Appendix A

Constant-offset prestack migration

Migration is often described as the inverse or pseudo-inverse of a forward-modeling process. For data collected along a line over a subsurface assumed to be independent of the cross-line direction, a high-frequency asymptotic forward-modeling operator gives 3-D wave propagation in 2-D structure (Bleistein, 1987).

$$
\begin{aligned}
D(\xi,\omega) \;=\; & \sqrt{2\pi\,|\omega|}\;\exp\left[\,3\pi i/4\;\mathrm{sgn}(\omega)\right] \int_C ds\; R(\mathbf{x})\; F(\omega)\; \frac{W(\xi+h;\mathbf{x})W(\xi-h;\mathbf{x})}{\sqrt{\sigma_{\xi+h}^{-1}+\sigma_{\xi-h}^{-1}}} \\
& \exp\Big[i\omega[\tau(\xi+h,\mathbf{x})+\tau(\xi-h,\mathbf{x})]\Big]\;\hat{n}\cdot\left[\nabla\tau(\xi+h,\mathbf{x})+\nabla\tau(\xi-h,\mathbf{x})\right]. \quad \text{(A.1)}
\end{aligned}
$$

$R(\mathbf{x})$ is the reflectivity of the subsurface; for velocity analysis purposes it can be assumed to have no variations with incidence angle. $W(\xi+h,\mathbf{x})$ is the amplitude of a WKBJ asymptotic Green's function and $\tau(\xi-h,\mathbf{x})$ is the phase of the asymptotic Green's function. $F(\omega)$ is the Fourier transform of source function. ξ is the source-receiver midpoint coordinate; h is the half-offset. σ is the independent variable with dimensions of length^2/time used to solve the ray equations by the method of characteristics (Bleistein, 1984). $\hat{n}$ is the normal to the reflector surface C. Equation (A.1) models constant-offset sections, but other representations can be obtained by modifying source and receiver parameterization.

Now consider inversion of this forward-modeling operator. A direct inverse can be found using the methods of Beylkin (1985). Another way to invert equation (A.1) is with the iterative method of LeBras and Clayton (1988). Iterative methods back-propagate or back-project the data onto the model space using the adjoint of the modeling operator and minimize the norm of the difference of the observed data and the data predicted by applying equation (A.1) to the model. The adjoint of equation (A.1) is given by

$$\hat{R}(\mathbf{x}') = \int_{\xi} d\xi \int_{-\infty}^{\infty} d\omega \, \sqrt{2\pi\,|\omega|}\; \overline{F(\omega)}\; D(\xi,\omega)\; \frac{W(\xi+h;\mathbf{x}')W(\xi-h;\mathbf{x}')}{\sqrt{\sigma_{\xi+h}^{-1}+\sigma_{\xi-h}^{-1}}}$$

$$\exp\Big[-i\omega[\tau(\xi-h,\mathbf{x}')+\tau(\xi+h,\mathbf{x}')]\Big]\; \hat{n}\cdot[\nabla\tau(\xi+h,\mathbf{x}')+\nabla\tau(\xi-h,\mathbf{x}')] \,. \qquad \text{(A.2)}$$

The adjoint is a kinematically correct migration operator since it has the conjugate of the phase of the operator in equation (A.1). Despite the being kinematically correct, the adjoint alone is often an inadequate pseudo-inverse to the modeling operator because different parts of the output image are scaled differently. LeBras and Clayton precondition the first back-projection iteration by multiplying the operator in equation (A.2) by the inverse of the diagonal of the Hessian of Equation (A.1).

$$\begin{aligned} H(\mathbf{x}') \;\; &= \;\; \int_{-\infty}^{\infty} d\omega \int_{\xi} d\xi \; 2\pi|\omega| \; \frac{F(\omega)\overline{F(\omega)}}{\sigma_{\xi+h}^{-1}+\sigma_{\xi-h}^{-1}} \; W^2(\xi+h,\mathbf{x}')W^2(\xi-h,\mathbf{x}') \\ & \quad [\hat{n}\cdot[\nabla\tau(\xi+h,\mathbf{x}')+\nabla\tau(\xi-h,\mathbf{x}')]]^2 \, d\xi \; d\omega \;\; . \qquad \text{(A.3)} \end{aligned}$$

For most purposes, post-migration scaling of the image by $1/H(\mathbf{x}')$, is sufficient to balance the amplitudes of the output image. Deregowski (1985) and Deregowski and Brown (1983) give similar expressions for amplitudes of modeling and migration derived in the time domain.

