The tomographic estimation of seismic velocities from reflected raypaths

William S. Harlan

ABSTRACT

To estimate seismic transmission velocities from traveltimes along reflected raypaths, one must first determine the positions of reflection points. The traveltimes between sources and receivers, if not picked directly from the data, are sufficiently constrained by picked stacking velocities and zero-offset traveltimes. A particular velocity model allows us to estimate reflection points which are most consistent with picked traveltimes and their spatial derivatives at the surface. Perturbations of velocities along the estimated raypaths can then minimize remaining errors in recorded traveltimes or stacking velocities. An improvement in the velocity model allows us iteratively to improve the estimated reflection geometries.

INTRODUCTION

The tomographic inversion of seismic velocities has largely been proposed for experiments with simple raypath geometries, particularly unreflected geometries, such as from well to well. These methods assume that the travel times of waves in the data can be linearly modeled by the spatial integration of the reciprocal of velocity (or slowness) along known raypaths. Estimation of local velocities requires only solving a least-squares inverse of the integrations, an iterative process often called tomographic back projection. At first, straight lines between known endpoints approximate the overlapping, unreflected raypaths reasonably well. A partially inverted velocity function can be used to make a revised estimate of the bent raypaths. An iterative estimation of rays and velocities converges because the raypaths are well constrained by their endpoints.

Such a method can be applied directly to reflection seismic surveys only if the location of reflections are known. Reflected rays could then be broken into two unreflected rays with known endpoints. Unfortunately, the depth and lateral position of reflection points cannot be estimated from the traveltimes in the data until something is known about the velocities.

Loinger (1983) and Toldi (1985) showed that picked stacking velocities and zero-offset traveltimes can be linearized as a function of perturbations in traveltimes at multiple offsets. These traveltime perturbations can be expressed, according to classical tomographic theory, as a linear function of velocity or slowness perturbations along the reflected raypaths. The combined linear operator was inverted by iterative least-squares methods to estimate local velocity anomalies from the picked stacking parameters. To avoid an unwieldy operator, they assumed straight rays and flat, continuous reflectors. Bishop et al (1985) modeled digitized multi-offset traveltimes by tracing rays through a model of rectangular bins (according to Snell's law) down to continuous reflecting surfaces. Gauss-Newton methods (iterative linearization and optimization) alternately optimized the velocities of bins and the depths of reflectors along midpoint. Sword (1987) picked traveltimes and their spatial derivatives directly from unstacked data by local slant stacks. He traced rays down to a depth which exactly satisfied measured times and derivatives, but without obliging endpoints to meet at a single reflection point. He then optimized local velocities in a way that minimized the gaps between the rays' endpoints. Sword allowed curved rays and discontinuous, reflecting structure, but without the advantage of shared reflection points and their sensitivity to relative perturbations in traveltime between high and low offsets. This paper aims to combine the robustness and sensitivity of stacking parameters with the generality of traced rays and discontinuous reflection points. In addition, I shall define the velocity function as a sum of basis functions whose resolution can be adjusted to the precision of raypaths. Dynamic ray-tracing from ray differential equations will facilitate the optimization of reflection points and simplify bookkeeping during backprojections.

I shall first discuss the tomographic estimation of velocities from traveltimes along known raypaths. The limited resolution of the local velocities will depend, above all, on the distribution and geometry of raypaths through the medium. Afterwards we can evaluate the effect of inaccurately positioned raypaths on estimated velocities.

This procedure will be generalized for reflected raypaths by iteratively estimating reflection points that are most consistent with the recorded data and updated velocity models. In addition to traveltimes I will also require indirect measurements of the spatial derivatives of traveltimes at the source and receiver locations. These derivatives effectively constrain the angle at which rays leave and reach the surface. Such data are identical to those used by Sword (1987), but I shall use a very different objective functions to estimate reflection points and to perturb the velocity of the medium.

Stacking velocity analysis, unlike local slant stacks, can compare the traveltimes of rays that reflect from the same physical reflector but begin and arrive at different source and receiver pairs. This redundancy will improve estimates of reflection points and increase robustness in the presence of noise.

I emphasize three major assumptions in this approach. First, velocities and reflection geometries will be parametrized independently. This approach avoids the familiar conceptual model of constant-velocity layers whose boundaries are always conveniently marked by reflections. Seismic stratigraphy argues that transmission velocities do not correlate particularly well with the structure of reflections. In sedimentary environments, the high frequency abrupt changes in impedance that create reflections result from surfaces with equal times of deposition. The magnitude of these changes, however, is not as large as smoother changes in velocity due to post-depositional processes such as compaction.

