Modeling anisotropic porous rocks

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ABSTRACT

Porous rocks may have anisotropic permeability. An algorithm, based on a Markov random field, creates a two-state (rock, pore) model with four degrees of freedom corresponding to porosity and the three components of an orthotropic permeability tensor. The model may find application in simulated flow experiments.

INTRODUCTION

In order to simulate fluid flow through rocks, a porosity model is necessary. The flow is usually modeled through a network of closed conduits or around particles forming a spatial array. Several models have been proposed (Carman-Kozeny, Purcell, Scheidegger, Cornell, Katz, etc.), the majority based on capillary flow. We propose a two-phase (rock, pore), four-parameter (porosity, permeability) built around a random field.

GENERATING THE MODEL

The porous medium generated by the proposed algorithm has four degrees of freedom:

1. The porosity of the rock modeled.

2. The three "directional" components k_i of the permeability tensor.

The porous model is a two-state model, composed of holes and rock. The distribution of the holes is random generated, but the parameters of the distribution are controlled. An initial random uniform distribution is mapped into a gaussian distribution with mean value μ=0. and standard deviation σ=1. Given the gaussian distribution, simply by shifting the values we can change the mean and by multiplication we can change the standard deviation to obtain any required value. As the pores are represented by negative values and solid matter is represented by positive values, an initial normal distribution around 0 will describe a porosity of 50%.

The second concern when generating the model is related to the permeability of the medium. For isotropic porous media, Darcy's law presents simple proportionality between the components of the volumetric flux and the corresponding components of the pressure gradient.

$$v_i = \frac{-k}{\mu}\frac{\partial \mathcal{P}}{\partial x_i} \tag{1}$$

However sedimentary porous media, such as sandstone, have a layered structure and the permeability parallel to the layers is greater than in the perpendicular direction. We arrive at the general formulation of the Darcy's law:

$$v_i = -\frac{1}{\mu}(k_{i1}\frac{\partial \mathcal{P}}{\partial x_1} + k_{i2}\frac{\partial \mathcal{P}}{\partial x_2} + k_{i3}\frac{\partial \mathcal{P}}{\partial x_3}) \tag{2}$$

where k_{ij} form the components of a second-order tensor, the values of which depend on the orientation of the medium with respect to the coordinate system.

Along the principal axes of the porous media the nine components tensor is reduced to a diagonal matrix. We can build then the model using only three values corresponding to the principal values of the permeability.

The three different values for permeability in the model are obtained by convolving the random values which we obtained in the first part of the algorithm by an exponential or gaussian filter. The shape of the filter will determine the level of connectivity of closely positioned pores. A slow decaying filter will generate a higher connectivity, while a fast decaying filter will generate a lower connectivity. The general form of the exponential filter will be:

$$A(x) = (a_0, a_0 c, a_0 c^2, a_0 c^3, \ldots) \tag{3}$$

where $a_0 = \sqrt{1-c^2}$ and $c < 1$. The exponential filter will tend to create a "square" shape of the pores, because the values of the exponential filter are not isotropic in two dimensions, in other words there is not a radial symmetry for two identical exponential filters considered in along two axes of coordinates.

A better filter for this case could be a gaussian filter, which has radial symmetry in two dimensions. The general form of the filter will be:

$$A(x) = (a_0, a_1, a_2, a_3, \ldots) \tag{4}$$

where $a_i = \frac{1}{D}\frac{1}{\sigma}e^{\frac{-i^2}{\sigma^2}}$ and $D = \frac{1}{\sum_i a_i}$.

