

Suggestion for a new formula to calculate Group-delay from frequency domain measurements

Itzhak Shapir, Senior Member

RAFAEL, Armament Development Authority Ltd.– Microelectronics Directorate.

Phone: +97248794951 Fax: +97248795576 email: itzhaksh@rafael.co.il

Abstract -- This paper presents a simple case study of a low Q band reject filter. Calculation based on phase derivative produces negative Group-delay for this circuit.

In the paper, a modified formula $D_g = [(\partial A/A)/\partial \omega]^2 + (\partial \Phi/\partial \omega)^2]^{1/2}$ is suggested, based on a normalized derivative of the full complex transmission function $T(\omega) = A(\omega)e^{-j\Phi(\omega)}$. This formula produces positive delay for all frequencies, having a maximum value at resonance. For flat amplitude response, the formula converges to the classic formula $D_g = -\partial \Phi/\partial \omega$.

Index terms -- Delay, group-delay, filters.

I. Introduction:

The investigation described in this paper started when a substantial negative group delay was measured for a simple band-stop filter within the rejection band, using a state of the art Automatic Network Analyzer. The surprise was even bigger when the exact result was repeatedly analyzed with popular circuit simulators. Being more aware of this anomaly, we started to notice “strange” group-delay characteristics also at some amplitude equalizers that use low-Q band reject sections. This effect was also reported for band-pass amplifiers realized by using a low-Q band reject section at their feedback network [12], [13]. In a following study, a new way to calculate the group delay was found, which complies with the standard way for most networks but predicts physically accepted positive values for these and similar unique cases.

II. Group-delay calculation and measurement:

Group-delay, also known as “Envelope Delay”, is the delay of the information modulating a sinusoidal carrier signal.

The modulated signal has a finite bandwidth, a complex time function and can be considered as the “wave group”. There can be a difference between the phase delay, which is calculated

from the ratio between the phase and frequency of a pure sine wave and the group-delay of the complex modulated signal. Theory of guided waves [1], [2] usually discusses the case of constant amplitude transmission function. The general way to calculate Group-Delay in that case is by:

$$(1) \quad D_G = \partial \beta(\omega)/\partial \omega = -\partial \Phi/\partial \omega$$

A true group-delay measurement would be the measurement of the delay of information, which modulates a sinusoidal carrier wave, is transmitted through the measured DUT and then is de-modulated. Some system test equipment like communication link analyzers perform this type of measurement. Some textbooks [3] differentiate between “Static Group-Delay” defined by (1) and “Dynamic Group-Delay” which is the true modulation delay. A simulator that would use a similar algorithm could calculate true group-delay as well. In the case where the full network structure with all the frequency dependent elements such as capacitors, inductors and transmission lines is available, then the following formula, which is thoroughly discussed in [4] can be used:

$$(2) \quad D_G = -\text{Im}[(1/V_0)(\partial V_0/\partial \omega)]$$

Where

$$(3) \quad \partial V_0/\partial \omega = \sum (\partial V_0/\partial Y_C)(\partial Y_C/\partial \omega) + \sum (\partial V_0/\partial Z_L)(\partial Z_L/\partial \omega) + \sum [(\partial V_0/\partial Z_0)(\partial Z_0/\partial \omega) + (\partial V_0/\partial \gamma)(\partial \gamma/\partial \omega)]$$

where: $Y_C = j\omega C$; $Z_L = j\omega L$; $\gamma = \alpha - j\beta$

This formula however is impractical where only input/output relations are available for the whole network or sub-elements that comprise that network.

Time domain simulators may be also used to analyze the true group-delay of a circuit or system by analysis of the envelope output arrival time.

(Note: in cases where the circuit changes the proportions within the spectral content of the modulated signal, the envelope shape will be distorted. Different selection of reference points on the envelope like peaks or zero-crossings, may result in different delay measurements in time-domain based delay measurements).

Unfortunately, microwave circuit simulators and measurement systems are operating mostly in the frequency domain, calculating or measuring S parameters for a steady state sinusoidal wave at a set of discrete frequencies, one at a time. Measurement systems and in many cases circuit simulators have to deal with “Black box” S Parameters only, where no data is available on the internal components of that “Black box”. The delay is then derived only from the signal phase data by using the well-known formula:

$$(4) \quad D_g = - \partial\Phi/\partial\omega$$

This formula is a simple form of (1) and is derived for the case of lossless and frequency independent transmission media. [Notice that for constant amplitude cases $\partial\Phi/\partial\omega$ is negative and the expression $D_g = \text{ABS}(\partial\Phi/\partial\omega)$ is equivalent to formula (3)]. All filter design reference books, like [5], [6] and [7] use this formula, where only in few, like [5], there is a comment that the physical meaning of group delay is not clear when the attenuation varies rapidly with frequency.

