Turning multi-component data into tensor fields

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ABSTRACT

Multi-component source and receiver data, so called 9-component data, can be treated as a tensor field only if the source and receiver radiation patterns are uniform. Careful design, such as the use of identical elements and Gal'perin geometry, are sufficient in the case of seismometers. Sources are another matter, since they interact in an unknown, variable and nonlinear manner with the earth's surface. These source radiation patterns are reduced to uniformity by a new technique. The method requires reasonably ideal geophones and fully reciprocal data, but is independent of structural complexity or other simplifying model assumptions.

INTRODUCTION

Three-component data are widely used in reflection seismology. However only a few data sets consist of so called 9-component recordings. One reason is that 3-component sources have not been available, although this deficiency has been recently overcome with sources like Omnipulse or ARIS, and to a lesser extent the horizontal vibrators.

Using three component sources and receivers a 9-component data set is generated which we would like to treat as a cartesian tensor wave field (that is, a vector-vector product field). If we had access to three identical and orthogonal source elements and a similar set of receiver elements, and if their isotropic radiation patterns did not vary within the prospect area, then we would have the fullest possible seismic description of the earth as is available at the surface.

In practice, while it is certainly feasible to design seismometers with these required properties, suitable sources seem unachievable, and we must rely on our seismometers, experimental design and the Reciprocal Theorem to get us out of trouble. Morse and Feshbach (1953) provide the theory for general vector fields, Knopoff and Gangi (1959) the seismic applications, and Fenati and Rocca (1984) provide some interesting practical examples.

In this paper we discuss practical methods of estimating and backing out the internal and source-to-source variations in radiation pattern.

NOMENCLATURE

When talking about multi-component source and receiver data, so called 9-component data, the question arises how to describe the data. A simple way to look at them, is to use the matrix notation

$$(X \quad Y \quad Z)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} Xx & Xy & Xz \\ Yx & Yy & Yz \\ Zx & Zy & Zz \end{pmatrix} \tag{1}$$

where X, Y, Z and x, y, z are the components in cartesian coordinates. Capital letters denote source type and small letters denote receiver type. One data sample in time results then from multiplying the source vector by the receiver vector. The physical meaning is that three shots with orthogonal components are recorded in a receiver having also three orthogonal components. It is useful to regard such a data matrix as a sample of a tensor wavefield. For the data matrix to be really a tensor sample, its components have to be generated and recorded uniformly. Otherwise the tensor elements cannot be directly compared with each other, since they contain still information about different source and receiver mechanisms.

Constructing a tensor sample is then equivalent to equalize source and receiver radiation patterns for the nine components in each shot and also equalize the radiation pattern variation from shot point to shot point.

RECIPROCITY

Seismic wave propagation obeys the reciprocity principle if the experiment is carried out properly. The differential equations which describe wave motion are self-adjoint, leading to a Green's function which is symmetric in it's arguments. Locations and components of source and receiver can be interchanged arbitrarily, yielding the same result. The theory is well known and described by Morse and Feshbach (1953) for general vector wave fields. Knopoff et al.(1959) show applications of the reciprocity principle to seismic waves, and make it that if wave generation and recording are carried out properly, then reciprocity is valid.

The meaning of reciprocity for a vector wave field can be illustrated in a very simple seismic experiment. As shown in Figure 1 imagine a source acting on a free surface of an elastic medium. The source vector (direction of force) at point *Source 1* excites the free surface and radiates elastic waves. The wave is scattered at an