Second, reflecting surfaces cannot, in general, be assumed either flat or continuous. Since a raypath is well constrained by its reflection point, let us parametrize the reflections as a collection of such points. We should enforce continuity only if the estimated velocities appear poorly constrained otherwise. For interpretation, a depth migration of the original data (with the estimated velocity function) would give the clearest view of reflecting surfaces.

Third, it will be admitted that high spatial frequencies of transmission velocities are uninvertible from traveltimes. The following section discusses how it is possible to parametrize these velocities with an adjustable degree of smoothness.

A SMOOTH VELOCITY MODEL

There are many reasons why we cannot expect to invert high spatial frequency changes in transmission velocities from traveltime information. First, the limited distribution of sources, receivers, and reflecting structure all limit the density of raypaths through the medium. Second, the positions of raypaths cannot be estimated perfectly, particularly when rays are reflected. Random errors in raypaths will blur detail in inverted velocities. Third, the traveltimes of waves between two points are affected by velocities in the region called the Fresnel zone. Inversions of traveltimes cannot hope to resolve structural detail on a smaller scale than the minimum thickness of this zone.

All of these reasons recognize that traveltimes contain integrated and averaged information. Nearby points are likely to affect the same raypaths. Whenever nonuniqueness appears in the inversion, it will be safest to assume that the medium is as homogeneous and featureless as possible. Otherwise, one risks creating a unreliably complicated velocity model that does not explain the data significantly better than a simple one.

Ray tracing will require that velocities be defined everywhere in the medium, not just at discrete sampling points. There is a strong tradition of interpolating a velocity model with "bins," a grid of rectangles with constant velocities. Velocities change only at the boundaries between bins, and then they change sharply. Since we do not expect to be able to invert rapid spatial changes in velocities, we should not allow them in our model. Instead, let us define a continuous velocity function as a sum of smooth basis functions—in effect, a generalization of bins. The maximum resolution will be limited by the width of smoothness parameters w_1 and w_2. These widths should not be less than the sampling rate Δx_1 and Δx_2.

$$v(x_1, x_2) = \sum_i \sum_j v_{ij} \frac{1}{w_1} f \left(\frac{x_1 - i\Delta x_1}{w_1} \right) \frac{1}{w_2} f \left(\frac{x_2 - j\Delta x_2}{w_2} \right) \tag{1}$$

$f(x)$ should be a function with unit area and unit width, such as

$$f(x) = e^{-\Pi x^2}.$$

Conventional bins can be created by a rectangular basis function with the same width as the sampling rate.

In theory, traveltimes should not be integrated along infinitesimally thin raypaths, but over a thicker weighted region called the Fresnel zone. (The thickness depends on the frequency bandwidth of the propagating wave.) The regional integration is as good as the infinitesimal integration only if the velocity function varies little over the width of the Fresnel zone. The basis functions can always be made wide enough to cover the Fresnel zone, without reducing the resolving power of an inversion.

TRAVELTIME TOMOGRAPHY FOR WELL-CONSTRAINED RAYPATHS

Ray tomography assumes that early estimates of velocity and raypaths are already close to a solution that explains recorded traveltimes. Perturbations of seismic velocity along a ray affect both the traveltime and position of the ray–a potentially costly calculation. Fortunately, Fermat's principle notes that a ray minimizes the traveltime between its endpoints, so perturbation of a ray's position affects the traveltime only to second order. Thus we need only consider how perturbations in velocity affect traveltime along the original raypaths.

First I shall define a conventional tomographic inversion such as used for unreflected rays. If the points of reflection are known, then rays can be divided into unreflected parts and inverted similarly. Briefly the steps are the following:

1. A set of traveltimes $\{\hat{t}^k\}$ are recorded along raypaths with known endpoints.

2. A preliminary velocity model $v(\underline{x})$ is chosen as a function of the spatial coordinates $\underline{x}$.

3. A collection of raypaths $\underline{x}^k(\sigma)$ are traced between endpoints as a function of distance σ (straight lines are used for early iterations).

4. The velocity model is reestimated by a "backprojection" that minimizes errors in the recorded traveltimes:

$$Min_{v(\underline{x})} \sum_k \left[\hat{t}^k - \int \frac{d\sigma}{v[\underline{x}^k(\sigma)]} \right]^2 .$$

5. Steps 3 and 4 are repeated until convergence is reached.

Even when velocities change rapidly, unreflected raypaths are unlikely to bend far from a straight line between the known endpoints. Thus, one can feel certain that initial approximate raypaths are enough to improve the velocity model. The improved velocity model can improve the raypaths, and so on.