III. A Low Q Band-reject section:

A low Q band-reject section is a good example that demonstrates the inaccuracy of the classic formula (4). In the example circuit (figure 1) a negative delay was calculated using a leading circuit simulator (figure 2) and measured (!) using a leading Automatic Vector Network Analyzer. This experiment can be easily repeated with any other similar software and measurement equipment. This non-physical result (can a passive networks be a causal system?) will repeat with most of the microwave circuit simulators and vector network analyzer since they all use the classic formula (4). The circuit is shown in figure 1 and the analysis results are given in figures 2. A polar chart of S21 is given in figure 3. It is clear that around the center frequency the amplitude does not change rapidly, yet the group delay calculated remains negative.

IV. Suggestion for a new formula:

It is easily observed from the polar chart at figure 3 that since the S21 amplitude rapidly declines with frequency and the center of the polar chart is left outside its circular pattern, the phase as measured from the polar chart center changes direction, producing negative delay values. Therefore, it is clear that for the general case of simultaneous amplitude and phase change a better formula is required for the group delay calculation.

The general transmission function $T(\omega)$ of a network (or any “black box”) in the frequency domain is of the form:

$$(5) \quad T(\omega) = A(\omega)e^{-j\Phi(\omega)}$$

Derivation by ω will give:

$$(6) \quad \begin{aligned} \partial T/\partial\omega &= (\partial A/\partial\omega)e^{-j\Phi} + A\partial(e^{-j\Phi})/\partial\omega \\ &= (\partial A/\partial\omega)e^{-j\Phi} + Ae^{-j\Phi}(-j\partial\Phi/\partial\omega) \end{aligned}$$

Normalizing at each frequency by $T(\omega) = A(\omega)e^{-j\Phi(\omega)}$ will give:

$$(7) \quad (\partial T/T)/\partial\omega = (\partial A/A)/\partial\omega - j\partial\Phi/\partial\omega \quad (\text{Sec})$$

Notice that the normalized derivative of the transmission function describes the changes in the output signal in reference to the input signal and is a complex delay expression. The imaginary part of this expression is the well-known formula (4) that describes the contribution of phase changes to the delay and the real part describes the contribution of the amplitude changes to the delay. It is easily observed from polar charts that these contributions are perpendicular and therefore the total Transmission Function differential is their sum phasor, hence the group delay is the magnitude of that complex delay expression:

$$(8) \quad D_g = [(\partial A/A)/\partial\omega]^2 + (\partial\Phi/\partial\omega)^2]^{1/2}$$

Notice that for the case of constant amplitude formulas (4) and (8) will have the same results, and that no negative delays could be calculated. Also for small amplitude variances the dominant factor would be the phase change, so this formula is fully consistent with formula (4) wherever that one is valid. The delay calculated for the band-stop section in figure 1 is presented in figure 4.

V. Notes about the normalized amplitude differential $\partial A/A$:

1. The normalized differential is dimensionless even if the transmission function is not dimensionless.
2. The amplitude normalization contributes to eliminate the influence of any ideal attenuation or amplification added to the network, while the phase normalization contributes to eliminate the influence of ideal phase shifters or phase inverters added to the network.
3. The normalization is local, means that $A(\omega)$ is the average of the two amplitudes forming the differential $\partial A(\omega)$. Ideal band reject resonators will have a singularity (infinite delay) at resonance. (From a philosophical standpoint, an infinite delay complies with the fact that the signal is fully rejected at resonance and therefore will never appear at the output.)

VI. Conclusions

The normalized complex frequency derivative of a transmission function results in a complex time value. The meaning of a complex time and the particular role of the real and its imaginary parts are yet to be investigated. However, it is clear that the imaginary part creates the classic formula for group delay, a formula that produce negative values in some cases, values which can not be physically true.

On the other hand, it is clear that the absolute value of the normalized complex derivative is identical to the classic formula for flat transmission function, but produces positive delay values, creating monotonous graph (unlike the absolute value of the classic formula calculation).

Therefore it is suggested to look further into the full derivative approach, in order to create a formula that will be valid for all types of transmission function.

References:

1. Robert E. Collin: "Field Theory of Guided Waves", IEEE Press, Copyright 1991, pp. 232 - 233.
2. Simon Ramo, John R. Whinnery and Theodore Van Duzer: "Fields and Waves in Communication Electronics", John Wiley and Sons, Inc., Copyright 1965, pp. 49 - 50.

