Data regularization by Inversion to Common Offset (ICO)

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SUMMARY

Driven by economic constraints, 3D surveys typically have sparse and irregular geometry that results in spatial aliasing. In this paper we present a new approach for imaging irregularly sampled 3D prestack data. The strategy is to regularize the coverage of 3D surveys and reduce the size of 3D prestack data by partial stacking. After regularization, 3D data become handy for prestack migration using any wave extrapolation methods including finite-differencing and wave-number domain techniques.

Posing partial stacking as an optimization process, we develop a new dealiasing technique named "inversion to common offset" (ICO) that solves for regularly-sampled models from irregularly sampled multifold 3D data. The matrix that relates the data to the reflectivity model is the azimuth-moveout operator (AMO). The technique can be viewed as a generalization of the inversion to zero offset (IZO) discussed by Ronen (1985). The main advantage of ICO, is that the modeling operator, AMO, is not restricted to a particular azimuth or offset. The model, in general, simulates a regular common-azimuth common-offset experiment. AMO is also very compact and consequently cheaper to apply than other wave-equation processes such as prestack depth migration. Since ICO is applied to normal-moveout corrected data, the iterative solution is less sensitive to the velocity field in comparison to least-squares migration.

We present the results of applying ICO to a field 3D land survey to regularize the geometry of the data and reduce the costs of its prestack imaging. The images obtained by prestack migration after regularization are superior to those obtained by migrating the irregularly sampled data. Furthermore, ICO provides a promising approach for reducing the costs of 3D acquisition by collecting data with sparse offset sampling.

INVERSION TO COMMON OFFSET

The offset dimension adds important aspects to reflection seismology. Mainly, it provides robust analysis of the velocity of seismic waves and enables enhancement of signal-to-noise ratio by stacking. We use the offset and azimuth dimensions to pose partial staking with AMO (Biondi et al., 1998) as an optimization process to regularize the coverage of 3D surveys. We define processing as the inverse of modeling irregularly sampled data from a regularly sampled model (Ronen, 1994; Chemingui and Biondi, 1997). The inverse of AMO-staking is AMO mapping from a regularly sampled common-azimuth and common-offset cube into an irregularly sampled data with a range of offsets and azimuths. The mapping represents a linear transformation based on integral formulation of AMO. The representation of the integral as discrete summation reduces to a matrix-vector multiplication. The relation between data and model is then given by the linear system of equations:

$$\mathbf{d} = \mathbf{L}\mathbf{m}. \tag{1}$$

where the vector **d** represents the irregular input data, **m** represents a regularly sampled model and **L** is the AMO operator. In general **L** can be any full or partial modeling operator.

Equation (1) represents a forward modeling relation, where the goal of processing is to perform the inverse of these calculations, i.e., to find models from the data. Mathematically, this is equivalent to estimating the inverse of **L** by solving the set of equations in (1). One way to solve such a system is to look for a solution that minimizes the average error in the set of equations. This minimization can be done in a least-squares sense where the norm $\|Lm - d\|_2$ is minimized. The choice of m that makes this error a minimum gives the least-squares solution

$$m = (L^T L)^{-1} L^T d. \tag{2}$$

PRACTICAL IMPLEMENTATION OF ICO

The inversion of 3D multichannel seismic data using wave-equation techniques is generally a cost prohibitive solution. The main reason for the high cost is the size of the prestack data and the time-variability of wave equation processes. The breakthrough with ICO is that the modeling operator, AMO, is very compact and the cost of applying it is negligeable compared to prestack migration. We present some implementation techniques and concepts from inverse theory that enable cost effective implementation of ICO on 3D data.

Log-stretch Fourier transformation

Bolondi et. al. (1984) described a logarithmic stretching of the time axis that can convert a non-Fourier transform implementation to a Fourier transform combined with a phase shift. The log-stretch transform makes AMO a time-invariant operator, which means it only depends on the difference between the input and output time. A transformation of the log-stretched data to the Fourier domain is then a convenient way to process the data in the $\Omega - X$ space. Furthermore, since each frequency inversion is completely independent, one needs to solve many small systems instead of solving one huge system of equations. In practice, several frequency bands from a useful bandwidth of the data are processed in parallel.

