Dix inversion constraned by L1-norm optimization

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Abstract

Two widely used classes of velocity models are blocky and smooth velocity models. The blocky characteristic in the intervel velocity can be represented in geologic environments such as carbonate layers, salt bodies and strong faultings. Using L1-norm optimization, we expect to find a blockier representation of the interval velocity. We make a simple 1-D synthetic 2-step interval velocity model to test the algorithm. A 1-D real data example is showed later in the paper.

INTRODUCTION

Dix inversion estimates interval velocities from picked stacking velocities. The result of constraint least-squared Dix inversion always gives you a smooth velocity model. However, to represent the geologic environment such as carbonate layers, salt bodies and strong faultings, we always need a blocky velocity model rather than smooth velocity model.

L1-norm optimization is well known as a powerful tool to yield sparse models. Another advantage of L1-norm optimization is that this method is not sensitive to outliers, thus the results produced by L1 always outperform those porduced by L2 when the data is noisy.

DIX INVERSION AS AN L1-OPTIMIZATION PROBLEM

The Dix Equation gives the linear relationship between the RMS velocity and the square of the interval velocity. We can write the Dix equation as an optimization fitting goal:

$$||W(Cu-d)||_1 \approx 0, \tag{1}$$

where **u** is the unknown model, a vector of squared interval velocities. **d** is the know data, a vector of squared RMS velocity multiplied by time. **C** is the causal integration operator. **W** is a data residual weighting function, which is proportional to our confidence in the RMS velocity.

Fitting goal fgoal itself cannot fully constrain the problem. Therefore, Clapp et al. (1998) supplement this system with a regularization term which enhances smoothness. In this case we use first order derivative:

$$\epsilon D_\tau u \approx 0, \tag{2}$$

where D_τ is the first-order finite differece derivative in time. ϵ is the weight of the model residual.

To speed up the convergence, we always introduce a preconditioning term into the system. Substituting $u = D^{-1} p$ into the system above, we get the precondition equations:

$$||W(CD^{-1}p - d)||_1 \approx 0, \tag{3}$$
$$\epsilon p \approx 0, \tag{4}$$
$$to$$

SYNTHETIC AND REAL DATA EXAMPLE

Figure 1 shows the input synthetic RMS velocities with and without random noise, and the true blocky interval velocity we try to invert for.

