

ANISOTROPIC SEISMIC WAVE PROPAGATION

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Stanford University, 1991

ABSTRACT

Traditionally, theoretical elastic-wave anisotropy has been studied analytically. While formal mathematical analysis can theoretically specify a wavefield exactly and completely, this very completeness often means that the results are expressed as pages of equations. These equations are often made more tractable by limiting the analysis to certain simple cases such as propagation along planes of symmetry or in highly symmetric media. Recent advances in computer power have made the study of theoretical anisotropy directly through *numerical* examples practical for the first time. To this end I present a gallery of examples of numerically calculated impulse-response surfaces and finite-difference wavefield snapshots. These examples are used to demonstrate and expand upon some of the theoretical properties of anisotropic elastic wave propagation predicted from geometrical or mathematical arguments. This philosophy of attack is applied to several varieties of anisotropy. Elliptical anisotropy can be completely modeled as linearly transformed isotropy. To the extent elliptical anisotropy is applicable, images of the subsurface generated by standard geophysical methods are sharp but distorted versions of the true depth picture. This is also true for the case of multiple dipping layers. For the case of two-dimensional transversely isotropic media I present examples spanning the wide range of wavefront behaviors possible in this symmetry system. I also present inequalities that can categorize the behavior from the elastic constants. Two-dimensional transversely isotropic equivalents of the isotropic wavetype-separation operators divergence and curl are derived and applied to finite-difference wavefields. The two-dimensional anisotropic operators work well although they are not as compact as the corresponding isotropic ones. The numerical examples show that mathematically tractable two-dimensional or symmetric cases are not representative of general three-dimensional anisotropy, however. In three dimensions wavetype-separation operators do not work for separating the two *qS* modes because of the obligatory presence of shear singularities tying the *qS* modes together. When a

transversely isotropic three-dimensional medium is perturbed to become orthorhombic a new event dubbed a “connection” can appear. This event acts to channel energy between the former qSV and SH modes outside of the symmetry planes, resulting in seismograms quite different in appearance from the unperturbed case.

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A fool may ask more questions in an hour than a wise man can answer in seven years. – J. Ray (1670), English Proverbs

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