Note that equations (A.1) – (A.3) require specification of the normal to the reflector surface $\hat{n}$. Usually this information is not known during migration because we don't know the shapes of the images of reflectors. Rather than using the unknown normal to the surface, I substitute $\hat{\nu}$, the bisector of the rays from source and receiver, for $\hat{n}$. This bisector *is* the normal to the reflector surface when the rays from source and receiver are the specular rays for the observed reflection. Thus, the amplitudes are correct for the specular rays which give the greatest contribution to the integral over the recording surface.

Equation (2.1) gives the compact form of the expression for forward modeling;

$$D(\omega,\xi) = \int_{\mathbf{X}} A(\xi,h,\mathbf{x},\omega)\ R(\mathbf{x})\ e^{i\varphi(\xi,h,\mathbf{x},\omega,v)}\ d\mathbf{x}\ . \tag{A.4}$$

The amplitude and phase of the forward-modeling operator of equation (2.1) are given by

$$\begin{aligned} A(\xi,h,\mathbf{x},\omega,v) &= \sqrt{2\pi\,|\omega|}\ F(\omega)\ \frac{W(\xi+h,\mathbf{x})W(\xi-h,\mathbf{x})}{\sqrt{\sigma_{\xi+h}^{-1}+\sigma_{\xi-h}^{-1}}} \\ &\qquad \hat{n}\cdot[\nabla\tau(\xi+h,\mathbf{x})+\nabla\tau(\xi-h,\mathbf{x})]\ ; && \text{(A.5)} \\ \varphi(\xi,h,\mathbf{x},\omega,v) &= \omega[\tau(\xi+h,\mathbf{x})+\tau(\xi-h,\mathbf{x})]+\frac{3\pi}{4}\ \mathrm{sgn}(\omega)\ . && \text{(A.6)} \end{aligned}$$

Equation (2.2) gives the compact expression for migration;

$$R'(\mathbf{x}') = \int_\xi \int_\omega B(\xi,h;\mathbf{x}',\omega)\ D(\omega,\xi)\ e^{-i\varphi(\xi,h,\mathbf{x}',\omega,v)}\ d\xi\ d\omega\ . \tag{A.7}$$

The amplitude and phase of the migration operator of equation (2.2) are given by

$$\begin{aligned} B(\xi,h,\mathbf{x}',\omega,v_n) &= 1/H(\mathbf{x}')\sqrt{2\pi\,|\omega|}\ \overline{F(\omega)}\ \frac{W(\xi+h,\mathbf{x}')W(\xi-h,\mathbf{x}')}{\sqrt{\sigma_{\xi+h}^{-1}+\sigma_{\xi-h}^{-1}}} \\ &\qquad \hat{\nu}\cdot[\nabla\tau(\xi+h,\mathbf{x}')+\nabla\tau(\xi-h,\mathbf{x}')]\ ; && \text{(A.8)} \\ \varphi(\xi,h,\mathbf{x}',\omega,v_n) &= -\omega[\tau(\xi+h,\mathbf{x}')+\tau(\xi-h,\mathbf{x}')]-\frac{3\pi}{4}\ \mathrm{sgn}(\omega)\ . && \text{(A.9)} \end{aligned}$$