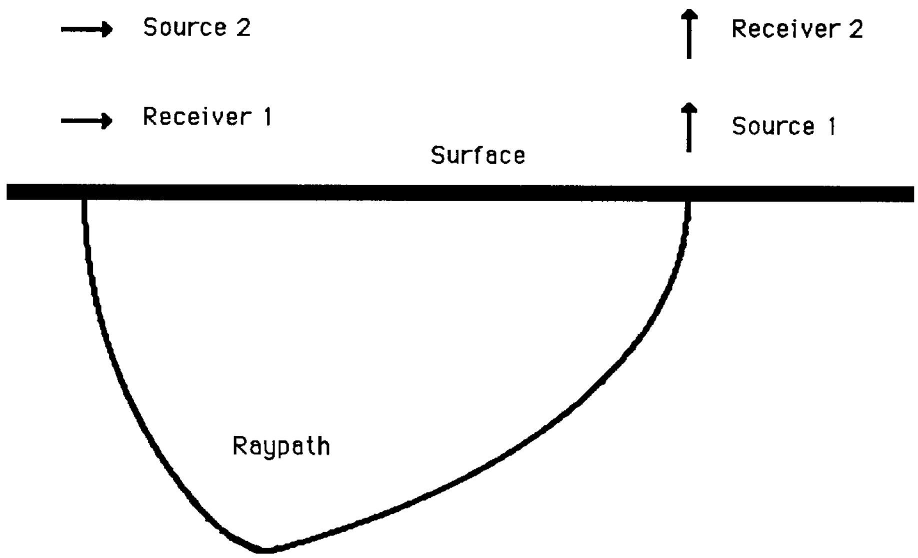

FIG. 1. A reciprocal experiment.

arbitrary point in the medium and detected in *Receiver 1* characterized by its receiver vector. Imagine now a second experiment with source and receiver positions interchanged and the same experiment repeated. Do we get identical results? With this setup we will not replicate the previously recorded trace. However, if we additionally interchange source and receiver vector, we will duplicate the previously recorded trace. The source at the previous receiver position now excites the free surface using the previous receiver vector and the scattered signal is detected at the previous source position using the previous source vector. There will be no differences between the two traces, arrival times and amplitudes will be identical.

It is important to realize that in this experiment the reciprocity relation will always be satisfied, however the medium is constituted. Because of the symmetry of stiffness constants in the elastic equations of motion, reciprocity is always guaranteed, even in the worst case of general anisotropy, arbitrary inhomogeneities and boundaries. Thus reciprocity is a general concept which is not tied to any particular subsurface model. Reciprocity can be cast into a mathematical expression

$$d_{\mathbf{X}\mathbf{x}}(\mathbf{s},\mathbf{r},t) \;=\; d_{\mathbf{x}\mathbf{X}}(\mathbf{r},\mathbf{s},t) \tag{2}$$

which says that data recorded on vector $\mathbf{x}$ at receiver position $\mathbf{r}$ originating from source vector $\mathbf{X}$ at position $\mathbf{s}$ is equivalent to recording data with source and receiver position interchanged and source and receiver vector interchanged.

RADIATION PATTERN

Reciprocity holds if source and receiver show identical behavior. A perfect seismometer records a seismic vector wave field on orthogonal components. Each component has an identical response characteristic to the wave field. The receiver does not influence the movement of the free earth's surface. Consequently, a uniform and isotropic radiation pattern characterizes the ideal seismometer. The ideal source has the same properties. Practically seismometers can be manufactured so as to have a reasonably uniform and isotropic radiation patterns. Sources are another matter; they interact in an unknown, non-linear way with the earth variable from location to location. Once the radiated pulse traveled a certain distance from the source, the wave motion is linear. From this point on the wave field carries with it the non-linear source signature, which we will call the effective radiation pattern in the far field of the source.

Reciprocity allows us to estimate the effective radiation pattern. There will be point to point variations in the high as well as in the low frequency components. Once the effective radiation patterns are determined we can equalize them for all locations. Variations within source components can be determined only to a constant radiation function. Since we have a wide experience with vertical sources and since they are perfectly symmetric, vertical sources can be used as a reference. The differential radiation pattern results from differences between source and receiver radiation patterns and is, in its most general form,

$$f(\omega, p(x), p(y))$$

a function of frequency and ray parameters. This differential radiation pattern function is a global description incorporating all the physical processes occurring in the near field of the source and represents a multidimensional filtering process.

A practical approach expands this function in a Taylor series near the vertical axis (zero dip). The first order approximation is cylindrically symmetrical and a function of frequency only. A one dimensional convolution operation is then necessary to match reciprocal with the original data.

$$d_{\mathbf{Xx}}(\mathbf{s}, \mathbf{r}, t) \;=\; f(t) \;*\; d_{\mathbf{xX}}(\mathbf{r}, \mathbf{s}, t) \tag{3}$$

In practice relationship 2 is rarely found to be exactly satisfied. There might be ambient noise or coupled air waves which are different in the two profiles. Another source of discrepancy is errors in positioning of shots and receivers. These are situations where this model is very likely to fail. In principle, however, reciprocity is not limited to any specific subsurface model.