CONCLUSIONS

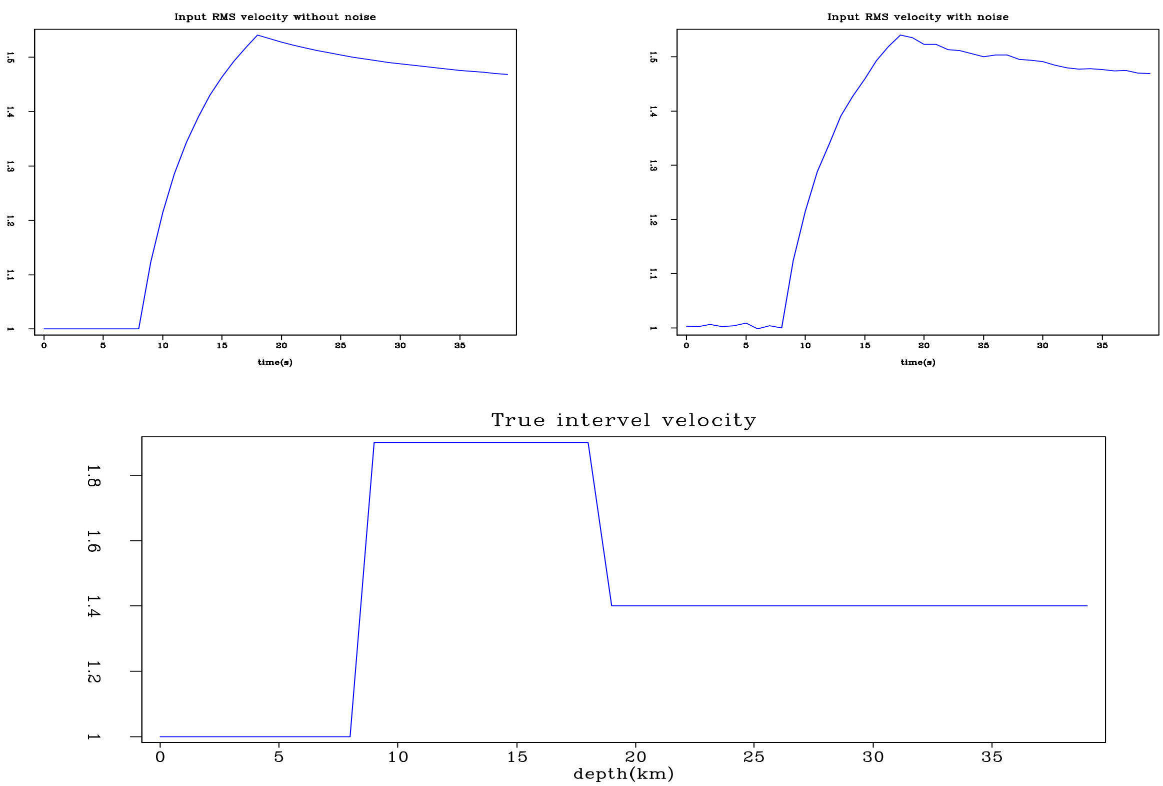

Figure 1: Input synthetic RMS velocity and true interval velocity. The two plots on the top row are the input RMS velocities with and without random noise, respectively. The plot on the bottom is the true interval velocity we try to invert for. **[ER]** **in** **[ER]**