Iterative solution for the pseudo-inverse

For each frequency component (or bandwidth) the systems of linear equations to be solved is still very large and one has to resort to iterative methods. This solves a huge set of simultaneous equations without the need to write down the matrix of coefficients. We use an iterative scheme based on the conjugate gradient solver. The algorithm generates a sequence of approximate solutions whose computations each involve the application of the adjoint followed by the forward operator. Both operations are AMO transformations and their computation is therefore reasonably cheap.

Diagonal weighting preconditioning

The convergence rate of the conjugate gradient algorithm depends on the condition number of the matrix to be inverted. For ill-conditioned matrices a preconditioner is often necessary. The design of a good preconditioner depends directly on the structure of the matrix. In the inversion relation (1), the number of equations is the number of traces in the input data and the number of unknowns is the number of output traces or bins. Since **L** is a Kirchhoff matrix, each row of **L** corresponds to a summation surface and each column corresponds to an impulse response. Due to irregular sampling, the rows and columns of **L** are badly scaled. To balance the coefficients of the matrix we apply a diagonal transformation to normalize its rows and columns. This involves pre- or post-multiplying the operator **L** by a diagonal matrix whose entries are the inverse of the sum of the rows or columns of **L**. The sum is always positive since Kirchhoff operators are associated with matrices that contain no negative elements. The diagonal weighting is essentially a calibration by a flat event, which is the operator's response to an input vector with all components equal to one.

Row-scaling preconditioning

Row scaling is equivalent to pre-multiplying the matrix **L** and the data vector **d** by a diagonal matrix $\mathbf{R^{-1}}$ and solving the system:

$$\mathbf{R}^{-1}\mathbf{d} = \mathbf{R}^{-1}\mathbf{L}\mathbf{m} \tag{3}$$

Since each row corresponds to a summation surface (impulse response of $\mathbf{L}^T$), $\mathbf{R^{-1}}$ is normalization of the data by the coverage before AMO.

Column-scaling preconditioning

This approach is based on a post-multiplication of the matrix **L** by a diagonal matrix $\mathbf{C^{-1}}$. The preconditioning operator introduces a new model $\mathbf{x}$ given by

$$\mathbf{x} = \mathbf{Cm} \tag{4}$$

By the preconditioning transformation, we have recast the original inversion relation (1) into

$$\mathbf{d} = \mathbf{LC}^{-1}\mathbf{x}. \tag{5}$$

After solving for $\mathbf{x}$ we easily compute $\mathbf{m} = \mathbf{C}^{-1}\mathbf{x}$.

Given that each column of $\mathbf{L}$ corresponds to an output bin, $\mathbf{C}^{-1}$ is normalization of the model by the coverage after AMO.

Row and column scaling

Proper balancing of the matrix $\mathbf{L}$ can be achieved by scaling in both data space and model space. However, applying either diagonal transformation ensures common magnitude of the elements of $\mathbf{L}$. The diagonal operators $\mathbf{R^{-1}}$ and $\mathbf{C^{-1}}$ have physical units inverse to $\mathbf{L}$. Therefore applying both of them results into an ill-conditioned system where the matrix $\mathbf{L}$ has the inverse of its original units. The solution is to scale the matrix $\mathbf{L}$ by the *square root* of $\mathbf{R^{-1}}$ and $\mathbf{C^{-1}}$ and solve the transformed system:

$$\mathbf{R}^{-1/2}\mathbf{d} = \mathbf{R}^{-1/2}\mathbf{LC}^{-1/2}\mathbf{x}. \tag{6}$$

As Figure 1 shows, the diagonal transformation has proved to be a suitable preconditioner for the linear system. The column scaling improved the convergence of the iterative solution and resulted into better convergence than the row scaling. The best convergence was achieved by properly scaling both the data space and model space. A good solution was obtained after 5 to 8 iterations of conjugate gradient solver.

