Fast, effective Kirchhoff time migration curved ray moveout corrections
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Summary

I present a family of solutions to the problem of including curved ray corrections into Kirchhoff prestack time migration. Parameterization of these alternate moveout formulas is obtained by least-squares fitting of traveltimes generated by fast local v(z) traveltime precomputations. The methods provide relatively accurate results at minimal extra cost beyond that of conventional Kirchhoff prestack time imaging.

Introduction

In typical Gulf Coast settings, conventional normal moveout visibly overcorrects arrivals near and beyond the 45º mark. The overcorrection tends to form dramatic “hockey-stick” features such as those highlighted in Figure 1. Such misalignment (and overstretch) directly interferes with our ability to stack image gathers or perform post-migration AVO analysis. Hence it can be desirable to quantitatively predict or fit the anomalous moveout and modify our imaging processes accordingly.

A number of authors, e.g. Sun and Martinez (2002), Sherrill and Bramblett (2002), or Causse, Haugen and Rommel (2000) [an excellent reference to the literature on series expansions of traveltimes], have deployed time migration algorithms that modify the standard double square root traveltime formula to better match wide-angle arrival times and avoid overcorrection. All of these entail several times the computational load of simple NMO, but are designed to be reasonably accurate within the limits of their underlying models. The result of Ghosh and Kumar (2002) that the famous Taner and Kohler (1969) traveltime power series is only an asymptotic expansion highlights the general need to formulate higher-order moveout methods within carefully restricted domains.

The practical aim of any such extension is basically to satisfy three, possibly competing, criteria:

1. better account for actual moveout,
2. avoid large computational penalty, and
3. preserve AVO amplitude anomalies.

Like the work of Sun and Martinez, the basis for the first method I will describe in the next section arises from the study of anisotropy, VTI media in particular. But instead of attributing all moveout deviations to anisotropy, I instead use the anisotropic parameter, \( \eta \) in this case, simply as a proxy for the effect of ordinary stratified layering.

A fast, effective method

Harlan (1995) gives the following formula which approximates nonhyperbolic VTI moveout as an NMO-like traveltme relationship with an offset-variant moveout velocity taking a particular, simple form:

\[
\begin{align*}
1 & = \frac{1}{V_h^{2\text{max}}(h)} \left[ 1 + 2\eta \left( \frac{h^2}{h_{\text{max}}^2 + V_h^2 t_0^2} - \frac{h^2}{h_{\text{max}}^2 + V_h^2 t_0^2} \right) \right]
\end{align*}
\]

Here \( \eta \) represents some vertically-averaged measure of the transverse isotropic parameter used in Tsvankin and Thomsen (1994) and \( h_{\text{max}} \) is a user-selected maximum time-variant offset of interest, e.g. a post-NMO stacking mute curve.

One beauty of this formula is that estimation of \( \eta \) from field data can be separately and stably done after initial (weighted) velocity analysis to tie down far-offset behavior. We then can adjust \( \eta \) to flex the predicted traveltme curve upwards or downwards over the intermediate offsets until it best matches the moveout in the actual CMP gather.

For the present purpose of prestack time imaging, however, I choose not to estimate \( \eta \) directly from the input dataset. Instead I fit the above Harlan moveout formula to traveltmes automatically calculated from a locally-planar v(z) medium at each velocity control point. This accounts at least for the first-order moveout deviations due to ray bending predicted by Snell’s law for a locally-stratified earth model. For the examples I will be showing, the algorithm I used takes the user-supplied time-variant maximum offset, selects about 10 intermediate offset locations for which v(z) traveltmes are calculated and then does a least-squares fit to come up with an \( \eta \) value for each specific CDP and time in my input RMS velocity table.

The potential bottleneck of calculating v(z) traveltmes turns out not to be an obstacle. I apply Newton’s method to a two-point traveltme calculation to rapidly converge an initial ray arrival at or beyond the offset of interest. By directly using Snell’s law, I avoid the divergence issues of offset expansion pointed out by Ghosh and Kumar. Ray times are computed top-down, far-to-near and only at velocity control points for efficiency.
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Fig 1. Common midpoint gather and simple time migration of a 1500+0.5z velocity gradient synthetic dataset. The three lines are tight, normal and wide stacking time-offset trends used for \( \eta \) estimation.