Appendix B

Asymptotic approximation to residual prestack migration

B.1 Residual constant-offset migration

Rewrite equation (2.3) changing the order of integration;

$$R'(\mathbf{x}') = \int_{\mathbf{X}} \int_{\omega} \int_{\xi} A(\xi, h, \mathbf{x}', \omega, v) B(\xi, h, \mathbf{x}', \omega, v_n) \\ e^{i[\varphi(\xi,h,\mathbf{x},\omega,v) - \varphi(\xi,h,\mathbf{x}',\omega,v_n)]} \; R(\mathbf{x}) \; d\xi \; d\omega \; d\mathbf{x} \; . \tag{B.1}$$

Using constant-velocity kinematics to get residual time migration, the total phase can be written as

$$\begin{aligned} \Phi &= \varphi(\xi, h, \mathbf{x}, \omega, v) - \varphi(\xi, h, \mathbf{x}', \omega, v_n) \\ &= \frac{\omega}{v}\left[\sqrt{(x-\xi-h)^2 + z^2} + \sqrt{(x-\xi+h)^2 + z^2}\right] \\ &\quad - \frac{\omega}{v_n}\left[\sqrt{(x'-\xi-h)^2 + z'^2} + \sqrt{(x'-\xi+h)^2 + z'^2}\right] \; . \end{aligned} \tag{B.2}$$

The largest contributions to the integrals over ξ (ignoring the endpoints of integration) come where the total phase of the integral is stationary with respect to ξ (Erdelyi, 1956),

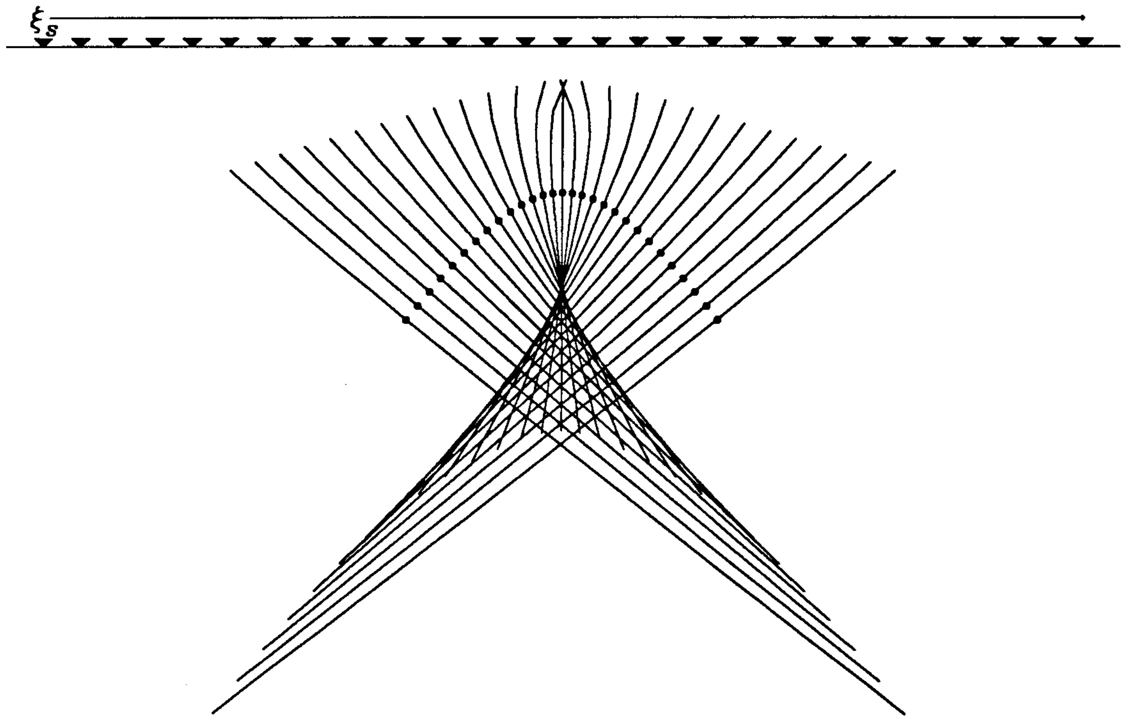

FIG. B.1. Trajectories in $\mathbf{x}$ of constant time dip for a given ξ. Each line corresponds to a different time dip and a different stationary ξ_0. Traveltime varies along each line. The dots correspond to the single points on each trajectory that also have zero total phase.

