Chapter 1

Introduction

1.1 Motivation

In the introduction to his classic 1954 paper on migration, J.G. Hagedoorn discusses the gap between the concept *trajectory* and the concept *wavefront.* He notes that while we visualize seismic energy interacting with a velocity field as a ray, the true interaction involves a bandlimited wavelet. Hagedoorn makes an effort to relate rays and waves by introducing the idea of beam width in a diagram similar to that of Figure 1.1. Writing that,

> [a]nalogous to the principle of Huygens-Fresnel, an energy quantum from the source S...can contribute to the first compression received in R if its trajectory does not exceed the minimum path by more than a half wavelength, corresponding roughly to the distance from A to B,

he defines a beam as the region falling within the first Fresnel zone. More specifically, for the constant-velocity transmission geometry of Figure 1.1, the beam width equals $\sqrt{d\lambda}$ (with d the source-receiver separation and λ the dominant source wavelength).

This thesis continues Hagedoorn's efforts to bridge the gap between rays and waves during the course of a comparison of ray-theoretic and wave-theoretic seismic tomography. Tomography names one broad group of inversion methods used for imaging two- and three-dimensional fields—tools that have become increasingly important as seismic data processing develops beyond a one-dimensional earth. Defined as the reconstruction of a field from integrals over paths through the field, tomographic methods are generally

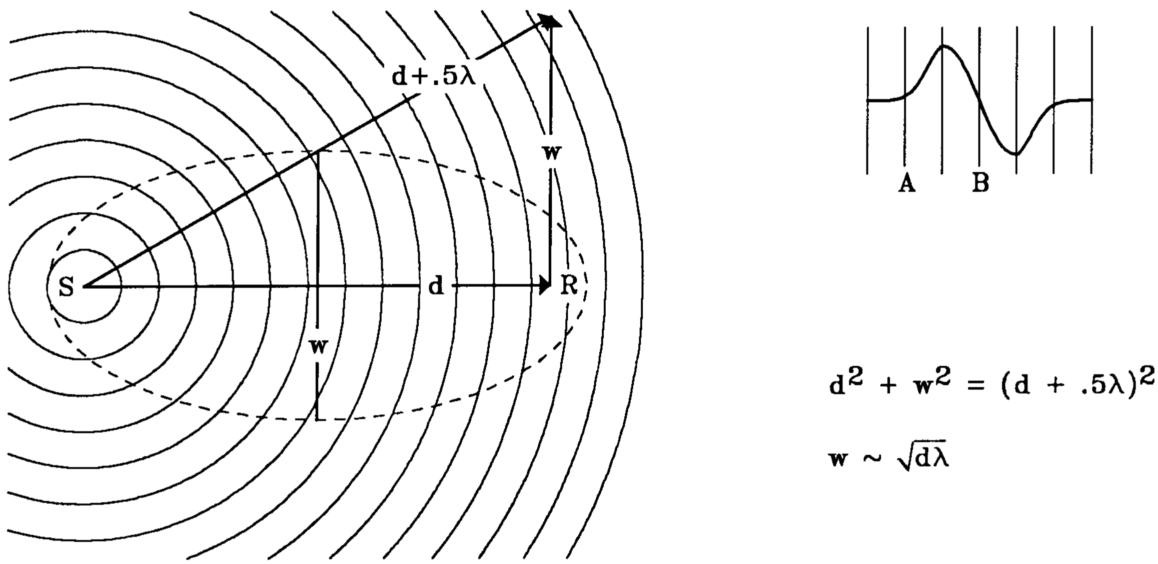

FIG. 1.1. Hagedoorn's beams. Adapted from Hagedoorn (1954).

implemented in four steps. First, the unknown field is illuminated by energy propagating from a known source (or sources) to a recording receiver (or receivers). Second, the recorded energy is compared to the expected energy, calculated by modeling though a first-guess background field. Third, a linear theory is formulated relating the resulting data perturbations to possible field perturbations. Fourth, an updated field is produced by projecting the data perturbations back through the background field—over the source-receiver propagation paths. These propagation paths are the integrals referred to in the definition of tomography: the equations describing the linear interaction of each source-receiver experiment with the field being examined. A comparison of these paths for ray-theoretic and wave-theoretic applications of seismic tomography clarifies the relation not only between the techniques but also between rays and waves themselves.

1.2 Organization

The comparison of ray-theoretic and wave-theoretic seismic tomography is presented in this thesis in four subsequent chapters. The discussion is confined to nonattenuating solutions of the scalar wave equation, although it could be extended to the full elastic case.

1.2.1 Ray-theoretic vs. wave-theoretic tomography: equations

Following this introduction, chapter 2 develops the equations of ray-theoretic and wave-theoretic tomography in a parallel fashion. While the two methods share the four fundamental steps of tomography, their backprojections are usually implemented in different domains. In ray-theoretic tomography the backprojections are most often performed in the space domain, under the title of ray-trace tomography; in wave-theoretic tomography they are most often performed in the frequency-wavenumber domain, under the title of diffraction tomography. This thesis reformulates wave-theoretic tomography in the frequency-space domain, under the title of *wave-equation tomography*. This reformulation not only facilitates the comparison of ray and wave propagation paths, but also makes wave-theoretic tomography more flexible in dealing with irregularly sampled surveys and inhomogeneous background media. The reformulation is described for both Born and Rytov linearizations of the scalar wave equation.

1.2.2 Raypaths vs. wavepaths

Chapter 3 compares the *raypath* and *wavepath* backprojection patterns of ray-trace and wave-equation tomography, as defined in chapter 2. Emphasizing the contrasting ways in which rays and waves interrogate velocity space, the comparison shows that rays and waves lie at two extremes of the uncertainty principle: one assuming infinite bandwidth, the other infinite time. Rytov wavepaths are linked to rays as wave-theoretic trajectories for transmitted energy: the monochromatic versions of Hagedoorn's beams. Born wavepaths are linked with migration as wave-theoretic trajectories for reflected energy.

1.2.3 Ray-trace vs. wave-equation tomography: inversion

Chapter 4 completes the parallel development of ray-trace and Rytov, wave-equation tomography with application of the methods to a synthetic transmission-geometry data set. As an implementation of full waveform inversion, wave-equation tomography is shown to make much fuller use of seismic information than ray-trace tomography—imaging the velocity field with far fewer source-receiver experiments.

1.2.4 Bandlimited raypaths

Chapter 5 returns to the observations of Hagedoorn: linking rays and waves through the uncertainty principle by defining ***bandlimited raypaths.*** A real data example is worked with bandlimited raypaths for a VSP geometry.