DOE BES Geosciences

Symposium on Fluid Flow

Gaithersburg, MD, March 11–12, 2010

EFFECTIVE STRESS FOR FLUID FLOW IN ANISOTROPIC FRACTURED ROCK: ISOTROPY TO ORTHOTROPY

James G. Berryman

Geophysics Department

Earth Sciences Division

Lawrence Berkeley National Laboratory

REFERENCES – SOME RECENT PAPERS

◦ J. G. Berryman, "Poroelastic measurement schemes resulting in complete data sets for granular and other anisotropic porous media," *Int. J. Engng. Sci.* **48**, 446–459 (2010).

◦ J. G. Berryman and S. Nakagawa, "Inverse problem in anisotropic poroelasticity: Drained constants from undrained ultrasound measurements," *J. Acoust. Soc. Am.* **127**, 720–729 (2010).

◦ J. G. Berryman, "Seismic waves in rocks with fluids and fractures," *Geophys. J. Int.* **171**, 954–974 (2007).

REFERENCES – SOME OLDER PAPERS

- J. B. Walsh, "Effect of pore pressure and confining pressure on fracture permeability," *Int. J. Rock Mech.* **18**, 429–435 (1981).
- K. B. Coyner, *Effects of Stress, Pore Pressure, and Pore Fluids on Bulk Strain*, Ph.D. Thesis, MIT (1984).
- J.G.B., "Effective stress for transport properties of inhomogeneous porous rock," *JGR* **97**, 17409 (1992).
- J.G.B., "Effective-stress rules for pore-fluid transport in rocks containing two minerals," *Int. J. Rock Mech.* **30**, 1165 (1993).

REFERENCES – OTHER OLDER PAPERS

◦ G. E. Backus, Long-wave elastic anisotropy produced by horizontal layering, *J. Geophys. Res.* **67**, 4427–4440 (1962).

◦ M. Schoenberg and F. Muir, A calculus for finely layered anisotropic media, *Geophysics* **54**, 581-589 (1989).

◦ T. M. Daley, M. A. Schoenberg, J. Rutqvist, and K. T. Nihei, Fractured reservoirs: An analysis of coupled elastodynamic and permeability changes from pore-pressure variation, *Geophysics* **71**, O33-O41 (2006).

OUTLINE

- Effective Stress Concept
  - Meaning of effective stress
  - Anisotropy to isotropy (combining measurements)
  - Review of isotropic results
- Orthotropy and Layered Systems
  - Backus and/or Schoenberg-Muir averaging
  - Results for layered heterogeneous anisotropy
- Discussion and Conclusions

MEANING OF EFFECTIVE STRESS (1)

The main idea of effective stress is to determine how strong the influence of fluid pressure is at counteracting the forces acting on the outside of a medium containing the fluid. Good commonplace examples are balloons or tire inner tubes, where air inside the balloon or tube is necessarily at higher pressure than on the outside – so the balloon or tire will inflate.

MEANING OF EFFECTIVE STRESS (2)

Fluid trapped inside a rock acts much like this. When the rock is squeezed, the pressure of the trapped interior fluid increases. If the fluid pressure is independently increased (like pumping up a tire), then the internal fluid pressure is counteracting the force of the external pressure to some extent. But the question is how effective is pressure really at counteracting the external forcing.

MEANING OF EFFECTIVE STRESS (3)

If the external confining pressure is p_c and internal fluid pressure is p_f, then the effective pressure is some pressure value having the general form

$$p_{eff} = p_c - Cp_f,$$

where C is called an "effective stress coefficient."

Typically $0 \le C \le 1$, but not always. Each physical property can have its own effective stress coefficient, and in some cases (permeability, fluid content) $1 < C$.

POROELASTIC COMPLIANCE MATRIX M

The compliance form of the poroelasticity matrix is:

$$\mathbf{M} = \begin{pmatrix} S_{11} & S_{12} & S_{13} & -\beta_1 \\ S_{12} & S_{22} & S_{23} & -\beta_2 \\ S_{13} & S_{23} & S_{33} & -\beta_3 \\ -\beta_1 & -\beta_2 & -\beta_3 & \gamma \end{pmatrix}$$

where S_{ij} are the drained compliances (no fluid present, for example), the off-diagonal terms are given by $\beta_i = \sum_j S_{ij} - 1/K_R^g$, for $i, j = 1, 2, 3$, and $\gamma = \sum_i \beta_i / B K_R^d$. B is Skempton's second coefficient, and is the only term in the 4×4 compliance matrix that includes fluid effects.

ORTHOTROPIC POROELASTICTY

$$\mathbf{M}\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ -p_f\end{pmatrix} = \begin{pmatrix}e_{11}\\ e_{22}\\ e_{33}\\ -\zeta\end{pmatrix}$$

where ζ is the increment of fluid content, p_f is fluid pressure, and $\mathbf{M}$ was defined previously. The stresses σ_{11}, etc., and the strains e_{11} etc., have their usual meanings, but for the *porous* solid matrix.