(Bleistein and Handelsman, 1975).

$$\frac{d\Phi}{d\xi} = \frac{d\varphi(\xi,h,\mathbf{x},\omega,v)}{d\xi} - \frac{d\varphi(\xi,h,\mathbf{x}',\omega,v)}{d\xi} = 0 \quad . \tag{B.3}$$

The stationarity condition on Φ is the same as equation (2.11), that the time dip of reflections imaged at $\mathbf{x}$ with velocity v and at $\mathbf{x}'$ with velocity v_n be the same. Solving equation (2.11) with the method in Chapter 2 section 3 finds stationary points ξ_0 given $\mathbf{x},\mathbf{x}',v,v_n$, and h. The line segments in Figure B.1 show trajectories in $\mathbf{x}$ that have the same time dip as the output point for a fixed ξ_s and the fixed output point $\mathbf{x}'$. Each different trajectory represents a different time dip and thus a different ξ_s, but all trajectories are for the same output point $\mathbf{x}'$. Any given ξ_0 makes the phase stationary for many points $\mathbf{x}$; different points along the same trajectory correspond to $\mathbf{x}$'s that have the same value of time dip for midpoint ξ_0 but different traveltimes. Looking near the center of Figure B.1 note that a given $\mathbf{x}$ can have multiple stationary ξ's; this occurs where the trajectories cross. Using the stationary phase formula, the formal expression for residual migration after integration over ξ is

$$R'(\mathbf{x}') \sim \int_{\mathbf{X}} d\mathbf{x} \int_{\omega_l}^{\omega_u} d\omega \sum_{\xi_0} A(\xi_0, h, \mathbf{x}, \omega, v) B(\xi_0, h, \mathbf{x}', \omega, v_n)\ R(\mathbf{x}_s) \sqrt{\frac{2\pi}{\left|\frac{d^2\Phi}{d\xi^2}\right|}}$$
$$\exp\left[i\varphi(\xi_0, h, \mathbf{x}, \omega, v) - i\varphi(\xi_0, h, \mathbf{x}', \omega, v_n) + \frac{i\pi}{4}\ \mathrm{sgn}\left(\frac{d^2\Phi}{d\xi^2}\right)\right] \quad ; \qquad \text{(B.4)}$$

where ξ_0 are stationary points of the integral over ξ for fixed $\mathbf{x}$. The correction to the amplitude is governed by the second derivative of the total phase with respect to ξ evaluated at the stationary point.

$$\begin{aligned} \frac{d^2\Phi}{d\xi^2} = & \frac{\omega}{v}\left[\frac{(x-\xi_0+h)^2}{((x-\xi_0+h)^2+z^2)^{\frac{3}{2}}} + \frac{1}{\sqrt{(x-\xi_0+h)^2+z^2}}\right. \\ & \left. + \frac{(x-\xi_0-h)^2}{((x-\xi_0-h)^2+z^2)^{\frac{3}{2}}} + \frac{1}{\sqrt{(x-\xi_0-h)^2+z^2}}\right] \\ & - \frac{\omega}{v_n}\left[\frac{(x'-\xi_0+h)^2}{((x'-\xi_0+h)^2+z'^2)^{\frac{3}{2}}} + \frac{1}{\sqrt{(x'-\xi_0+h)^2+z'^2}}\right. \\ & \left. + \frac{(x'-\xi_0-h)^2}{((x'-\xi_0-h)^2+z'^2)^{\frac{3}{2}}} + \frac{1}{\sqrt{(x'-\xi_0-h)^2+z'^2}}\right] . \qquad \text{(B.5)} \end{aligned}$$