By filtering the normal distribution in three independent directions, three different values for connectivity and implicit for permeability are obtained, as related to the shape of the filter used. Because filtering is a linear operation, the output of the gaussian distribution after convolution will be also a gaussian distribution, so we don't affect the value of the porosity. The actual algorithm does the convolution by multiplication in Fourier domain. This is done in order to have a continuous wraparound model which can be "tiled", so an infinite medium can be created by just adding the same dataset which represents the model. The multiplication in Fourier domain also increases the computation speed. An example of an anisotropic porosity model can be seen in Figure 1, where the exponential filter is fast decaying in the x direction with $c_x = 0.1$ and slow decaying in the direction of the other two axes with $(c_y, c_z) = 0.9$. A second example in Figure 2, shows an isotropic media, with the same exponential filter used in all three principal directions. The gaussian filter is represented in Figure 3 where the model has more connectivity in the horizontal plane, given by higher values for σ. An isotropic medium obtained by filtering with a gaussian filter is shown in Figure 4.

CONCLUSIONS

There are many attempts in literature to establish correlations between various dynamical properties of porous media. The simplest way to try to establish correlations theoretically, is to have a theoretical model of the porous media which can be treated mathematically. By trying to simulate different phenomena on the given medium it can be seen which models exhibit the characteristic behavior of the porous medium and which do not. If a proper model is found, it can be substituted for an actual porous medium and then one can predict by calculation how the medium will behave under yet untried conditions.

ACKNOWLEDGMENTS

Special thanks to Rick Ottolini for teaching and helping the primary author to convert and display the data for 3-D visualization on the Ardent computer.

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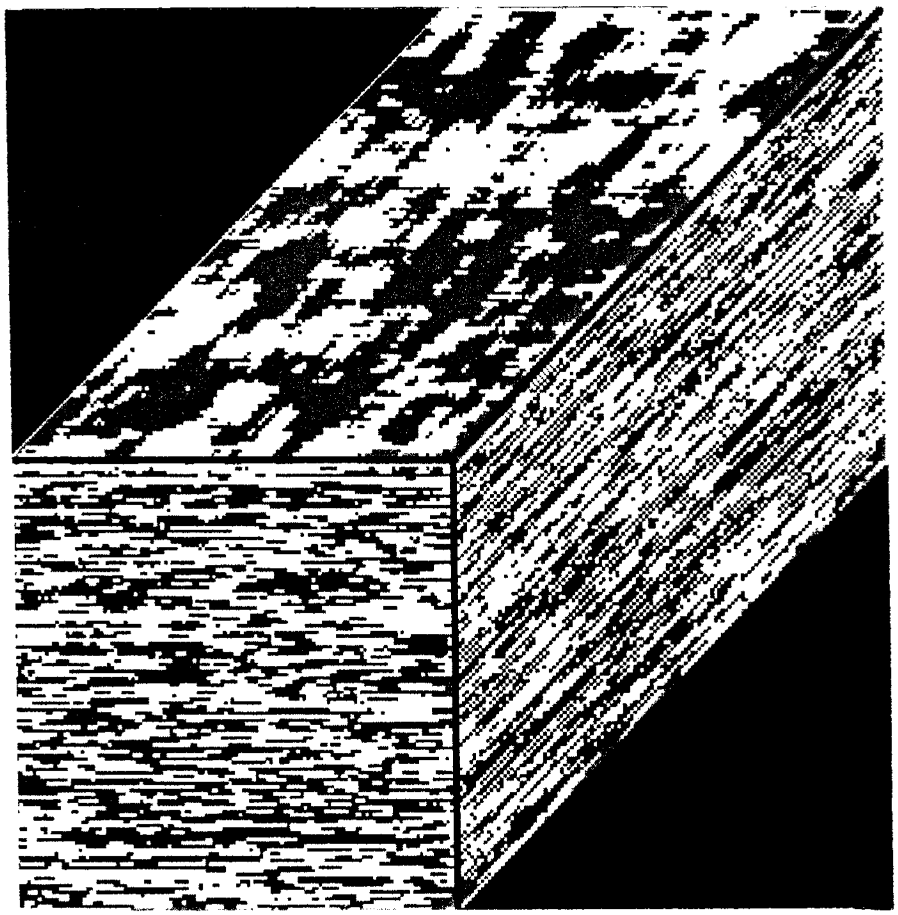

FIG. 1. Porosity model with higher horizontal permeability, exponential filter.