Figure 2 shows a shot gather and its reciprocal. It is part of a 9-component data set. The gather was recorded on the vertical component in the receiver and the shot fired the vertical component, thus in the above mentioned notation it is the Zz-gather.

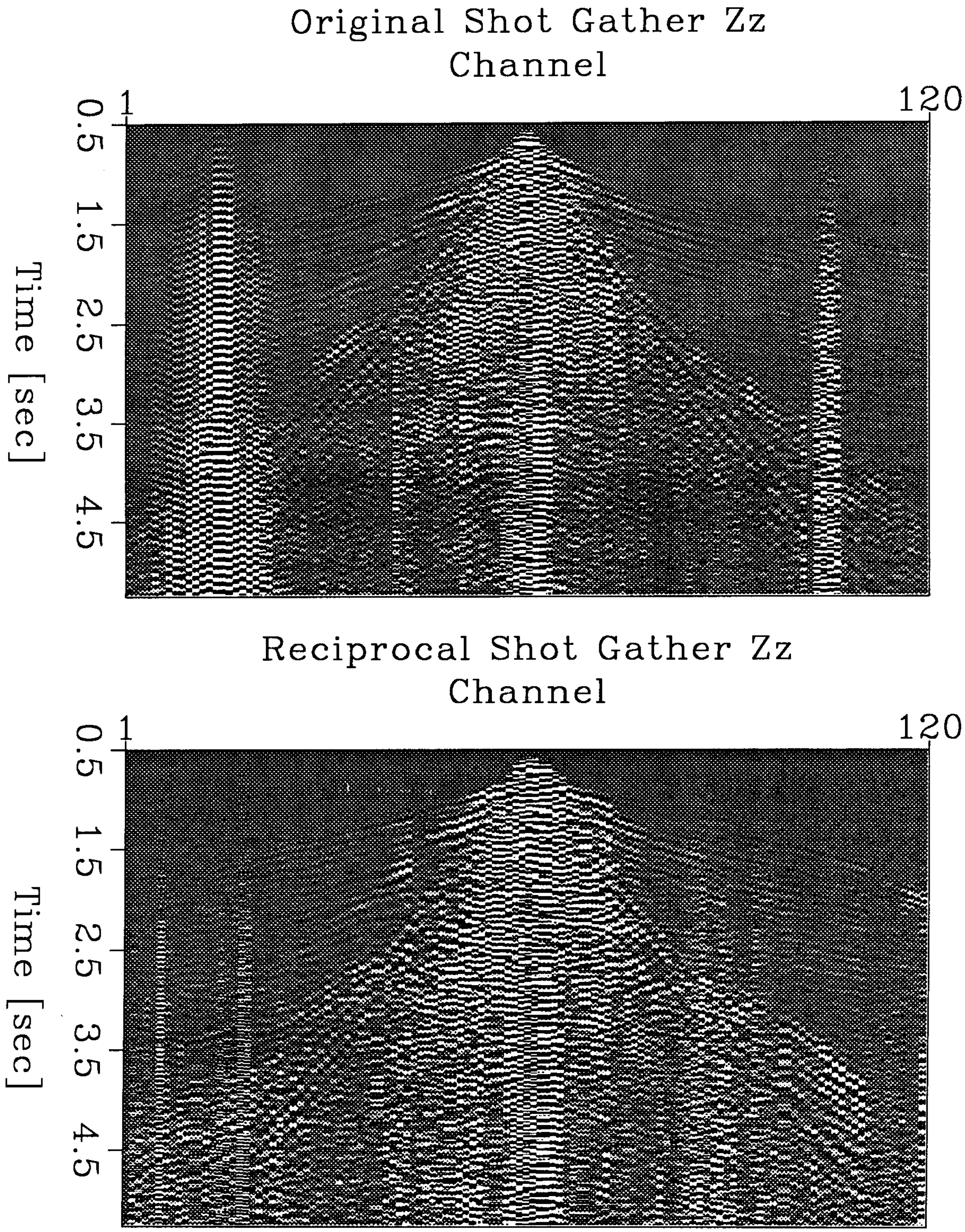

FIG. 2. Shot gather taken from the Lost Hills data set (*Zz*-gather)