The integral over ω is nearly the Fourier transform of a sifting integral. The phase of this integral is monotonic in ω and is governed by the difference in traveltimes implied by $\Phi = \varphi(\xi_0, h, \mathbf{x}, \omega, v) - \varphi(\xi_0, h, \mathbf{x}', \omega, v_n)$. If there were no other factors of ω in the equation, this integral would be non-zero only for combinations of $\mathbf{x}$, $\mathbf{x}'$, and ξ_0 where $\Phi = 0$ for given v and v_n. For combinations of ξ_0, $\mathbf{x}$, and $\mathbf{x}'$ where $\Phi \neq 0$, the integration over ω would be zero. In Figure B.1, the fat point on each trajectory is the single $\mathbf{x}$ location for which $\Phi = 0$ for the ξ corresponding to that trajectory. These combinations of $\mathbf{x}$ and ξ are the only points for which $\Phi = 0$ and $\partial\Phi/\partial\xi = 0$; they define the integration path of the residual-migration operator.

Extracting the dependencies on ω found in the amplitudes of modeling and migration

(A and B) and from the stationary phase calculation write

$$R'(\mathbf{x}') \sim \int_{\mathbf{X}} d\mathbf{x} \int_{\omega_l}^{\omega_u} d\omega \sum_{\xi_0} \hat{A}(\xi_0, h, \mathbf{x}, v)\hat{B}(\xi_0, h, \mathbf{x}', v_n)\ R(\mathbf{x}) \sqrt{\frac{2\pi|\omega|}{\left|\frac{d^2\Phi}{d\xi^2}\right|}}$$

$$\exp\left[i\varphi(\xi_0, h, \mathbf{x}, \omega, v) - i\varphi(\xi_0, h, \mathbf{x}', \omega, v_n) + \frac{i\pi}{4}\,\mathrm{sgn}\left(\omega\frac{d^2\hat{\Phi}}{d\xi^2}\right)\right] \ ; \qquad \text{(B.6)}$$

where

$$\Phi = \omega\hat{\Phi} \ ; \qquad \text{(B.7)}$$

$$A(\xi, h, \mathbf{x}, \omega, v) = \sqrt{|\omega|}\ \hat{A}(\xi, h, \mathbf{x}, v) \ ; \qquad \text{(B.8)}$$

$$B(\xi, h, \mathbf{x}', \omega, v_n) = \sqrt{|\omega|}\ \hat{B}(\xi, h, \mathbf{x}', v_n) \ . \qquad \text{(B.9)}$$

Now recognize the integral over ω as taking a one-half order directional derivative of the image in $\mathbf{x}$ evaluated at the points where $\Phi = 0$ and write

$$R'(\mathbf{x}') \sim \int_{\mathbf{X_s}} \hat{A}(\xi_0, h, \mathbf{x}_s)\hat{B}(\xi_0, h, \mathbf{x}') \sqrt{\frac{2\pi}{\left|\frac{d^2\hat{\Phi}}{d\xi^2}\right|}} D^{\frac{1}{2}}_{\pm t} R(\mathbf{x}_s) d\mathbf{x}_s \ . \qquad \text{(B.10)}$$

The directional derivative $D^{\frac{1}{2}}_{\pm t}$ is taken in the direction of constant time dip and variable traveltime. Depending on the sign of equation (B.5) the derivative is either causal or anti-causal. Residual constant-offset migration now only requires integrating over $\mathbf{X}_s$, the trajectory of points satisfying the stationarity condition and traveltime condition. In terms of equation (2.5),

$$K(\mathbf{x}', \mathbf{x}_s) = \hat{A}(\xi_0, h, \mathbf{x}_s)\hat{B}(\xi_0, h, \mathbf{x}') \sqrt{\frac{2\pi}{\left|\frac{d^2\hat{\Phi}}{d\xi^2}\right|}} \ . \qquad \text{(B.11)}$$

B.1.1 Higher-order stationary points

Some of the stationary points in equation (B.6) can be second or higher-order stationary points. These points correspond to the cusps on the sides and kinks on the top of the summation operators seen in Figures 2.4 and 2.5. For a second-order stationary point on a cusp

$$\frac{d\Phi}{d\xi} = 0 \; ; \; \frac{d^2\Phi}{d\xi^2} = 0 \; ; \; \frac{d^3\Phi}{d\xi^3} \neq 0 \; . \tag{B.12}$$

