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TIME-REVERSAL PROCESSING OF REFLECTION SEISMIC DATA

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Outline

- What and Why of Time-Reversal Acoustics
  - Eigenfunctions of the scattering operator
  - Applications to reflection seismic data
- Locating a Single Isolated Target
  - Frequency domain
  - Time domain
- Locating Many Targets and/or Horizons
  - Homogeneous v. random medium
  - Time domain
- Conclusions

Wave Propagator in 3D

The reduced wave equation with radiation condition for acoustics in a random medium:

$$\nabla^2 G + k^2 n^2(x) G = -\delta(x)$$

$$lim_{|x| \to \infty} |x| \cdot \left(\frac{\partial G}{\partial |x|} - iknG\right).$$

The variable refractive index is $n(x)$ assumed to be of the form

$$n(x) = 1 + \epsilon \mu(x).$$

When μ vanishes, the solution is well-known to be

$$G_0(x) = \frac{e^{ik|x|}}{4\pi|x|},$$

which is the propagator in 3D homogeneous media.

Wave Propagator in Random Media

J. B. Keller (1961) has shown that for the random case, the propagator is modified to

$$\langle G(x) \rangle = \frac{e^{ikn^*|x|}}{4\pi|x|},$$

where n^* is an effective index of refraction, given explicitly by

$$(n^*)^2 = 1 + \epsilon^2 \left\langle \mu^2 \right\rangle f(k),$$

where

$$f(k) = 1 - 2ik \int_0^\infty (e^{2ikr} - 1) N(r) dr$$

and $N(r)$ is the correlation function

$$N(r) \equiv N(x - x') = \langle \mu(x)\mu(x') \rangle / \left\langle \mu^2 \right\rangle.$$

Attenuation Coefficient in Random Media

The attenuation coefficient for the random medium is then given by

$$\alpha \equiv Im(kn^*) = \epsilon^2 \left\langle \mu^2 \right\rangle k^2 \int_0^\infty (1 - cos\, 2kr) N(r) dr.$$

So one important difference between the propagator for the homogeneous case, and the one for the random case, is that amplitude decay is expected due – not only to spherical divergence – but also to scattering caused by the fluctuations in wave speed of the medium.

Acoustic Wave Scattering (1)

We assume that the problems of interest are well-approximated by the inhomogeneous Helmholtz equation:

$$\left[\Delta + k_0^2 n^2(x)\right] u(x) = s(x),$$

where $u(x)$ is the wave amplitude, $s(x)$ is a localized source function, $k_0 = \omega/c_0 = 2\pi f/\lambda$,
with f being frequency, λ wavelength, and c_0 the assumed homogeneous background wave speed, while $n(x)$ is the acoustic index of refraction such that

$$n(x) = \tfrac{c_0}{c(x)}.$$

Thus, $n^2(x) = 1$ in the background and $a(x) = n^2(x) - 1$ measures the change in wave speed at the scatterers.

Acoustic Wave Scattering (2)

Pertinent fundamental solutions for this problem satisfy:

$$\left[\Delta + k_0^2\right] G_0(x, x') = -\delta(x - x'),$$

and

$$\left[\Delta + k_0^2 n^2(x)\right] G(x, x') = -\delta(x - x')$$

for the homogeneous and inhomogeneous media, respectively.

The solution for the homogeneous medium is well-known in 3D to be

$$G_0(x, x') = \frac{e^{ik_0|x-x'|}}{4\pi|x-x'|}.$$

Acoustic Wave Scattering (3)

The fundamental solution for the inhomogeneous problem can be written in terms of that for the homogeneous one in the usual way as:

$$G(x, x') = G_0(x, x') + k_0^2 \int a(y) G_0(x, y) G(y, x') d^3y.$$

Note that the right hand side depends also on G.

The regions of nonzero $a(x)$ are assumed to be finite in number, denoted by N, in compact domains Ω_n, small compared to the wavelength λ. Then, there will be some position y_n (usually) inside each domain, characterizing the location of each of the N scatterers.