To define optimal discretized velocities, we need only substitute function (1) into objective function (2). Velocity is, in fact, less convenient to optimize than either slowness (reciprocal velocity) or squared slowness. Whatever function of velocity we choose to define our medium, all can be discretized by a linear superposition in the form of (1).

It is easy to optimize objective function (2) with respect to slownesses because the integrated traveltime is a linear function of reciprocal velocities. The objective function is a quadratic function of discretized slownesses and can be optimized by a gradient descent method such as steepest descent or conjugate gradients (Luenberger, 1984).

ESTIMATION OF REFLECTION GEOMETRIES

The tomographic estimation of local velocities cannot achieve a resolution greater than the accuracy with which raypaths are traced through the medium. Unreflected rays can be well approximated by straight lines without any knowledge of local velocities. Reflected raypaths, on the other hand, suffer from much greater error if their points of reflection are estimated accurately. These reflection points, unlike intermediate points, cannot be approximated without some knowledge of velocities. Thus, the accuracy of estimated reflection points and estimated velocities are mutually dependent.

Reflected rays require an additional iterative loop:

- Find those reflection points most consistent with the data and an assumed velocity model.

- Trace raypaths from sources and receivers to those reflection points.
- Revise the velocity model by tomographic backprojection along the estimated raypaths.
- Repeat these steps until convergence is reached.

The width of velocity basis functions should be at least as large as the expected spatial error in estimated raypaths. When the lowest spatial frequencies of velocity have been estimated with accuracy, we can expect improved accuracy in the estimated raypaths and reflections points. The width of basis functions can then be reduced accordingly.

Dynamic ray tracing

Ray tracing through bins of constant velocity is deceptively simple. A ray passes in a straight line from one boundary of a bin to another. At a boundary the ray bends according to Snell's Law and then passes straight again to the next boundary. The method is simpler to describe, however, than to implement. Rays pass a different distance through every bin. Rays can pass through sides as well as tops and can clip through corners. The integration of traveltime must scale by these varying distances; backprojections must weight velocity perturbations by them.

Dynamic ray tracing – the extrapolation of ray differential equations by finite differences – avoids both these problems. Ray differential equations all derive from the Eikonal equation, a high-frequency approximation of the acoustic wave equation. The Eikonal equation says that the unreflected traveltime $T(\underline{x})$ from a source to any point $\underline{x}$ is constrained by

$$\sum_i \left[\frac{\partial}{\partial x_i} T(\underline{x})\right]^2 = v(\underline{x})^{-2} \equiv m(\underline{x}). \tag{3}$$

The squared magnitude of the gradient of traveltime equals the squared reciprocal of local velocity, to be called squared slowness $m(\underline{x})$.

To rewrite this equation, let us parametrize any particular (unreflected) raypath $\underline{x}(r)$ as a function, not of time t or distance σ, but of r, where infinitesimally

$$dr \equiv v^2 dt \equiv v d\sigma \equiv m^{-1} dt \equiv m^{-1/2} d\sigma. \tag{4}$$

An integral of r along a ray divided by an integral of traveltime is equal to the mean-square velocity. Traveltime becomes a linear integral of squared slowness because $dt = m dr$. We shall, hereafter, find squared slowness most convenient to work with.

Now define a slowness vector equal to the gradient of traveltime. This vector will always point in the direction of the ray.

$$p_i(r) \equiv \frac{\partial}{\partial x_i} T\left[\underline{x}(r)\right] \tag{5}$$

Equations (3), (4), and (5) imply the following ray equations:

$$p_i(r) = \frac{d}{dr} x_i(r), \quad \text{and} \tag{6a}$$

$$\frac{d}{dr} p_i(r) = \frac{1}{2} \frac{\partial}{\partial x_i} m[\underline{x}(r)]. \tag{6b}$$

These two first-order equations can be extrapolated by finite-differences in r, if we know values of $\underline{x}(r)$ and $\underline{a}p(r)$ at one point. If $r = 0$ at the surface then we certainly know $\underline{x}(0)$. We can also measure the lateral spatial derivative of traveltime $p_1(0) = \frac{\partial T\,[\underline{x}(0)]}{\partial x_1}$ by comparing adjacent shots or receivers. The vertical derivative can be solved from a combination of the Eikonal (3) and the definition of the slowness vector (5):

$$p_2(0) = \sqrt{m[\underline{x}(0)] - p_1(0)^2}. \tag{7}$$

Thus we can trace raypaths down from both sources and receivers if we measure the spatial derivatives of traveltime at these points. If the data and squared slowness model are perfect, then these rays should cross at the correct reflection point and give the correct total traveltime. Optimizing a reflection point will require a compromise between the possible errors.