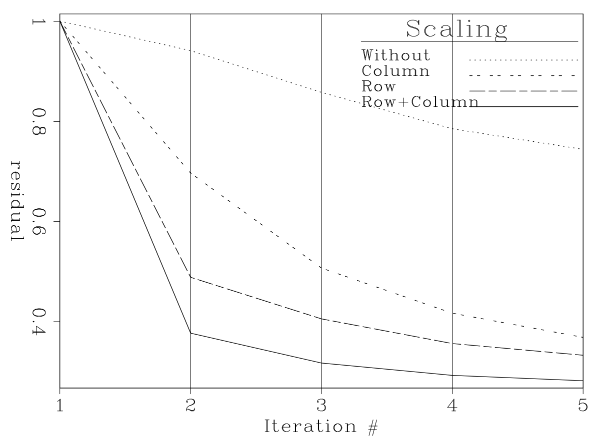

Figure 1: Convergence of ICO for one frequency inversion using different preconditioners.

Regularization of the inversion

In order to compute stable solutions to ill-conditioned systems it is often necessary to apply regularization methods. One way to achieve such objective is to augment the problem with a second regression that adds assumptions about the model. We now seek a solution to the system of regressions:

$$0 \approx \mathbf{r}_d = \mathbf{L\,m} - \mathbf{d} \tag{7}$$

$$0 \approx \mathbf{r}_m = \lambda\,\mathbf{P\,m} \tag{8}$$

To impose a smoothness constraint on the solution, we chose $\mathbf{P}$ to be the Laplacian operator, which represents spatial differentiation in the midpoint-space. Setting the penalty operator, $\mathbf{P}$, to be the identity matrix reduces to the standard Tikhonov regularization. The parameter λ controls the smoothness of the solution and is function of the smallest resolved singular value of $\mathbf{L}$.

APPLICATION TO 3D REAL DATA

This section presents the results of applying ICO to a 3D land survey recorded in the Shorncliff region of Canada. The dominant geology of the area is flat. One of the objective targets is quite shallow (approximately .71 seconds) and consists of a fluvial deposit system marked by a buried meandering channel. Previous studies of the dataset focused on post-stack processing techniques i.e, DMO processing and inversion to zero offset (Ronen and Goodway, 1998). In our application, we address the problems related to prestack depth imaging and the effects of irregular sampling on the image quality.

The survey was designed with the aim of obtaining a high fold coverage for a good quality final stack. The image from the over-sampled survey can then be used to assess the quality of images obtained by simulating more economic acquisition geometries. We therefore decimated the original dataset to create a sparse geometry that results into aliasing problems. The shot lines from the over-sampled survey were spaced at 140 meters, whereas in the decimated experiment they alternate between 280 and 420m for an average spacing of 350m. We also extracted every fifth receiver line to simulate a cross-spread geometry with 350m line-spacing. The 3D subset used in the simulations consisted of 11300 traces whose source-receiver azimuth is between -60^o and 60^o with an absolute-offset range from 400 to 1000 meters. Figure 2a shows the fold distribution for the subset binned at the survey nominal CMP spacing of 35 m. The variations in coverage between different bins vary substantially from 0 to a maximum of 7. Figure 2b represents a fold chart for the same offset and azimuth range from the original survey. The densely-sampled subset contains 148,000 traces, and therefore, is 13 times larger than the decimated survey.