Drained bulk modulus $1/K_R^d = \sum_{ij} S_{ij}$. Grain (or mineral) bulk modulus $1/K_R^g = \sum_{ij} S_{ij}^g$, where S_{ij}^g are the compliances of nonporous grains or minerals.

MEASUREMENT REDUCTIONS (1)

Matrix **M** has 10 independent coefficients: six drained compliances, three off-diagonal poroelastic-coupling compliances, and one purely fluid-fluid coefficient.

We can reduce this burden by our choices of measurement schemes.

This leads to the concept that I call "telescoping."

MEASUREMENT REDUCTIONS (2)

Telescoping concept:

We can reduce the size of the problem from 4×4 to 2×2 in two steps.

Step 1: Setting $\sigma_{11} = \sigma_{22} = \sigma_{33} = -p_c$,
which is uniform confining stress (negative of confining pressure).

Step 2: Summing $e = e_{11} + e_{22} + e_{33}$,
which is total strain.

MEASUREMENT REDUCTIONS (3)

Setting $\sigma_{11} = \sigma_{22} = \sigma_{33} = -p_c$

allows us to write the 4×2 matrix:

$$\mathbf{M}_1 = \begin{pmatrix} S_{11} + S_{12} + S_{13} & -\beta_1 \\ S_{12} + S_{22} + S_{23} & -\beta_2 \\ S_{13} + S_{23} + S_{33} & -\beta_3 \\ -\beta_1 - \beta_2 - \beta_3 & \gamma \end{pmatrix}.$$

Summing $e = e_{11} + e_{22} + e_{33}$,

allows us to write the 2×2 matrix:

$$\mathbf{M}_2 = \begin{pmatrix} 1/K_R^d & -\alpha/K_R^d \\ -\alpha/K_R^d & \gamma \end{pmatrix}.$$

MEASUREMENT REDUCTIONS (4)

Finally, we have the general statement (true for orthotropy, isotropy, and all possibilities in between:

$$\begin{pmatrix} 1/K_R^d & -\alpha/K_R^d \\ -\alpha/K_R^d & \gamma \end{pmatrix} \begin{pmatrix} -p_c \\ -p_f \end{pmatrix} = \begin{pmatrix} e \\ -\zeta \end{pmatrix}.$$

This formula implies two effective stress statements. One for volume is:

$$e = -(1/K_R^d)(p_c - \alpha p_f).$$

Another for fluid content is:

$$\zeta = -(\alpha/K_R^d)(p_c - p_f/B).$$

ELASTIC LAYER AVERAGING USING BACKUS OR SCHOENBERG-MUIR SCHEMES

$$\begin{pmatrix} E_T \\ E_N \end{pmatrix} = \begin{pmatrix} S_{TT} & S_{TN} \\ S_{NT} & S_{NN} \end{pmatrix} \begin{pmatrix} \Pi_T \\ \Pi_N \end{pmatrix},$$

where

$$E_T = \begin{pmatrix} e_{11} \\ e_{22} \\ e_{12} \end{pmatrix} \text{ and } E_N = \begin{pmatrix} e_{33} \\ e_{32} \\ e_{31} \end{pmatrix}$$

and

$$\Pi_T = \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{pmatrix} \text{ and } \Pi_N = \begin{pmatrix} \sigma_{33} \\ \sigma_{32} \\ \sigma_{31} \end{pmatrix}.$$

LAYER COMPLIANCE REARRANGEMENTS

$$S_{TT} = \begin{pmatrix} s_{11} & s_{12} & \\ s_{21} & s_{22} & \\ & & s_{66} \end{pmatrix},$$

$$S_{NN} = \begin{pmatrix} s_{33} & & \\ & s_{44} & \\ & & s_{55} \end{pmatrix},$$

and

$$S_{NT} = \begin{pmatrix} s_{31} & s_{32} & \\ & 0 & \\ & & 0 \end{pmatrix} = S_{TN}^{T}.$$

DISTINGUISH FAST AND SLOW VARIABLES

$$\begin{pmatrix} E_T \\ < E_N > \end{pmatrix} = \begin{pmatrix} S^*_{TT} & S^*_{TN} \\ S^*_{NT} & S^*_{NN} \end{pmatrix} \begin{pmatrix} < \Pi_T > \\ \Pi_N \end{pmatrix},$$

where E_N and Π_T are the fast variables (changing across boundaries), and therefore need to be averaged.