Acoustic Wave Scattering (4)

There is also a constant

$$q_n = k_0^2 \int_{\Omega_n} a(y) d^3 y$$

that characterizes the strength of the scatterer. Then,

$$G(x, x') \simeq G_0(x, x') + \sum_{n=1}^{N} q_n G_0(x, y_n) G(y_n, x').$$

Furthermore, if the scatterers are sufficiently far apart, and the scattering strengths q_n are not too large, then $G(y_n, x')$ on the far right can be replaced by $G_0(y_n, x')$, and we finally have

$$G(x, x') \simeq G_0(x, x') + \sum_{n=1}^{N} q_n G_0(x, y_n) G_0(y_n, x').$$

No Multiples

The approximation just made is equivalent to assuming that multiple scattering is negligible. Or, for realistic applications to seismic data — where multiples clearly occur, we must assume that the multiples have been eliminated or that their effects have at least been minimized in the data before we can proceed to the next step in the analysis. The technical term for this approximation is the "first Born approximation," implying only one iteration of the original integral equation.

Transfer Matrix (1)

In the absence of clutter (such as known scatterers with high albedo), we have the elements of the transfer matrix K in the form

$$K_{l'l}(\omega) = \sum_{n=1}^{N} q_n G_{nl'} G_{nl} = \sum_{n=1}^{N} \sigma_n \frac{(q_n/|q_n|) G_{nl'} G_{nl}}{\sum_{r=1}^{L} |G_{nr}|^2},$$

where G_{nl} is a component of the vector of propagators

$$\mathcal{G}_n = \begin{pmatrix} G_{n,1} \\ G_{n,2} \\ \vdots \\ G_{n,L} \end{pmatrix}.$$

Main (assumed localized for now) scattering targets are labelled by n, while array elements are labelled by l.

Transfer Matrix (2)

The full transfer matrix can therefore be written as

$$K(\omega) = \sum_{n=1}^{N} q_n \mathcal{G}_n \mathcal{G}_n^T,$$

for N reflectors. So, when $N = 1$, the vector $\mathcal{G}_1$ is clearly the unique (within a phase factor and normalization factor) singular vector of the rank one matrix $K(\omega)$. Thus, we expect these vectors $\mathcal{G}_n$ associated with particular scattering points $1 \le n \le N$ to play a very important role in the analysis, and in particular to be reasonable approximations in many cases of the singular vectors of the transfer matrix.

Singular Value Decomposition of K

The singular-vectors for $K(\omega)$ are approximately given by the time-reversed propagator vectors G^*_{il}, since

$$\sum_{l=1}^{L} K_{l'l}(\omega) G^*_{nl} = \sum_{n'=1}^{N} q_{n'} G_{n'l'} \sum_{l=1}^{L} G_{n'l} G^*_{nl} \simeq \sigma_n \frac{q_n}{|q_n|} G_{nl'}.$$

The real singular-value (square root of the eigenvalue of K^*K) is

$$\sigma_n = |q_n| \sum_{r=1}^{L} |G_{nr}|^2,$$

and where, for simplicity, we assumed that the localized and relatively small scatterers are well-separated so that

$$\mathcal{G}^T_{n'} \mathcal{G}^*_n = \sum_{l=1}^{L} G_{n'l} G^*_{nl} \simeq \left(\sum_{l=1}^{L} |G_{n'l}|^2 \right) \delta_{nn'}.$$

This statement is exactly right only for a single scatterer.

Lanczos Version of SVD for K

Another way to understand the singular value decomposition for this problem is to consider the eigenvalue problem for the Hermitian matrix associated with K obtained by completing the square (Lanczos, 1961):

$$\begin{pmatrix} & K \\ K^* & \end{pmatrix} \begin{pmatrix} (q_n/|q_n|)\mathcal{G}_n \\ \mathcal{G}_n^* \end{pmatrix} = \sigma_n \begin{pmatrix} (q_n/|q_n|)\mathcal{G}_n \\ \mathcal{G}_n^* \end{pmatrix}.$$

When the scattering coefficient is real, this form of the singular-vector (and also eigenvector) is symmetric in $\mathcal{G}_n$ and $\mathcal{G}_n^*$.

MUSIC: Theme & Variations (1)

MUSIC is a method for determining whether or not each vector in a set of vectors is fully or only partially in the range of an operator.

MUSIC stands for MUltiple SIgnal Classification.