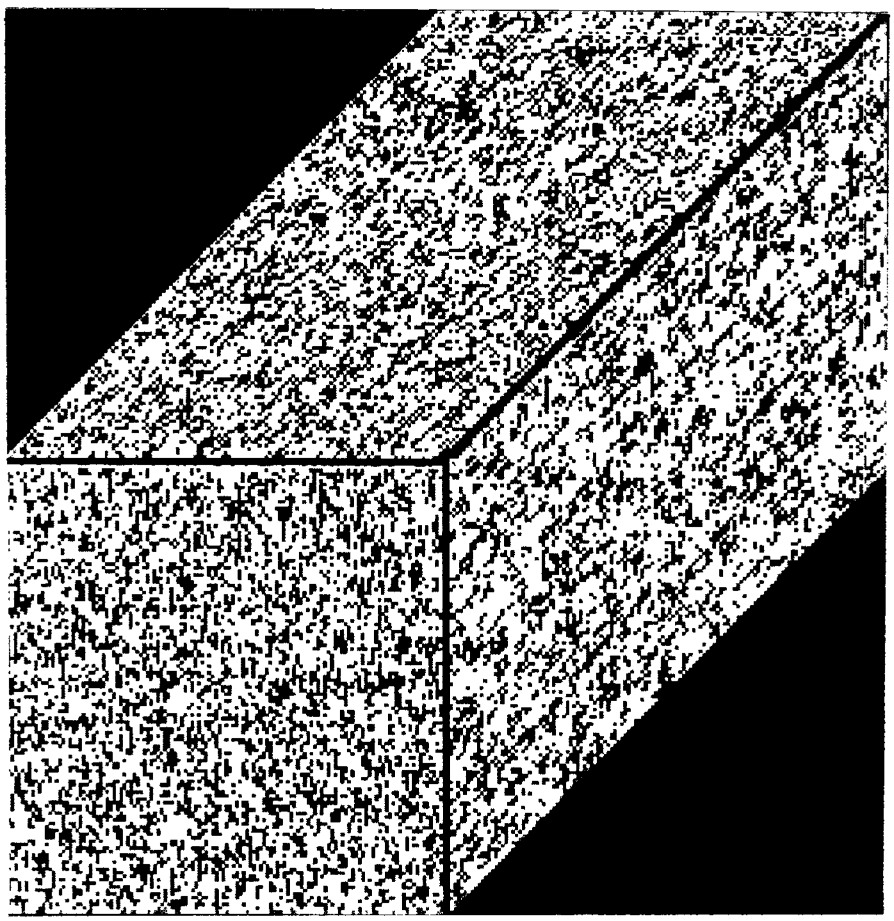

FIG. 2. Isotropic porosity model, exponential filter.