The matrix coefficients are:

$$S^*_{TT} =< S^{-1}_{TT} >^{-1},$$

$$S^*_{TN} = (S^*_{NT})^T = S^*_{TT} < S^{-1}_{TT} S_{TN} >, \text{ and}$$

$$S^*_{NN} =< S_{NN} > - < S_{NT} S^{-1}_{TT} S_{TN} > + S^*_{NT} (S^*_{TT})^{-1} S^*_{TN}.$$

POROELASTIC LAYER AVERAGING FOR DRAINED SYSTEMS: USING SCHEMES LIKE BACKUS OR SCHOENBERG-MUIR

$$\begin{pmatrix} E_T \\ -\zeta \\ E_N \end{pmatrix} = \begin{pmatrix} S_{TT} & -g_{12} & S_{TN} \\ -g_{12}^T & \gamma & -g_3^T \\ S_{NT} & -g_3 & S_{NN} \end{pmatrix} \begin{pmatrix} \Pi_T \\ -p_f \\ \Pi_N \end{pmatrix},$$

where

$$g_{12}^T = (\beta_1, \beta_2, 0) \text{ and } g_3^T = (\beta_3, 0, 0).$$

Then, drained conditions ($p_f = 0$) give the same equations as the elastic case, but for porous media.

POROELASTIC LAYER AVERAGING FOR UNDRAINED SYSTEMS: USING SCHEMES LIKE BACKUS OR SCHOENBERG-MUIR

However, undrained conditions ($\zeta = 0$) result in the undrained version of the original elastic equation (I am not showing this work here!), which is:

$$\begin{pmatrix} E_T \\ E_N \end{pmatrix} = \begin{pmatrix} S^u_{TT} & S^u_{TN} \\ S^u_{NT} & S^u_{NN} \end{pmatrix} \begin{pmatrix} \Pi_T \\ \Pi_N \end{pmatrix}.$$

So the analysis is the same as before, but now using the undrained constants in the appropriate (locally undrained) layers.

GASSMANN'S EQUATIONS (1)

$$K_R^u = K_R^d + \frac{\alpha^2}{(\alpha-\phi)/K_R^g+\phi/K_f} = \frac{K_R^d}{1-\alpha B},$$

where K_R^u is the undrained bulk modulus. K_R^d is the drained bulk modulus; K_R^g is the mineral (or solid) modulus; K_f is the pore fluid bulk modulus; ϕ is the porosity; $\alpha = 1 - K_R^d/K_R^g$ is (volume) effective stress coefficient; B is Skempton's second coefficient.

GASSMANN'S EQUATIONS (2)

Rearranging into compliance form, we have

$$\frac{1}{K_R^d} - \frac{1}{K_R^u} = \frac{\alpha}{K_R^d} \times$$

$$\left[1 + \frac{\phi K_R^d}{\alpha K_f}\left(1 - \frac{K_f}{K_\phi}\right)\right]^{-1} = \frac{\alpha}{K_R^d} B,$$

which shows explicitly how Skempton's B coefficient is related to all the other constants.

GASSMANN'S ORTHOTROPIC EQUATIONS (3)

Compliance correction for undrained fluid inclusions:

$$\Delta_f S_{ij} = -\gamma^{-1} \begin{pmatrix} \beta_1^2 & \beta_1\beta_2 & \beta_1\beta_3 & & & \\ \beta_1\beta_2 & \beta_2^2 & \beta_2\beta_3 & & & \\ \beta_1\beta_3 & \beta_2\beta_3 & \beta_3^2 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix}$$

The fluid effects (through K_f or B) appear only in the overall factor $\gamma = \alpha/BK_R^d$. The coefficients β_i, $i = 1, 2, 3$, satisfy a sumrule of the form

$$\beta_1 + \beta_2 + \beta_3 = 1/K_R^d - 1/K_R^g \equiv \alpha/K_R^d.$$

CONCLUSIONS

- Effective stress coefficients are very well understood for homogeneous (one solid material) porous media, whether isotropic or anisotropic.

- Effective stress coefficients for heterogeneous materials (rocks in particular) are **not** so well understood yet.

- Effective stress of scale invariant properties are also better understood than those for non-scale-invariant properties, including fluid permeability.

OLDER REFERENCES (1)

- A. M. Nur, J. D. Walls, K. Winkler, and J. DeVilbiss, Effects of fluid saturation on waves in porous rock and relation to hydraulic permeability, *Soc. Petr. Eng. J.* **20**, 450 (1980).
- J. B. Walsh and W. F. Brace, The effect of pressure on porosity and the transport properties of rock, *JGR* **89**, 9425-9431 (1984).

OLDER REFERENCES (2)

- R. W. Zimmerman, 1991, *Compressibility of Sandstones*, Elsevier, Amsterdam.
- J. G. Berryman, 1995, Mixture theories for rock properties: in *American Geophysical Union Handbook of Physical Constants*, edited by T. J. Ahrens, AGU, New York, pp. 205–228.

ACKNOWLEDGMENT

This work was performed under the auspices of the U.S. Dept. of Energy (DOE) by the UC Lawrence Berkeley National Lab. under contract # DE-AC02-05CH-11231 and supported specifically by the Geosciences Research Program of the DOE Office of Basic Energy Sciences, Div. of Chemical Sciences, Geosciences and Biosciences. All support of the work is gratefully acknowledged.