Integrating traveltimes

Once a traced ray has been evenly and sufficiently sampled in r, we can perform the integration of traveltime by summing the squared slownesses at the sampled points and scaling by the sampling rate dr:

$$t = \int m[\underline{x}(r)]dr. \tag{8}$$

Perturbing squared slownesses affects not only the traveltime but also the difference in r between fixed points. So we must write

$$\delta t = \int \delta m[\underline{x}(r)]dr + \int m[\underline{x}(r)]\delta(dr) = \frac{1}{2} \int \delta m[\underline{x}(r)]dr. \tag{9}$$

The second equality is derived by assuming that the position of (and distance between) traced points remains constant. That is, $\delta(d\sigma) = \delta m[\underline{x}(r)]^{1/2}dr = 0$ is expanded and solved for $\delta(dr)$.

Equation (9) allows us to rewrite and optimize objective function (2) for linearized perturbations in squared slowness along traced rays:

$$Min_{\delta m(\underline{x})} \{\sum_k \hat{t}^k - \int m[\underline{x}^k](r)dr - \frac{1}{2} \int \delta m[\underline{x}^k(r)]dr\}^2. \tag{10}$$

Again the superscript k indexes different raypaths. A similar linearization could allow us to optimize other parametrization of the velocity model.

Optimizing raypaths from surface data

To estimate the reflection point of a raypath, we need more than just source and receiver locations and traveltime. We also need the lateral spatial derivatives of traveltime (horizontal slowness) recorded at the source and receiver locations. Assume then that we have five measurements for every raypath (some raypaths may share common reflection points):

$$\left[\underline{x}^{-(0)}, \underline{x}^{+(0)}, \hat{t}, \hat{p}_1^{+(0)}, \hat{p}_1^{-(0)}\right].$$

Minus and plus signs indicate the unreflected rays that begin at the source and receiver respectively – one the downgoing raypath, and one the upcoming raypath. Assume that r increases with depth and equals zero at the source and receiver positions. The hats indicate measurements with certain statistical errors.

These data parameters are those used by Sword (1987). He picked spatial derivatives of traveltime directly on unstacked data by local slant stacks, also called the method of controlled directional receptivity (CDR). Sword first traces rays to a depth at which measured traveltimes and horizontal slownesses are satisfied exactly, but without obliging endpoints meet at a single reflection point. He then optimizes local velocities in a way that reduces the gaps between the endpoints rays.

Instead I propose optimizing raypaths that always unite at chosen reflection points, but which satisfy neither the recorded times or horizontal slownesses exactly. If estimated raypaths always fit the picked traveltimes perfectly, backprojections could not improve the velocity model. Fitting the horizontal slownesses perfectly is also dangerous: a small error in these quantities can create enormous errors in the traveltime. Consider the following algorithm, which optimizes each ray's reflection point individually.

1. Trace rays from sources and receivers to a depth with the correct total traveltimes. These endpoints need not, in general, coincide.

2. Assume that the waves appear planar in the vicinity of these endpoints. The difference in traveltime between nearby points can be calculated from the slowness vectors at the endpoints.

3. While tracing downward, calculate a linear perturbation of the ray for other values of horizontal slowness. Thus one can calculate the initial horizontal slowness that would send the ray through nearby points.

4. Find a common reflection point that minimizes least-squares errors in the picked traveltime and horizontal slownesses. Calculate the corresponding perturbation of the horizontal slownesses and return step one if unsatisfied.

5. This procedure can be carried out simultaneously for a number of rays with the same reflection point.

Step 1 requires only the dynamic raytracing we have already seen. The first iteration can extrapolate with the recorded values of horizontal slowness. Later iterations will begin with perturbed values.

