GP211. Closed book. 10 minutes. November 2, 2009. Your name ____________________

1. Given the autoregression $0 \approx r_t = y_t + \sum_{\tau>0} a_\tau y_{t-\tau}$ Prove that when the output power is minimum, the residual r_t is orthogonal to the fitting function y_{t-5}.

2.

$$\mathbf{0} \approx \mathbf{r}_a = \begin{bmatrix} y_1 & 0 & 0 \\ y_2 & y_1 & 0 \\ y_3 & y_2 & y_1 \\ y_4 & y_3 & y_2 \\ y_5 & y_4 & y_3 \\ 0 & y_5 & y_4 \\ 0 & 0 & y_5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ a_2 \end{bmatrix} ; \qquad \mathbf{0} \approx \mathbf{r}_b = \begin{bmatrix} 0 & 0 & 0 \\ y_1 & 0 & 0 \\ y_2 & y_1 & 0 \\ y_3 & y_2 & y_1 \\ y_4 & y_3 & y_2 \\ y_5 & y_4 & y_3 \\ 0 & y_5 & y_4 \\ 0 & 0 & y_5 \end{bmatrix} \begin{bmatrix} 1 \\ b_1 \\ b_2 \end{bmatrix}$$

(a) Cross off all columns perpendicular to $\mathbf{r}_a$.

(b) If we want to neglect the column that multiplies b_2, what claim must we make?

(c) Assuming we can neglect the column multiplying b_2 why is $\mathbf{r}_a \cdot \mathbf{r}_b = 0$?

(d) Why does $\mathbf{a} = \mathbf{b}$?

(e) $\mathbf{r}_a$ is perpendicular to $\mathbf{r}_b$, but given $\mathbf{r}_a$ how can you find $\mathbf{r}_b$?

(f) How is $\mathbf{r}_a \cdot \mathbf{r}_b$ related to the autocorrelation of $\mathbf{r}_a$?

(g) What is the autocorrelation of white noise?

3. Given that the residual is orthogonal to the fitting functions, show that the output at A is orthogonal to the output of B thereby proving that one lag of the output autocorrelation vanishes.

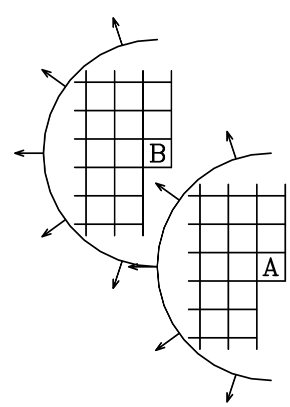

Figure 1: A 2-D whitening filter template, and itself lagged. At output locations "A" and "B" the filter coefficient is constrained to be "1". Viewing the semicircles as having infinite radius, the B filter is contained in the A filter. whitepruf [NR]