Rewrite equation (B.6) separating contributions from simple and non-simple stationary points, denote a second-order stationary point as ξ_{00}, and apply the appropriate stationary phase formula.

$$\begin{aligned} R'(\mathbf{x}') \sim \int_{\mathbf{X}} d\mathbf{x} \int_{\omega_l}^{\omega_u} d\omega & \left[\sum_{\xi_0} A(\xi_0, h, \mathbf{x}, \omega, v) B(\xi_0, h, \mathbf{x}', \omega, v_n) \; R(\mathbf{x}) \sqrt{\frac{2\pi}{\left|\frac{d^2\Phi}{d\xi^2}\right|}} \right. \\ & \exp\left[i\varphi(\xi_0, h, \mathbf{x}, \omega, v) - i\varphi(\xi_0, h, \mathbf{x}', \omega, v_n) \; + \; \frac{i\pi}{4} \operatorname{sgn}\left(\frac{d^2\Phi}{d\xi^2}\right)\right] \\ & + \sum_{\xi_{00}} A(\xi_{00}, h, \mathbf{x}, \omega, v) B(\xi_{00}, h, \mathbf{x}', \omega, v_n) \; R(\mathbf{x}) \sqrt[3]{\frac{1}{\left|\frac{d^3\Phi}{d\xi^3}\right|}} \left(\frac{1}{3}\right) \Gamma\left(\frac{1}{3}\right) \\ & \left. \exp\left[i\varphi(\xi_{00}, h, \mathbf{x}, \omega, v) - i\varphi(\xi_{00}, h, \mathbf{x}', \omega, v_n) \; + \; \frac{i\pi}{6} \operatorname{sgn}\left(\frac{d^3\Phi}{d\xi^3}\right)\right]\right] \; . \end{aligned} \tag{B.13}$$

Now the integral over ω can be evaluated as in equation (B.10).

B.2 Residual NMO+DMO

Begin by rewriting equation (2.22), the cascade of constant-offset modeling, constant-offset migration, zero-offset modeling and zero-offset migration.

$$\begin{aligned} R''(\mathbf{x}'') \; = \; & \int_{\omega'} \int_{\xi'} \int_{\mathbf{X}'} \int_{\omega} \int_{\xi} \int_{\mathbf{X}} \Lambda(\mathbf{x}, \mathbf{x}', \mathbf{x}'', \omega, \omega', \xi, \xi', v, v_n, h) \; R(\mathbf{x}) \\ & e^{i\Psi(\mathbf{x}, \mathbf{x}', \mathbf{x}'', \omega, \omega', \xi, \xi', v, v_n, h)} \; d\mathbf{x} \; d\xi \; d\omega \; d\mathbf{x}' \; d\xi' \; d\omega' \; . \end{aligned} \tag{B.14}$$

As with residual constant-offset migration above, use the kinematics of constant velocity to get the total phase Ψ.

$$\begin{aligned} \Psi(\mathbf{x}, \xi, \omega, \mathbf{x}', \xi', \omega') \; = \; & \frac{\omega}{v} \left[\sqrt{(x - \xi - h)^2 + z^2} + \sqrt{(x - \xi + h)^2 + z^2} \right] \\ & - \frac{\omega}{v_n} \left[\sqrt{(x' - \xi - h)^2 + z'^2} + \sqrt{(x' - \xi + h)^2 + z'^2} \right] \end{aligned}$$

$$+\frac{2\omega'}{v_n}\sqrt{(x'-\xi')^2+z'^2}-\frac{2\omega'}{v}\sqrt{(x''-\xi')^2+z''^2} \quad . \quad \text{(B.15)}$$

The amplitude Λ is the product of the amplitudes for constant-offset modeling, constant-offset migration, zero-offset modeling and zero-offset migration.

