduce crossline and inline migration aperture in order to make shot profile migration economically feasible for marine streamer data. The disadvantage of decimating the shots in shot profile

migration is particularly evident in the quality of prestack volumes for migration velocity analysis or amplitude variation with angle. Even if a decimation factor of 1 to 10 produces little deterioration in the stacked image (particularly on synthetics) it creates a huge problem in the prestack image. The danger of limiting aperture in shot profile migration is that important information is lost. Restricting aperture in shot profile migration (or more precisely stated, the volume into which the shot record is extrapolated) can severely limit steep dip resolution.

ComAz is based on the observation that marine streamer data are collected along relatively narrow streamer arrays, and makes the assumption that multi-streamer data can be represented by an equivalent (after rebinning or azimuth moveout) data set that is purely zero azimuth. The method further assumes that migrated energy does not rotate in azimuth during the downward continuation process of migration imaging. These assumptions are generally good, but an exception occurs for the case of steeply dipping imaging targets that are at 45 degrees azimuth to the data acquisition geometry. Under these conditions the Common Azimuth assumptions break down, and the resulting image is degraded (this image degradation is typically manifest as a reflector mispositioning or as an apparent velocity error).

Narrow Azimuth Migration (NarAz) addresses this particular issue by allowing the data to retain the narrow azimuth range with which it is acquired. Instead of assuming that the data are all zero-azimuth and are not allowed to rotate during downward continuation, NarAz assumes data are acquired over a narrow crossline azimuth range, and that the data are allowed to rotate over the given azimuth range. When NarAz is implemented to allow an adequate number of crossline azimuths (typically from three to sixteen), it will capture all recorded propagation events and image them accurately for a computational cost that is substantially less than that of SPM.

Aside from the significant (order of magnitude) speed issue, Common Azimuth and Narrow Azimuth migration have substantial advantages in terms of amplitudes for attribute analysis (Sava et al., 2001), and the ability to generate angle gathers at no additional cost for migration velocity analysis and residual moveout (Liu et al, 2001).

The greater speed of processing offered by NarAz over SPM translates into shorter turn-around times. Provided the turn-around times are sufficiently short, the processing, depth imaging and perhaps the interpretation phases of a 3D seismic survey may allow for several, very significant iterations and consequently better results. This latter speedof-processing advantage and access to much larger blocks of survey data may enable a significant change in imaging, target definition and characterization. This is not feasible with conventional older and slower algorithms (such as SPM).

### Examples

The Sigsbee synthetic data set produced by the SMAART JV simulates a deepwater Gulf of Mexico imaging objective with steep dips and significant velocity contrast (Figure 1). SPM and DSR migrations of Sigsbee are presented in Figures 2 and 3. SPM uses a Hale-McClellan finite difference propagator while DSR uses an extended split-step high-order propagator. Both migrations produce similar results, imaging subsalt sediment, diffractor targets, and flat reflector targets. Steep dips are better imaged in the DSR result, and runtime for DSR on 3-D data is typically 60 times faster than SPM.

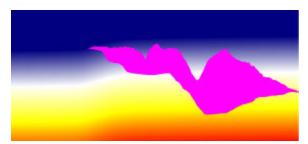


Figure 1. Sigsbee migration velocity.

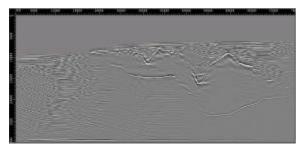


Figure 2. SPM image of Sigsbee.

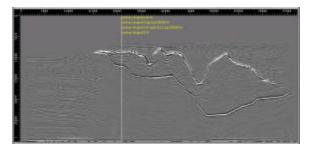


Figure 3. DSR migration image of Sigsbee. The vertical line shows the location of gathers of Figure 4.

One of the major advantages of DSR migration is the ability to generate angle domain image gathers at no additional computational cost (Prucha et al., 1999). Figure 4 illustrates an angle domain image gather extracted from the Sigsbee DSR migration in a region with salt, and Figure 5 illustrates a gather in a region without salt. The figures illustrate that mutes and prestack postmigration processing can be easily applied to these ACIGs. By contrast, gathers generated by SPM (Figures 6 and 7) do not lend themselves as well to muting and prestack image enhancement. The gathers in Figures 6 and 7 are extracted at different locations, have different salt thickness, and different mute patterns. Picking mute patterns and distinguishing between signal, noise, and artifact is not as straight forward in the SPM gathers as in ACIGs or offset gathers. Similarly, the SPM gathers are not as useful for attribute analysis and MVA as ACIGs or offset gathers. ACIGs or offset gathers can be generated from the SPM gathers, but at substantial computational cost. Generating offset gathers from SPM is typically 6 times the computational cost of straight migration, and generating angle gathers equivalent to Figures 4 and 5 is typically 2 to 3 times the computational cost (Stork et al., 2002).

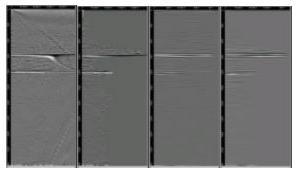


Figure 4. Angle-domain image gathers from DSR migration in a region with salt. Raw gather on the left, followed by muted gather, postmigration processed gather, and postmigration processed muted gather.

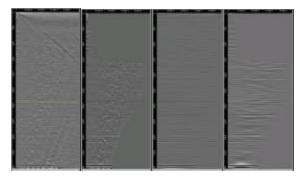


Figure 5. Angle-domain image gathers in a part of the model without salt. Raw gather with noise spikes on the left, followed by

muted gather, postmigration processed gather, and postmigration processed muted gather.

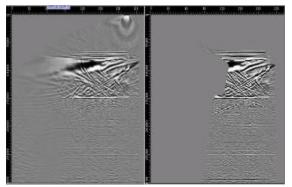


Figure 6. Migrated shot profile for Sigsbee before and after mute.

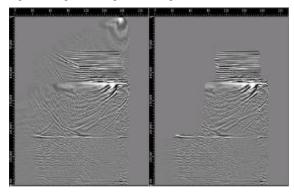


Figure 7. Migrated shot profile for Sigsbee before and after mute.

#### Conclusions

Based on algorithmic considerations and imaging results, there are different areas of applicability for different imaging formulations. Shot profile wave equation algorithms are well suited for land and ocean bottom data, while DSR-based wave equation migration is best for marine streamer data. The prospective explorationist should be aware of the strengths and weaknesses of the various imaging methods, the approximations and assumptions that are invoked, and what effect these will have on the desired outcome.

Considerations when selecting wave-equation migration include:

- Proper preprocessing regularization (AMO)
- Correct amplitude treatment
- High order extrapolation for downward continuation
- Handling of lateral velocity variations accurately
- Inclusion of all recorded data in the migration
- Aperture in shot profile to capture steep dips

The key to depth imaging is the velocity model, and to get the correct velocity model, it is critical to be able to output prestack gathers so that wave equation MVA can be performed with angle or offset domain image gathers. It is also critical to perform the velocity updating in a manor that is consistent with the migration engine that will be ultimately used for the final image, and to be able to perform as many iterations as are necessary.

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