Appendix B

Bandlimited-ray width

The equation describing the envelope (A) of the bandlimited raypaths shown in Figure 5.2 is:

$$
\begin{aligned}
A &= \frac{b}{\Delta\omega}\left\{\frac{2}{\Delta t^4}\left[1-\cos(\Delta t\Delta\omega)\right]-\frac{2}{\Delta t^3}\Delta\omega\sin(\Delta t\Delta\omega)\right. \\
&\quad \left. -\frac{2}{\Delta t^2}\left[\omega_{max}\omega_{min}\cos(\Delta t\Delta\omega)+\omega_{max}^2+\omega_{min}^2\right]\right\}^{1/2}. \qquad \text{(B.1)}
\end{aligned}
$$

Here b is the product of the Green's function geometrical-spreading terms in equations 2.8 and 2.16:

$$b=\frac{4\pi d}{x^2+d^2/4}. \qquad \text{(B.2)}$$

d is the source-receiver separation; x is the offset from the source-receiver axis; and Δt is the time delay between the direct and x-scattered arrivals:

$$\Delta t=\frac{1}{v}\left(2\sqrt{x^2+\frac{d^2}{4}}-d\right). \qquad \text{(B.3)}$$

The width of the bandlimited raypath is determined by the term dominating A's behavior for small x. In equation B.1 this is the factor multiplying $2/\Delta t^4$: $1-\cos(\Delta t\Delta\omega)$. The first zero of this term marks the boundary of the first main lobe in A, echoing the uncertainty relation discussion of chapter 5 with:

$$\Delta t \Delta\omega = 2\pi. \tag{B.4}$$

The simple inverse relation between Δt and $\Delta\omega$ yields a more complicated inverse relation when solved for the half width x:

$$x = \sqrt{\frac{\pi v}{\Delta\omega}\left(\frac{\pi v}{\Delta\omega} + d\right)}. \tag{B.5}$$

For large $\Delta\omega$ (or $d >> \pi v/\Delta\omega$) this simplifies to:

$$x = \sqrt{\frac{\pi v d}{\Delta\omega}}. \tag{B.6}$$

Redefining $2\pi v/\Delta\omega$ as λ_Δ yields

$$x = \sqrt{d\lambda_\Delta/2} \tag{B.7}$$

—a form reminiscent of the $\sqrt{d\lambda}$ width of Hagedoorn's beams in Figure 1.1.