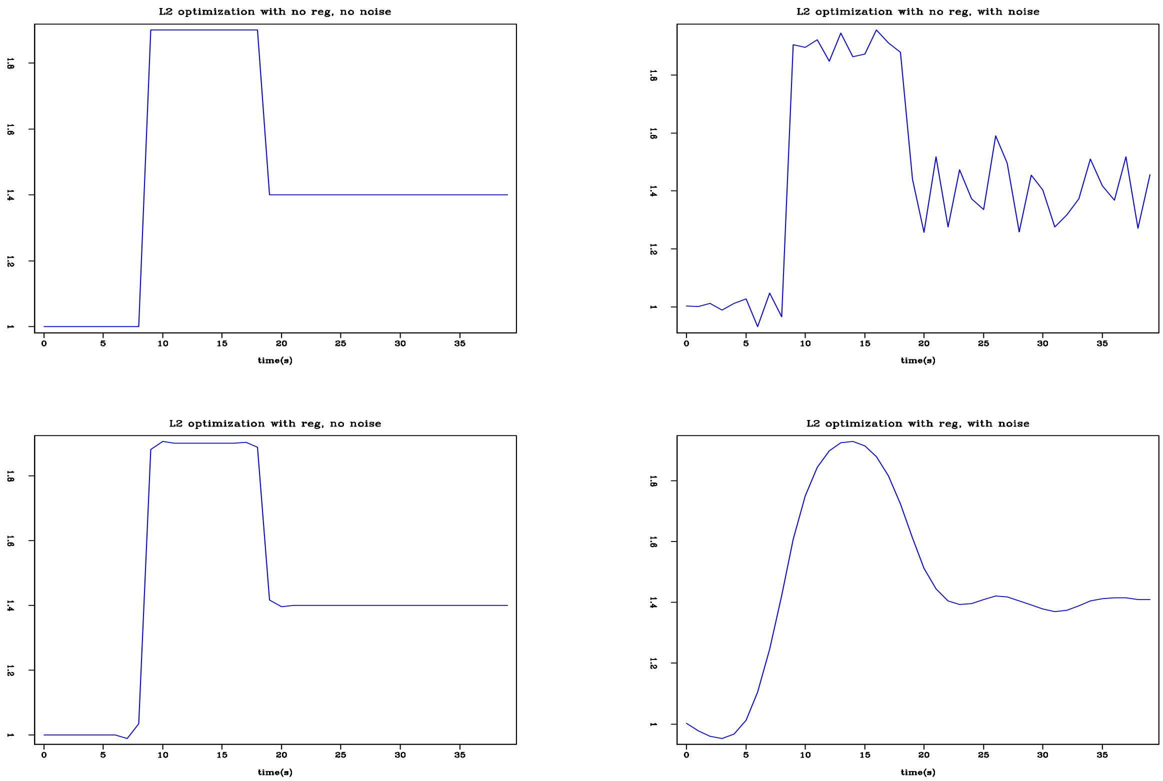

Figure 2: Inversion results using L2 optimization. **[ER]** **l2** **[ER]**

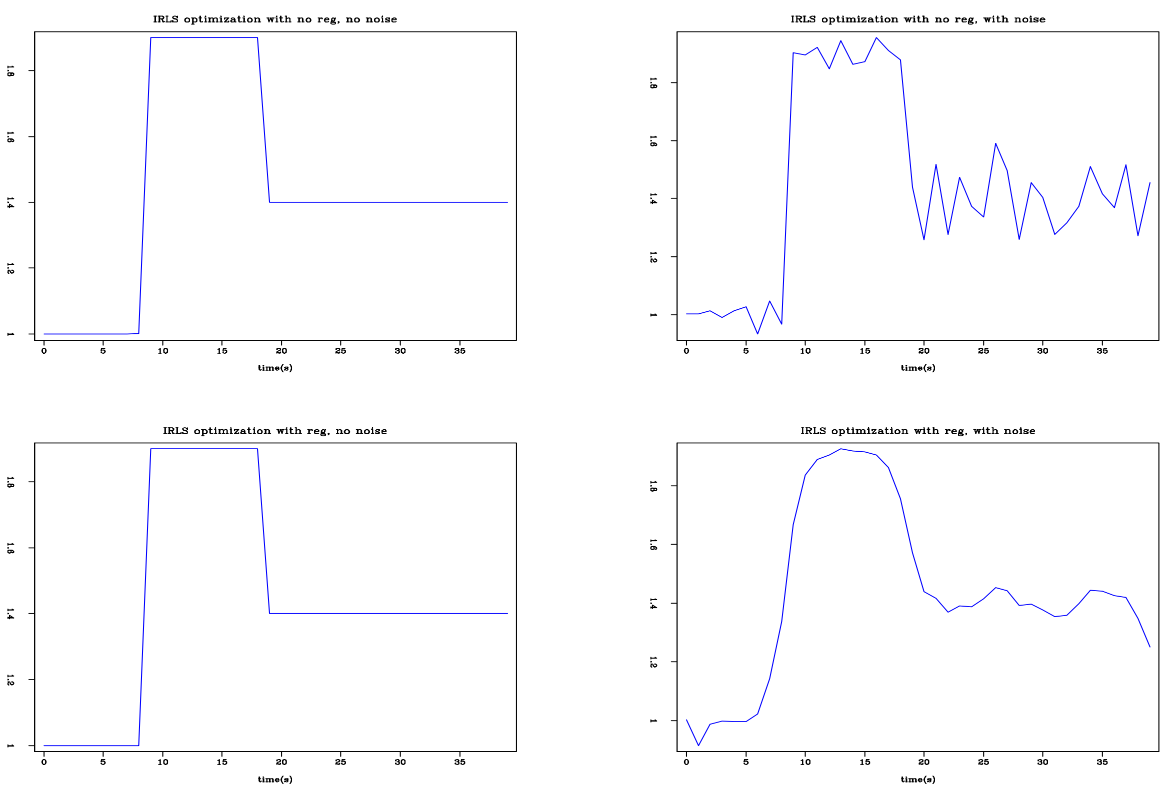

Figure 3: Inversion results using IRLS. **[ER]** **irls** **[ER]**