Fig 2. Common midpoint gather of Figure 1 migrated with curved ray corrections designed over tight, normal and wide design time-offset corridors. One may observe that the wider the design corridor, the more the far-offset hockey stick artifacts are flattened.
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Example

Figure 1 shows a simple Gulf Coast style linear gradient synthetic CMP gather and the output of 3D prestack Kirchhoff time migration without curved ray correction. In Figure 2, I have automatically estimated $\eta$ for each of the three design time-offset lines shown in Figure 1 and generated corresponding migrated image gathers at that same CMP location. As expected, the farther offset images are progressively pushed down as the $\eta$ design corridor widens. We can also see that even the tightest design offset, where conventional time migration appears to have well-aligned events already, produces an image gather that is visibly flatter at farther offsets after curved-ray time migration. Finally, I note that the traveltine misfit of the Harlan formula to the original common midpoint gather is less than 5 msec for even the widest design offset, and is, on average, less than 2 msec.

Another variation

The Harlan formula, can be applied in prestack time imaging at quite a low cost, especially with pretabulation of the terms depending only on $h_{\text{max}}$. This pretabulation comes at the cost of extra storage, which can, at times, degrade migration performance. It also depends upon an RMS-like velocity associated with the largest offset of the design window, which is not the same as the near-offset velocity $V_o$ normally obtained by conventional velocity analysis.

An alternative formula that employs $V_o$ is

$$\frac{1}{V^2_s(h)} = \frac{1}{V^2_o} \left[ 1 - 2\eta \left( \frac{h^2}{h^2 + V^2_o t_0^2} \right) \right]$$

This variation agrees, after some tedious algebraic manipulation, with the original Harlan formula to first order in $\eta$. As Figure 3 shows, it is also extremely similar in practice, with its main effect being to apply a smidgen more far-offset correction.

Yet another variation

To try to get an even faster curved-ray traveltime formula, we can turn to a method due originally to Stolt (1978), later explicated in Levin (1985). This alternate formula is given by:

$$t = (1 - \delta) t_0 + \delta \sqrt{t_0^2 + \frac{h^2}{2V^2_o}}$$

where $\delta$ is an adjustable parameter. Physically, this traveltime model is an average of a horizontal line and a hyperbola with a reduced velocity and has the desirable property that its apex curvature is identical to that of the normal NMO hyperbola, but its linear asymptotes are not quite as steep. In other words, the flanks are raised and can be adjusted by varying $\delta$. Setting $s = 1 - 2\eta$ makes the asymptotic slope match that of the previous traveltime formula.

As Figure 4 shows, in practice this formula tends to undercorrect wider angle arrivals and, indeed, I had to boost $\eta$ by 50% to try to better align the farther offset arrivals for this example. It would appear that a separate least-squares estimation for $s$ is appropriate in order to stabilize this fastest of the three methods.

Conclusions

I have presented three fast, effective methods to implement approximate curved-ray corrections in Kirchhoff time migration. These formulas are simple and can be intuitively understood as smoothly adjusting arrival times between conventional near-offset hyperbolas and faster wider offset arrival times and slopes.

Due to the the explicit near and far-offset controls given by velocity analysis, the key adjustable parameter in these methods is in essence an intermediate offset correction and thus well-constrained when being estimated manually or automatically fit by the fast $s(z)$ ray-tracing I have used.

Finally, within the range over which the time anisotropic parameter $\eta$ is fit, I have found that both methods based on the Harlan formula perform very similarly and are empirically better suited to curved-ray time imaging than that based on Stolt stretch.

References


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Fast, effective PSTM curved ray corrections


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Fig. 3. Comparison of the migrated image gather produced using the modified Harlan formula (L) with the original Harlan formula (R). The differences are slight, with the modified approximation applying slightly more far-offset moveout correction than the original.

Fig. 4. Migrate image gather using the Stolt-derived traveltime formula. Here the \( \eta \) values were boosted by 50% to improve far-offset alignment.