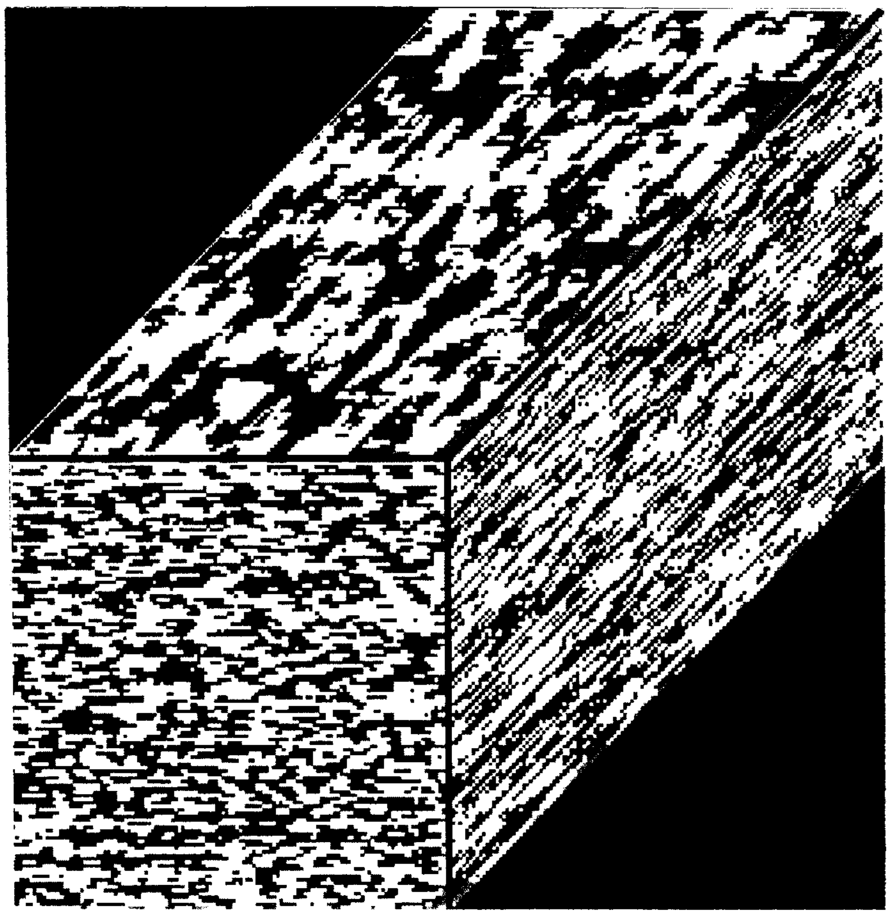

FIG. 3. Porosity model with higher horizontal permeability, gaussian filter.

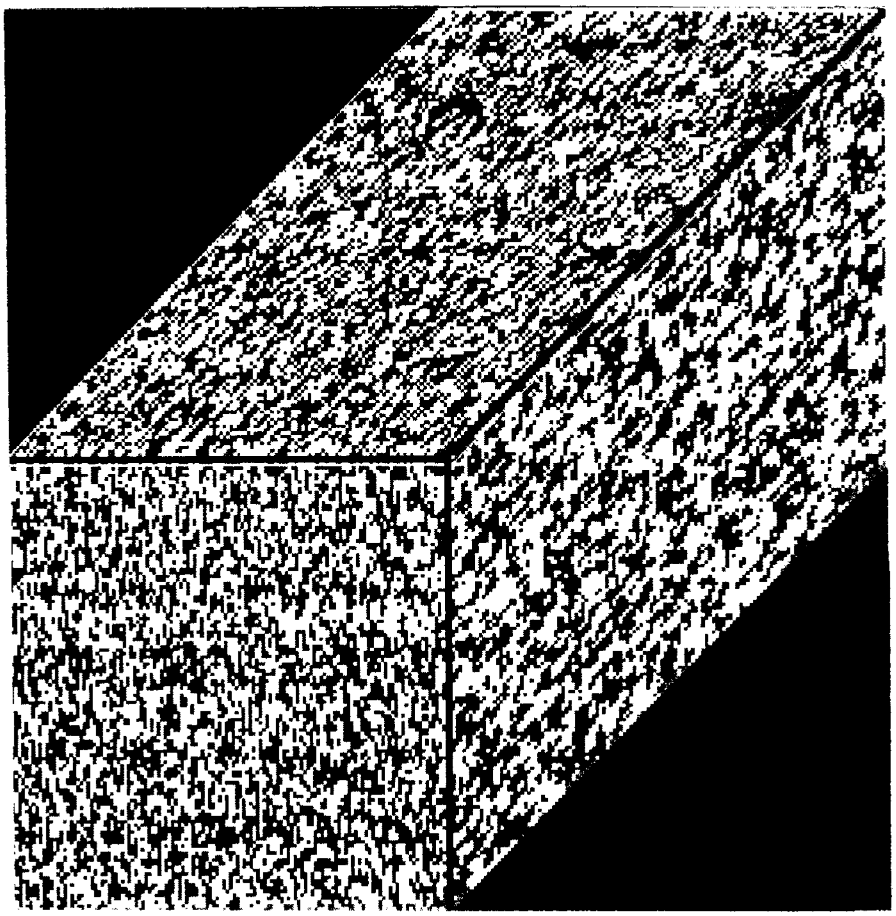

FIG. 4. Isotropic porosity model, gaussian filter.