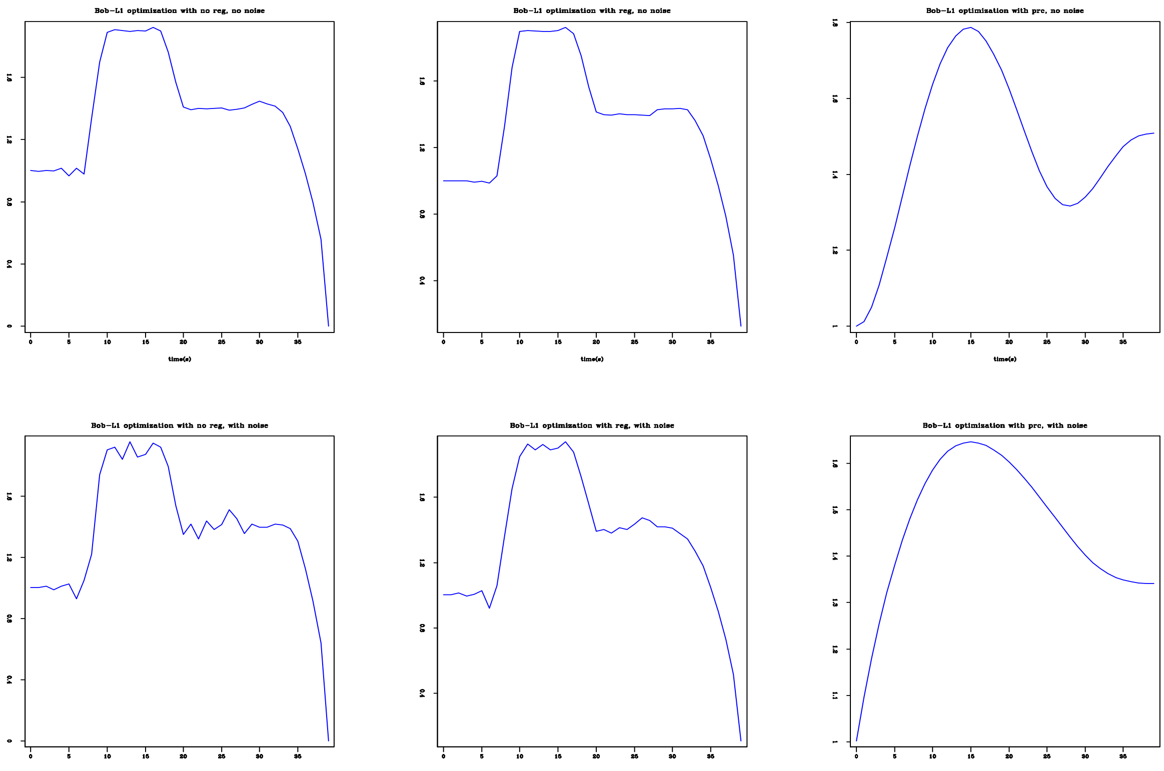

Figure 4: Inversion results using Bob-L1step code. **[ER]** **bob** **[ER]**

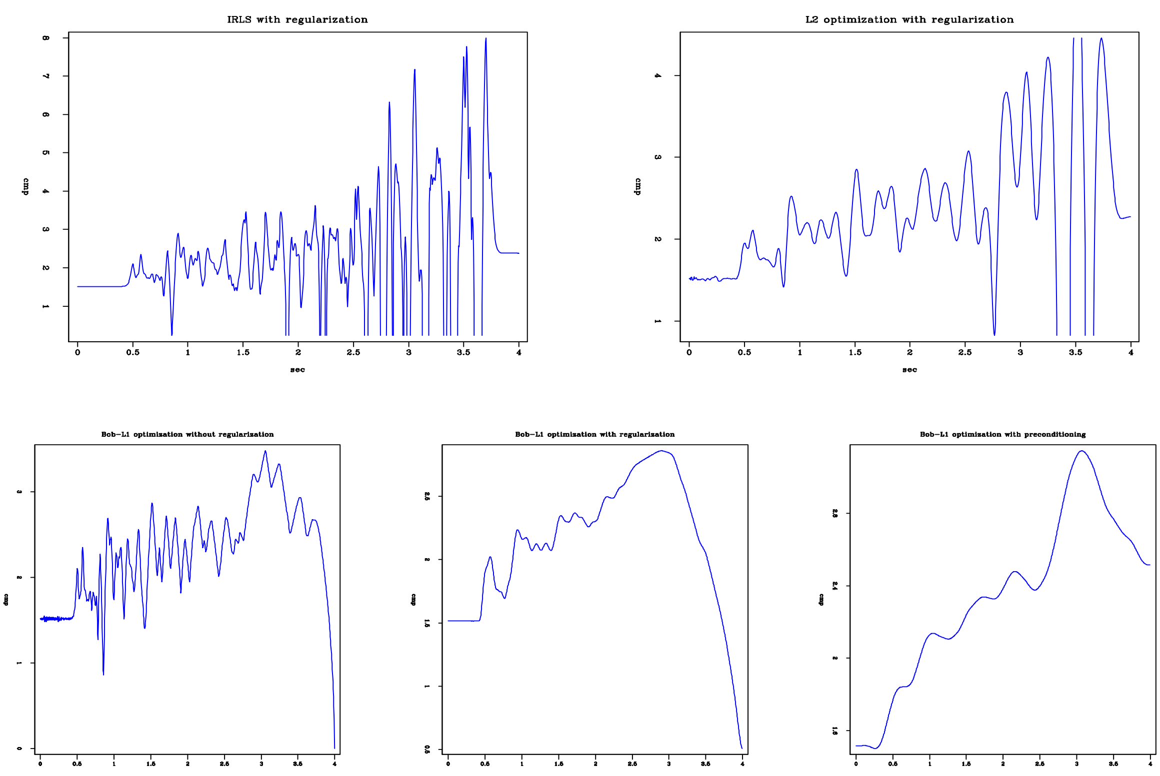

Figure 5: Real data inversion results using different methods. **[ER]** **real** **[ER]**