Step 2 requires only that we remember the slowness vector at each endpoint. To calculate the effect of perturbing an endpoint on the traveltime, we can use a plane wave approximation and the definition of the slowness vector (5):

$$\delta t = \sum_i p_i(r)\delta x_i(r). \tag{11}$$

Step 3, sometimes called the "shooting method," requires a linearization of the ray equations (6) with respect to perturbed ray coordinates and slowness vectors:

$$\frac{d}{dr}\delta p_i(r) = \frac{1}{2}\sum_j \frac{\partial^2}{\partial x_i \partial x_j} m[\underline{x}(r)]\ \delta x_j(r) \quad \text{and} \tag{13a}$$

$$\frac{d}{dr}\delta x_i(r) = \delta p_i(r). \tag{13b}$$

To extrapolate by finite differences, set the boundary conditions:

$$\delta x_1(0) = \delta x_2(0) = 0, \tag{14a}$$

$$\delta p_1(0) = 1, \text{ and } \delta p_2(0) = -\frac{p_1(0)}{p_2(0)}\ \delta p_1(0). \tag{14b}$$

The last equality is a perturbation of equation (7). Extrapolation of (13) and (14) gives a function $F_i(r)$ such that

$$\delta x_i(r) = F_i(r)\delta p_1(0). \tag{15}$$

The least-squares inverse of this equation can find the perturbation of the horizontal slowness at $r = 0$ that will allow the ray to pass through a nearby point $\underline{x}(r) + \delta\underline{x}(r)$:

$$\delta p_1(0) = \sum_i G_i(r)\delta x_i(r), \qquad \text{where} \quad G_i(r) = \frac{F_i(r)}{\sum_j F_j(r)^2} \ . \tag{16}$$

Step 4 uses the results of the previous steps to estimate an optimal reflection point. Assume that we have traced rays with approximately the correct horizontal slownesses at the source and receiver down to two nearby points, $\underline{x}^+(r^+)$ and $\underline{x}(r^-)$, with approximately the correct total travel time $t^-(r^-) + t^+(r^+)$. If these rays are perturbed so that they pass through a common reflection point, then traveltimes will be perturbed by equation (12) and horizontal slownesses by equation (16). An optimum reflection point $\underline{x}^{refl}$ should minimize the following errors between recorded and perturbed traveltimes and horizontal slownesses:

$$\begin{aligned}\Delta t = \hat{t} - t^-(r^-) - t^+(r^+) \\ - \textstyle\sum_i p_i^-(r^-)[x_i^{refl} - x_i^-(r^-)] - \sum_i p_i^+(r^+)[x_i^{refl} - x_i^+(r^+)],\end{aligned} \tag{17a}$$

$$\Delta p_1^- = \hat{p}_1^-(0) - p_1^-(0) - \sum_i G_i^-(r^-)[x_i^{refl} - x_i^-(r^-)], \quad \text{and} \tag{17b}$$

$$\Delta p_1^+ = \hat{p}_1^+(0) - p_1^+(0) - \sum_i G_i^+(r^+)[x_i^{refl} - x_i^+(r^+)]. \tag{17c}$$

It is easiest to minimize these errors in a least-squares sense:

$$Min_{\underline{x}^{refl}} \quad = \quad \sum_i [(\Delta t)^2 + \alpha\,(\Delta p_1^-)^2 + \alpha\,(\Delta p_1^+)^2]. \tag{18}$$

α weights the relative importance of accuracy in time and horizontal slowness. Specifically, α should equal the variance of expected errors in time divided by the variance of expected errors in horizontal slownesses. One can choose these variances so that both correspond to an equal variance in the positions of the endpoints.

The minimization can be performed explicitly by differentiating the quadratic objective function with respect to the coordinates of the reflection point, setting the derivatives to zero, and solving the two-by-two system of equations (or a three-by-three system for three dimensions).

We can estimate a common reflection point for several different offsets simply by summing the objective function over all the source and receiver pairs. Allowing multiple offsets to share reflection points is almost always advisable. Lower-offset reflections become useful only if one assumes that they result from the same structure as high offsets. Zero-offset reflections do not constrain a velocity model at all because downgoing and upgoing rays are identical. Changing the velocity merely moves the estimated reflection point up or down the identical rays.

INVERSION OF STACKING VELOCITIES AND ZERO-OFFSET TIMES

The preceding estimation of reflection points required a minimal set of data (9) for each raypath: the horizontal spatial derivative of time measured at the source and receiver points, and a total traveltime. Rather than make these measurements directly (as did Sword with CDR stacks), we can also attempt to invert data which are defined in terms of these measurements. Stacking velocities are one of the few measurements always available that describe the shape of unstacked reflections. Loinger (1983) and Toldi (1985) have already explored the connections between stacking velocities and multi-offset traveltimes. We need only generalize their work slightly to consider dipping reflections.