3. Kai Chang: "Handbook of Microwave and Optical Components", John Wiley and Sons Inc., 1977, Vol. 1, pp. 493 - 495.
4. Gabor C. Temes, Jack W. LaPatra: "Introduction to Circuit Synthesis and Design", McGRAW-HILL Book Company, Copyright 1977, pp. 426 - 431.
5. George L. Matthaei, Leo Young and E. M. T. Jones: "Microwave Filters, Impedance-Matching Networks and Coupling Structures", Artech House Books, Copyright 1980, pp. 9-12, 343-345.
6. Anatol I. Zverev: "Handbook of filter Synthesis", John Wiley & Sons, Copyright 1967, pp. 61-63.
7. Grant E. Hansell: "Filter Design and Evaluation", Van Nostrand Reinhold Company, Copyright 1969, pp. 3-4.
8. Frank S. Crawford Jr.: "Waves", Berkeley Physics Course - Volume 3, McGraw-Hill Book Co., Copyright 1968, pp. 268-273.
9. George W. Chantry: "Long Wave Optics", Academic Press 1984, Vol. 1, pp.208 - 225.
10. John David Jackson: "Classical Electrodynamics", John Wiley & Sons, Inc., Copyright 1962, pp. 202-231.
11. Julius Adam Stratton: "Electromagnetic Theory", McGraw-Hill Inc. 1941, pp. 301-340.
12. Morgan Mitchell and Raymond Y. Chiao: "Causality and Negative Group Delays in a simple band-pass amplifier", American Journal of Physics, Vol. 66 no. 1, January 1998.
13. R. Y. Chiao, E. L. Bolda, et al. : "Superluminality and Amplifiers", Prog. Cristal Growth and Charact. Vol 33, pp. 319-325, 1996, Elsevier Science LTD.
14. Eric L. Bolda, Raymond Y. Chiao and John C. Garrison: "Two Theorems for the Group Velocity in Dispersive Media", Physical Review, vol. 48 no. 5 November 1993.

Figures:

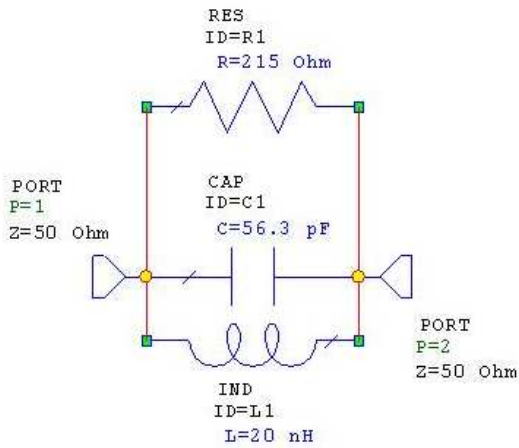


Figure1: Test-case circuit

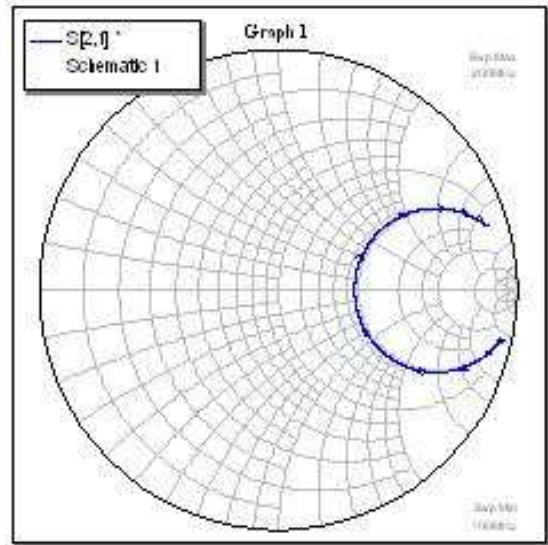


Figure 3: Polar chart (on smith chart) of band-stop section transmission function

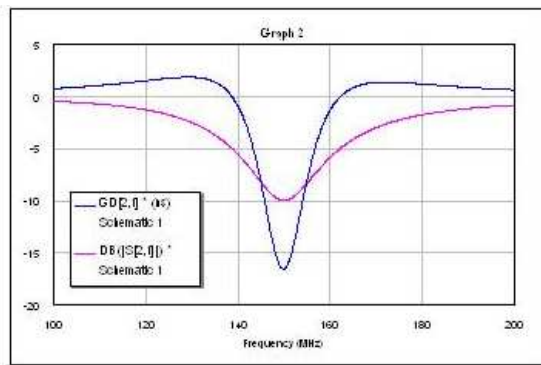


Figure2: Standard analysis of amplitude and group-delay transmission

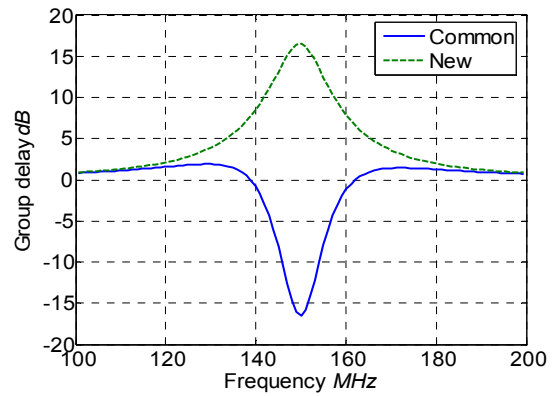


Figure 4: Group delay calculated for common and new formulas