If $T = KK^*$ is the operator of interest,

V_i is a known eigenvector of T, and

H_r is a vector from the test set

(i.e., a vector of Green's function propagators

from test point r to all the members of the array).

MUSIC: Theme & Variations (2)

Next, we consider the "noise space" operator

$$n = \mathcal{I} - \sum_{i=1}^{N} V_i^* V_i^T = \mathcal{I} - \mathcal{R},$$

where $\mathcal{R}$ is the resolution operator (projecting onto the range space of the operators T and K). We want to determine whether the test vector H_r is orthogonal to the noise space (and therefore in the reflector set). To do this we simply consider

$$H_r^T n H_r^* \simeq 0.$$

Think of this as a "fitting goal" for a reflection point in the model space. Also, define the square of the direction cosine

$$cos^2(V_i, H_r) = |V_i^T \cdot H_r^*|^2 / |H_r|^2.$$

MUSIC: Theme & Variations (3)

Assuming that r is a parameter or vector ranging over locations in space, then there are several related functionals we can plot in order to "image" the MUSIC classification of vector character, including:

$$cosec^2(\tilde{V}, H_r) = \frac{1}{1-\sum_{i=1}^{N} cos^2(V_i, H_r)}$$

and

$$cotan^2(\tilde{V}, H_r) = \frac{\sum_{i=1}^{N} cos^2(V_i, H_r)}{1-\sum_{i=1}^{N} cos^2(V_i, H_r)},$$

where $\tilde{V}$ is the set of vectors $V_1, \ldots, V_N$.

MUSIC: Theme & Variations (4)

Another variation on the MUSIC classification scheme is to consider a subset of the eigenvectors, and plot the incomplete versions of the previous choices

$$cosec^2(\hat{V}, H_r) = \frac{1}{1 - \sum_{i=1}^{N'} cos^2(V_i, H_r)}$$

and

$$cotan^2(\hat{V}, H_r) = \frac{\sum_{i=1}^{N'} cos^2(V_i, H_r)}{1 - \sum_{i=1}^{N'} cos^2(V_i, H_r)},$$

where $\hat{V}$ is the set of vectors $V_1, \ldots, V_{N'}$, and $N' \leq N$.

MUSIC: Theme & Variations (5)

In particular, this scheme could be used for just a few eigenvectors at a time. Viewing eigenvectors as measurements, we see that using fewer eigenvectors will produce poorer resolution as less information is available to constrain the images.

Key point: It is not even necessary to know the eigenvectors. It is sufficient to know any set of orthogonal vectors in the range of the operator. What is important is to have information about the resolution operator $\mathcal{R}$, so the noise space operator can be determined by

$$n = \mathcal{I} - \mathcal{R}.$$

MUSIC: Theme & Variations (6)

In prior work here at SEP some of us have shown that it is possible to compute estimates of the resolution operators from a set of vectors coming from an iterative (Krylov subspace) scheme. If the orthonormal vectors coming from such as scheme (CG, LSQR, etc.) are θ_i, then

$$\mathcal{R} = \sum_i \theta_i^* \theta_i^T,$$

and, for example, the cosecant version of MUSIC is

$$cosec^2(\tilde{\theta}, H_r) = \frac{1}{1 - \sum_{i=1}^{N} cos^2(\theta_i, H_r)}.$$

This alternative is very advantageous for large data sets, as for example will always be present in 3D surveys.

Frequency Domain Caveat

The method just described will work pretty well
for homogeneous media with a just a few scatterers.
In heterogeneous media like the earth, we expect that
this approach will need some serious modifications
along the lines of work published earlier by the
Stanford group, including Borcea, Papanicolaou, and Tsogka.
The problem is that everything I have presented here is
valid in the frequency domain. But the methods work
better for random media (like the earth) after transforming
back to the time domain. But that is another seminar.

Conclusions

• Methods of analyzing time-reversal data are progressing rapidly. Two types of analysis are the main ones being considered at the moment: (1) SVD and (2) signal processing schemes much like those used in reflection seismology.

• Locating a single target can be done quite easily using either time-domain or frequency-domain methods.

• Locating multiple targets is most easily accomplished in the time-domain, and especially so in a weakly random propagating medium.