Data regularization

To equalize the coverage of the irregularly sampled subset, we applied three different regularization methods: conventional binning after NMO, partial stacking by calibrated AMO, and inversion to common offset (ICO). The model is a common offset section sampled at 17.5 m spacing, with zero effective azimuth and 700 m constant offset. The results are compared in Figure 3. As expected, NMO-stacking nicely preserved the continuity of flat events. Due to the low fold of the decimated subset and the fast varying coverage between CMP bins, the signal to noise ratio is not homogeneous across the section. With normalized AMO (one iteration of ICO, calibrated by a flat-event response), amplitudes are not balanced and aliasing noise dominates the seismic sections. The result of regularized ICO with row and column scaling, after 7 iterations, is substantially better than calibrated AMO. Amplitudes along the flat reflectors are well equalized and aliasing noise is mostly eliminated.

Migration after regularization

The next step after regularizing the coverage of the 3D subset is to apply 3D migration to the partial stack. Although at this stage any wave-extrapolation technique can be applied to the regularly sampled subset, we chose Kirchhoff migration for consistency in comparing the results of imaging before and after regularization.

Figure 4 compares the results of migrating the 3D subset using different imaging flows. The Figure represents a particular depth slice (910 m) where differences are most noticeable between the results. The migration of the oversampled survey indicates a complex morphology of a meandering river system marked by ramification of the major channel. The output of migrating the irregularly sampled subset is very noisy and distorted by strong artifacts that make the interpretation of the channels difficult. The result of migrating the NMO-stack showed a surprisingly bad resolution at the observed depth. In contrast, migration after regularization with ICO unveiled much of the details in the image and resolved the different branches of the channel. Given the dominantly flat geology of the survey, it was expected that migration after NMO would still provide a good image. That was indeed the case for many depth sections especially at the floor of the river channel. A

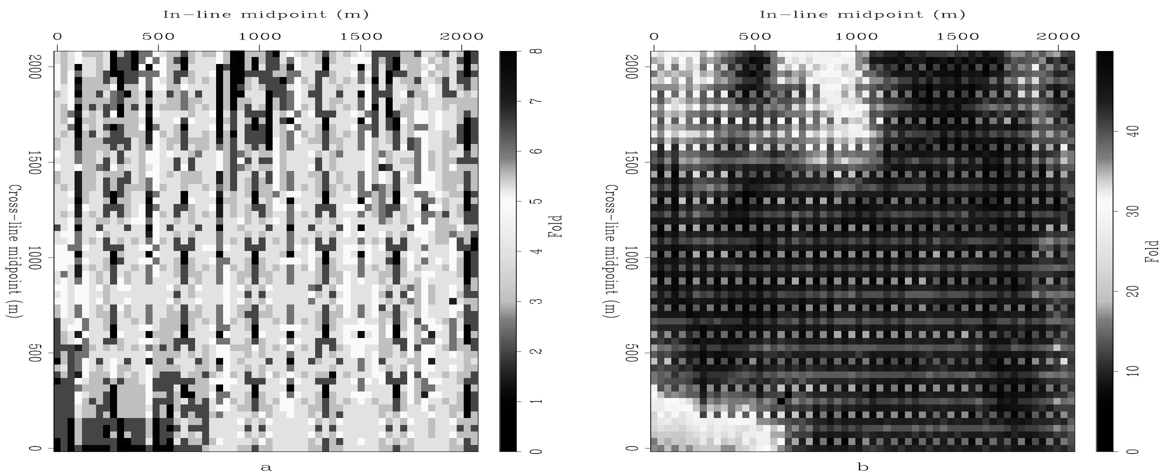

Figure 2: Fold distribution of the 3D subset: (a) decimated survey; (b) over-sampled survey