FILTER DESIGN TECHNIQUE

Our goal is to find (3-component) source filters that will allow source and reciprocal gathers to be matched in some sense. Such a filter can be implemented in various domains. However, the noise problems associated with raw land data, missing traces, etc. and an interest in interactivity all point towards schemes that operate in the physical domain of time and space. As a further bound on complexity we also choose to limit our filters to the time domain, thinking of this as the first term in a Taylor series expansion about zero dip. Filtering in (x, ω) implies then finding the filter by

$$f(\omega) = \frac{d_{\mathbf{Xx}}(\mathbf{s},\mathbf{r},\omega)\ d^{*}_{\mathbf{XX}}(\mathbf{r},\mathbf{s},\omega)}{d_{\mathbf{Xx}}(\mathbf{s},\mathbf{r},\omega)\ d^{*}_{\mathbf{Xx}}(\mathbf{s},\mathbf{r},\omega)}, \tag{4}$$

where the numerator is the cross-correlation and the denominator is the auto-correlation. Instead of using this global matching method, we can estimate a filter in the time domain, which typically has only a few points. Since we don't expect large differences between the original and the reciprocal, maybe small time shifts or different amplitude scaling, we can construct a short, *best fitting*, filter. Using a lattice technique, we will find a filter which minimizes the total error for a filter with pre-specified length. This is in contrast to a truncated filter, where the error increases with decreasing length. In the spatial direction we can take a robust average to finally end up with a single filter for the whole gather.

A practical approach in removing the radiation pattern, is to look at the diagonal elements in equation 1. These data are acquired using the same component for source as well as receiver. A cross equalization between the data and their reciprocal counterparts yields a filter which when applied to data generated by same source type removes radiation effects on all these components. I.e., the filter which equalizes the original Zz gather with its reciprocal, is applied to Zx, Zy, and Zz; like wise for Xx, Yy and their complementary components.

Once we estimated and removed intra-source-component effects, we can try to find filters which equalize inter-source-components. Since vertical sources are symmetric, we will use it as a reference source. Consequently, we have to cross-equalize corresponding off-diagonal elements in equation 1. I.e., the original gather Xz is matched with Zx reciprocal and the filter applied to Xx, Xy and Xz. Similarly, the original Yz is cross-equalized with Zy reciprocal and the resulting filter applied to Yx, Yy and Yz. Now the data is normalized to have the same radiation pattern for all its various components.

CONCLUSIONS

We propose a method for reducing to uniformity the variable and anisotropic radiation patterns of multi-component sources. The reciprocal principle provides the means, but places constraints on the seismometers (reducible to orthogonal, isotropic) and the recording geometry (split spreads). In paraxial approximation the correcting filters are functions of frequency only, and their calculation amounts to a straight-forward cross-equalization process.

ACKNOWLEDGMENTS

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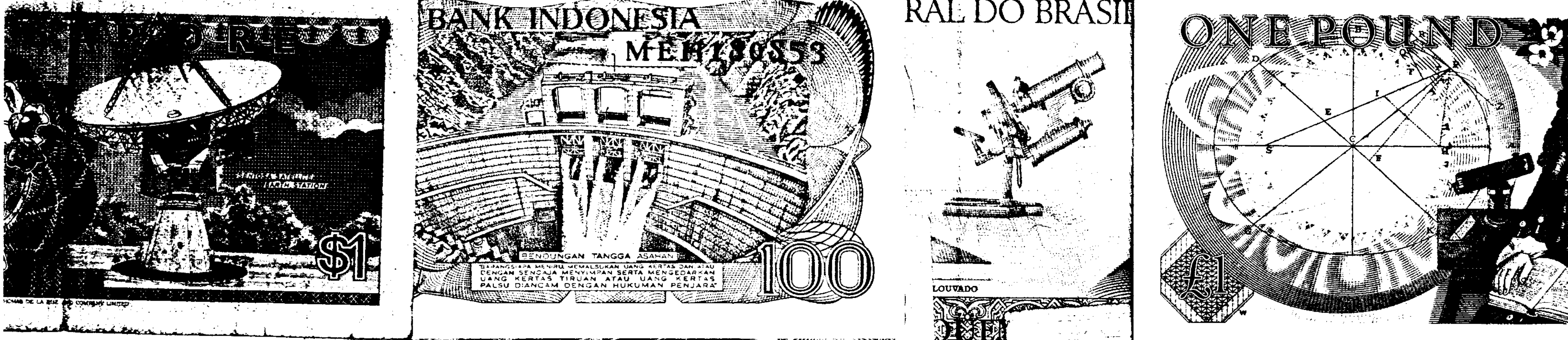

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