$$\begin{aligned}\Lambda(\mathbf{x},\xi,\omega,\mathbf{x}',\xi',\omega',\mathbf{x}'') &= A(\xi,h,\mathbf{x},\omega,v)B(\xi,h,\mathbf{x}',\omega,v_n) \\ &\quad \times A(\xi',h=0,\mathbf{x}',\omega',v_n)B(\xi',h=0,\mathbf{x}'',\omega',v) \quad . \quad \text{(B.16)}\end{aligned}$$

Since residual NMO+DMO is composed of two residual-migration operators, it is obvious that the largest contributions to the ξ and ξ' integrals come from the stationary points of the two residual-migration operators. Furthermore, the leading order contributions to the integral over $\mathbf{x}'$ come where the traveltime functions of constant-offset migration and constant-offset modeling are tangent. To express this condition, I perform the $\mathbf{x}'$ integration in (r,θ) coordinates by substituting $r\cos\theta = x' - \xi$ and $r\sin\theta = z'$. The multidimensional stationarity condition states that the gradient of Ψ with respect to the variables of interest must vanish. (Again, I ignore the endpoints of the integrations in ξ and ξ'.)

$$\nabla\Psi(\xi,\xi',\theta) = 0 \quad . \quad \text{(B.17)}$$

$$\frac{\partial\Psi}{\partial\xi} = 0 \; ; \; \frac{\partial\Psi}{\partial\xi'} = 0 \; ; \; \frac{\partial\Psi}{\partial\theta} = 0 \; . \quad \text{(B.18)}$$

Formally write residual NMO+DMO after evaluating these integrals with the multidimensional stationary phase formula (Bleistein and Handelsman, 1975) as

$$\begin{aligned}R''(\mathbf{x}'') \;\sim\; & \int_{\mathbf{X}}\int_r\int_{\omega'}\int_{\omega}\sum_{(\xi,\xi',\theta)_s}\Lambda((\xi,\xi',\theta)_s,\mathbf{x},r,\mathbf{x}'',\omega,\omega')\; R(\mathbf{x})\;\left|\det\left(\frac{\partial\mathbf{x}'}{\partial r}\;\frac{\partial\mathbf{x}'}{\partial\theta}\right)\right| \\ & \exp\left[i\Psi((\xi,\xi',\theta)_s,\mathbf{x},r,\mathbf{x}'',\omega,\omega') \;+\frac{i\sigma\pi}{4}\right]\sqrt{\frac{(2\pi)^3}{|\det(\mathbf{J})|}}\;d\omega\;d\omega'\;dr\;d\mathbf{x}\;; \quad \text{(B.19)}\end{aligned}$$

where $(\xi,\xi',\theta)_s$ denotes a stationary triple. $\partial\mathbf{x}/\partial r$ and $\partial\mathbf{x}/\partial\theta$ build the Jacobian of the change of variables for the $\mathbf{x}'$ integration. The new amplitude factor is governed by the determinant of the matrix

$$\mathbf{J} = \begin{pmatrix} \frac{\partial^2\Psi}{\partial\xi^2} & \frac{\partial^2\Psi}{\partial\xi\partial\xi'} & \frac{\partial^2\Psi}{\partial\xi\partial\theta} \\ \frac{\partial^2\Psi}{\partial\xi\partial\xi'} & \frac{\partial^2\Psi}{\partial\xi'^2} & \frac{\partial^2\Psi}{\partial\xi'\partial\theta} \\ \frac{\partial^2\Psi}{\partial\xi\partial\theta} & \frac{\partial^2\Psi}{\partial\xi'\partial\theta} & \frac{\partial^2\Psi}{\partial\theta^2} \end{pmatrix} . \tag{B.20}$$

The additional phase term σ is the number of positive eigenvalues of minus the number of negative eigenvalues of $\mathbf{J}$.