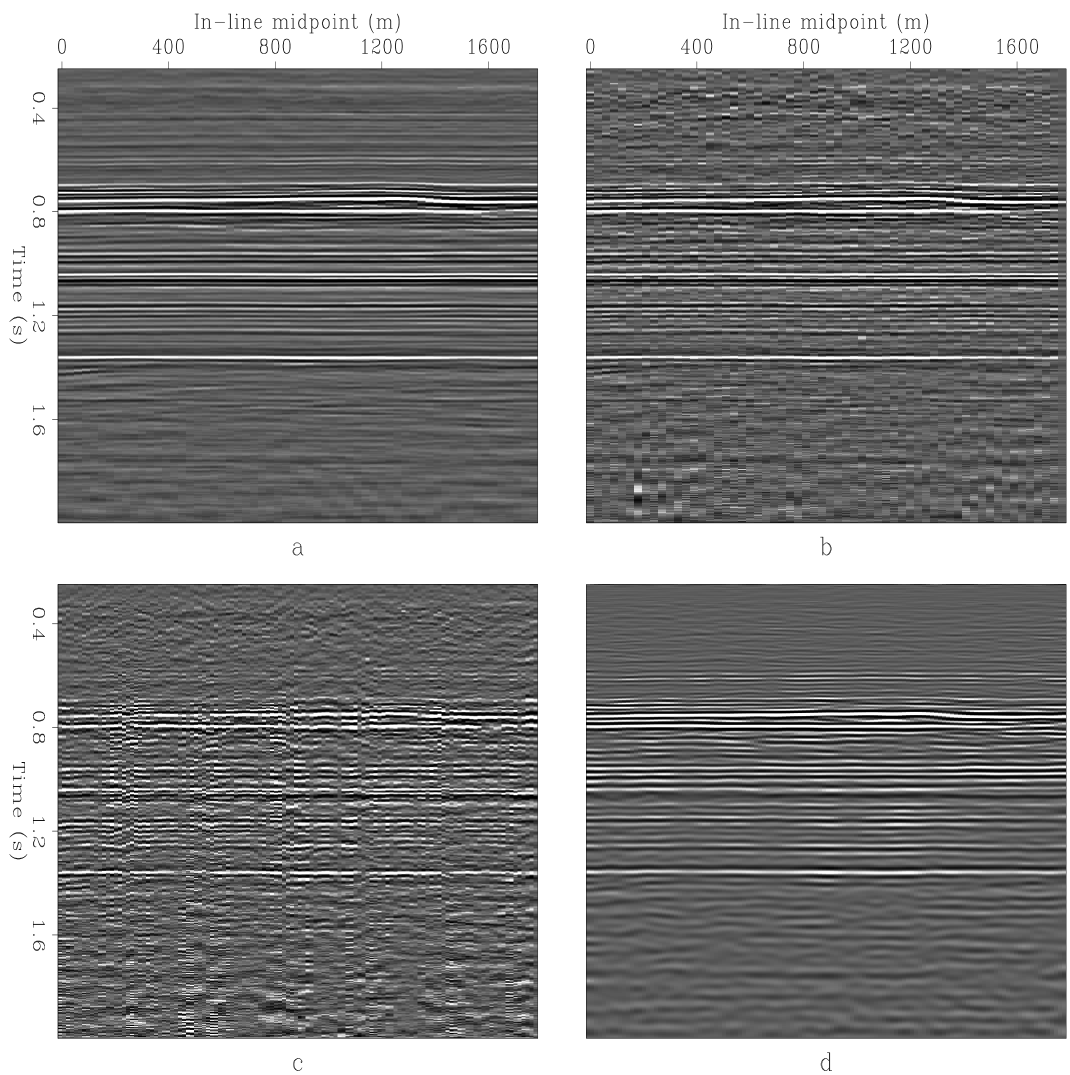

Figure 3: In-line sections at 1 km obtained by: a) NMO-Stack of over-sampled subset, b) NMO-Stack of decimated subset, c) Calibrated AMO-Stack, d) ICO partial stack.

