Logarithm Bidirectional Method With Regularization

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When we consider regularization in the logarithm bidirectional method, we need to change our objective function into

\[ J = \text{hyp}(r) + \varepsilon \|u\| = \sum_t H(r_t) + \varepsilon \sum_n u^2_n \tag{1} \]

Take the gradient of the penalty function assuming there is only one variable, \(u_3\) giving a single regression equation:

\[ 0 \approx \frac{\partial J}{\partial u_3} = \sum_t \frac{\partial H}{\partial r} \frac{\partial r}{\partial u_3} + 2\varepsilon u_3 \tag{2} \]

Then the gradient for all nonzero lags is:

\[ 0 \approx \Delta u_{\text{new}} = \sum_t r_{t+\tau}H'(r_t) + 2\varepsilon u \tag{3} \]

Although the gradient is changed, we still want to keep \(\Delta r\) unchanged:

\[ \Delta u_{\text{old}} = \sum_t r_{t+\tau}H'(r_t) \tag{4} \]

\[ \Delta r = r \ast \Delta u_{\text{old}} \tag{5} \]

Now let us figure out how to find the scalar factor \(\alpha\). By taylor expansion,

\[ J = \sum_t \left( H_t + \alpha \Delta r_t H_t' + \alpha^2 \Delta r_t^2 H_t'' \right) + \varepsilon \sum_n ((u_{\text{new}} + \alpha \Delta u_{\text{new}})^2)_n \tag{6} \]

Setting \(\frac{\partial J}{\partial \alpha} = 0\) we get,

\[ \alpha = -\frac{\sum_t \Delta r_t H_t' + 2\varepsilon \sum_n (u_{\text{new}} \Delta u_{\text{new}})_n}{\sum_t (\Delta r_t)^2 H_t'' + 2\varepsilon \sum_n ((\Delta u_{\text{new}})^2)_n} \tag{7} \]