The integrals over ω, ω', and r can be evaluated using conditions on traveltimes. Only certain combinations of stationary triples $(\xi, \xi', \theta)_s$ and $\mathbf{x}$ and r give non-zero contribution after integration over ω and ω'. To get non-zero contribution to the integral over ω, we have the same condition as in equation (B.10). To get non-zero contribution from the ω' integral similarly requires that the two zero-offset contributions to the phase cancel. I state without proof that to get both of the phase components to cancel at the same time and also satisfy the stationarity conditions simplifies all integrals to one integral over a trajectory in $\mathbf{x}$.

To get the amplitudes, now extract the functions of ω and ω' from Λ and a factor $1/\sqrt{\alpha\omega^2\omega' + \beta\omega\omega'^2}$ from $|\det\mathbf{J}|$ and write

$$R''(\mathbf{x}'') \sim \int_{\mathbf{X}}\int_r\int_{\omega'}\int_\omega \sum_{(\xi,\xi',\theta)_s} \hat{\Lambda}((\xi,\xi',\theta)_s,\mathbf{x},r,\mathbf{x}'') \frac{|\omega||\omega'|}{\sqrt{\alpha\omega^2\omega'+\beta\omega\omega'^2}} \left|\det\left(\frac{\partial\mathbf{x}'}{\partial r}\ \frac{\partial\mathbf{x}'}{\partial\theta}\right)\right|$$

$$R(\mathbf{x})\ \exp\left[i\Psi((\xi,\xi',\theta)_s,\mathbf{x},r,\mathbf{x}'',\omega,\omega') + \frac{i\sigma\pi}{4}\right]\sqrt{\frac{(2\pi)^3}{\left|\det\left(\hat{\mathbf{J}}\right)\right|}}\ d\omega\ d\omega'\ dr\ d\mathbf{x}\ . \tag{B.21}$$

Again, we recognize that equation (B.21) contains the Fourier transform of a derivative-like convolutional operator acting on the image in $\mathbf{x}$ and evaluated at points that simultaneously satisfy the stationarity condition and $\Psi = 0$. Write the operator for residual NMO+DMO as

$$R''(\mathbf{x}'') \sim \int_{\mathbf{X}_s} \hat{\Lambda}((\xi,\xi',\theta)_s,\mathbf{x}_s,r,\mathbf{x}'')\ \mathbf{FT}^{-1}\left[\frac{|\omega||\omega'|}{\sqrt{\alpha\omega^2\omega'+\beta\omega\omega'^2}}\ e^{\frac{i\sigma\pi}{4}}\right] R(\mathbf{x})$$

$$\left|\det\left(\frac{\partial\mathbf{x}'}{\partial r}\ \frac{\partial\mathbf{x}'}{\partial\theta}\right)\right|\sqrt{\frac{(2\pi)^3}{\left|\det\left(\hat{\mathbf{J}}\right)\right|}}\ d\mathbf{x}_s\ . \tag{B.22}$$

In terms of equation (2.25),

$$R''(\mathbf{x}'') = \int_{\mathbf{X}_s} H(\mathbf{x}'', \mathbf{x}_s) \Upsilon_{t,t'} \, R(\mathbf{x}_s) \, d\mathbf{x}_s \quad ; \tag{B.23}$$

the integration weights are given by

$$H(\mathbf{x}'', \mathbf{x}_s) = \hat{\Lambda}((\xi, \xi', \theta)_s, \mathbf{x}_s, r, \mathbf{x}'') \left| \det \left(\frac{\partial \mathbf{x}'}{\partial r} \; \frac{\partial \mathbf{x}'}{\partial \theta} \right) \right| \sqrt{\frac{(2\pi)^3}{\left| \det \left(\hat{\mathbf{J}} \right) \right|}} \quad ; \tag{B.24}$$

the convolutional operator is given by

$$\Upsilon_{t,t'} = \mathbf{FT}^{-1} \left[\frac{|\omega||\omega'|}{\sqrt{\alpha \omega^2 \omega' + \beta \omega \omega'^2}} \, e^{\frac{i\sigma\pi}{4}} \right] \quad . \tag{B.25}$$

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