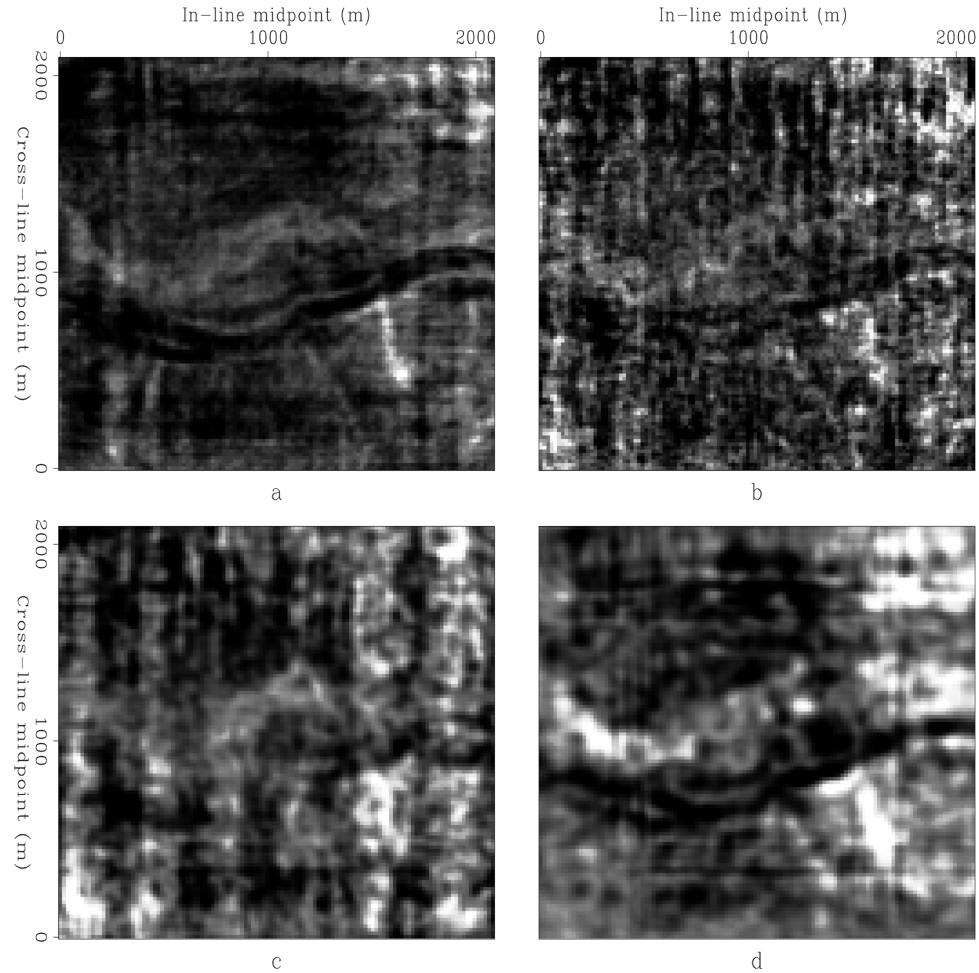

Figure 4: Depth slices at 910 m obtained by different imaging flows a) Migration of over-sampled subset, b) Migration of decimated subset, c) Migration after binning, d) Migration after regularization with ICO.

plausible explanation for this phenomenon is that the morphology of the river system becomes more complex towards the top of the deposition sequence. This results into diffractions from the edges of levees and from possible barrier islands. While these diffractions were nicely preserved by ICO, they were destroyed during stacking by NMO.

CONCLUSIONS

We presented a new approach for imaging irregularly sampled 3D prestack data. The method poses partial stacking with AMO as an optimization process to reduce the size of prestack data and regularize its coverage before migration. The new inversion, named ICO is applied to normal-moveout corrected data. It enables prestack analysis of the reflectivity function since the output models are partial stacks at non-zero offset. The partial stacks can be migrated separately and, either stacked together to form the final image, or, individually analyzed for amplitude and velocity variations.

We present a cost effective implementation of ICO in the Log-stretch Fourier domain with proper preconditioning and regularization of the inversion for iterative solvers. Results of applying ICO to a land 3D survey showed that regularizing the coverage before imaging helps preserve the amplitude information and the high frequency components of the reflectivity function. Furthermore, ICO provides a promising approach for reducing the costs of 3D acquisition.

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