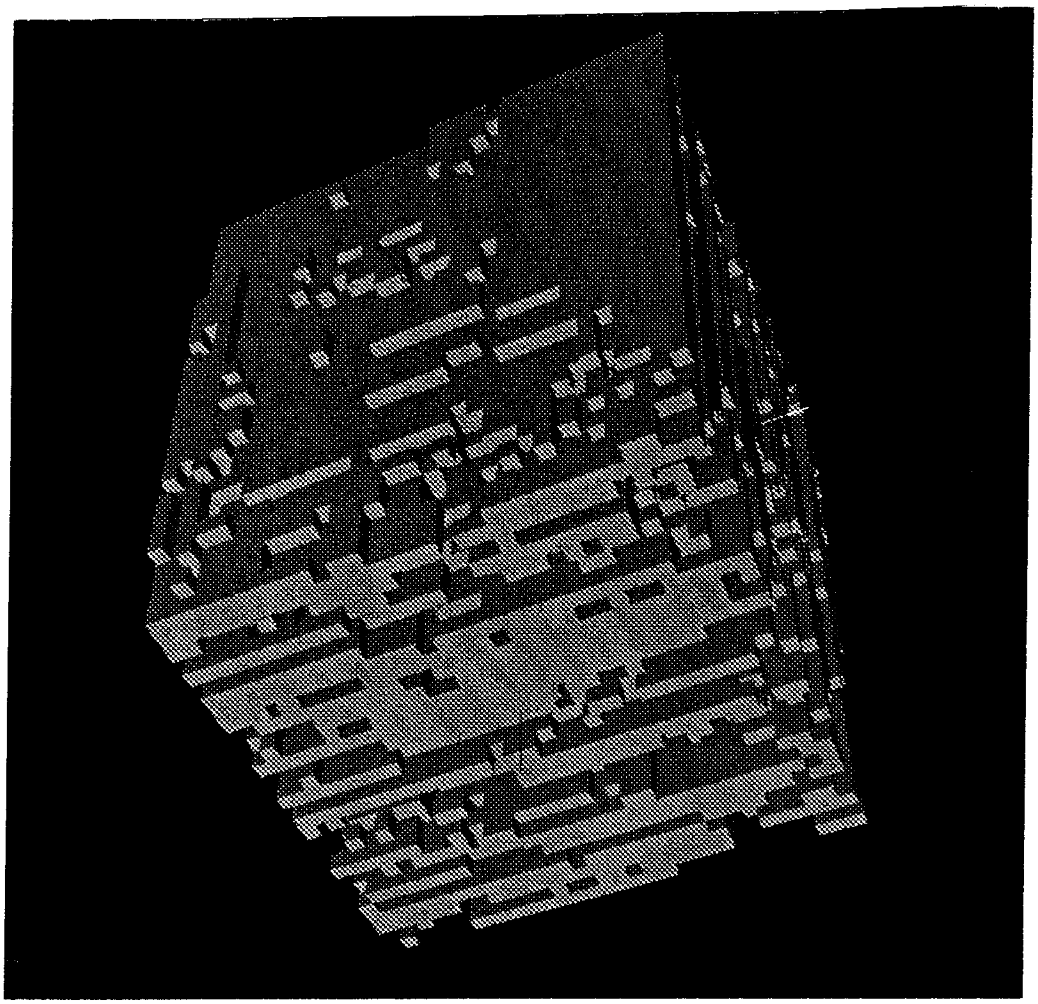

FIG. 5. Smaller data set displayed on Ardent.