Stacking velocity analyses (now of many varieties) attempt to find hyperbolic curves that best match reflection traveltimes $t(h)$ over offset h for each reflection and midpoint. Each best fitting hyperbola will be described by two parameters: a squared stacking slowness μ (squared reciprocal of stacking velocity) and a zero-offset traveltime τ:

$$t(h) = \sqrt{\tau^2 + \mu h^2}. \tag{19}$$

Squared stacking slowness and zero-offset traveltime do not have clear physical significance unless reflection geometries are very simple; they merely describe the best-fitting hyperbola. Even when the traveltimes of reflections are not hyperbolic over offset, the best-fitting hyperbola can recognize important relative changes in traveltime over offset.

Estimation of common reflection points

Instead of the five parameters of (11), let us make the following measurements for each midpoint and reflection:

$$\left[y, \{h\}, \hat{\tau}, \frac{\partial \hat{\tau}}{\partial y}, \hat{\mu}, \right] .$$

These measurements are, in order, the midpoint, the set of offsets used in the velocity analysis, the best-fitting zero-offset traveltime, the derivative of zero-offset traveltime over midpoint (best measured from a stacked section), and the best-fitting squared stacking slowness.

If indeed the reflections are hyperbolic, then the measurements (11) can be calculated (20) for every offset included in the velocity analysis:

$$\underline{x}^{-(0)} = s = y - h/2 \ , \qquad \underline{x}^{+(0)} = g = y + h/2 \ , \tag{21a}$$

$$\hat{t} = \sqrt{\hat{\tau}^2 + \hat{\mu} h^2} \ , \tag{21b}$$

$$\hat{p}_1^-(0) = \frac{\partial t}{\partial s} = \frac{1}{2}\frac{\partial t}{\partial y} - \frac{\partial t}{\partial h} \ , \text{ and } \quad \hat{p}_1^+(0) = \frac{\partial t}{\partial g} = \frac{1}{2}\frac{\partial t}{\partial y} + \frac{\partial t}{\partial h}, \tag{21c}$$

where, ignoring small changes in squared slowness over midpoint,

$$\frac{\partial t}{\partial y} = \frac{\hat{\tau}}{\sqrt{\hat{\tau}^2 + \hat{\mu} h^2}} \frac{\hat{\partial \tau}}{\partial y}, \qquad \text{and} \qquad \frac{\partial t}{\partial h} = \frac{\hat{\mu} h}{\sqrt{\hat{\tau}^2 + \hat{\mu} h^2}}. \tag{22}$$

These converted measurements can be used to estimate a single common reflection point for each midpoint and reflection. Objective function (18) need only include a sum over offsets. The redundancy of multiple offsets should make this estimate robust even when data do not satisfy the hyperbolic assumption perfectly. Raypaths estimated independently for each offset would be too sensitive to non-hyperbolic errors.

Tomographic inversion

We could now attempt to use the traveltimes calculated from equation (21b) for all offsets and midpoints to optimize a squared slowness model with objective function (10). Experiments show, however, that a model can fit converted traveltimes well without fitting the original stacking parameters. The problem is that positive or negative errors in traveltime have equivalent effects on the objective function, but not on the best-fitting hyperbola to the perturbed times. Let us instead attempt to optimize the measured stacking parameters directly.

I shall assume, as do Loinger and Toldi, that the best-fitting hyperbola fits recorded traveltimes $t(h)$ in a least-squares sense (although with a weighting different from theirs):

$$Min_{\mu,\tau} \quad \sum_h [t(h) - \sqrt{\tau^2 + \mu h^2}]^2. \tag{23}$$

Both the data and the stacking parameters are implicit functions of midpoint. If we replace this traveltime by an perturbed time $t(h) + \delta t(h)$ corresponding perturbations occur in the stacking parameters $\mu + \delta\mu$ and $\tau + \delta\tau$. These perturbations can be found by a least-squares solution of the following overdetermined system of equations:

$$\delta t(h) = \frac{1}{2} \frac{h^2}{\sqrt{\tau^2 + \mu h^2}} \delta\mu + \frac{\tau}{\sqrt{\tau^2 + \mu h^2}} \delta\tau. \tag{24}$$

The perturbations $\delta\mu$ and $\delta\tau$ can be solved as an explicit function of $\delta t(h)$ using Cramer's rule, as in

$$\delta\mu = \sum_h H_1(h)\delta t(h) \qquad \text{and} \qquad \delta\tau = \sum_h H_2(h)\delta t(h). \tag{25}$$

So far we have been able conveniently to work with every midpoint independently. Now we must make explicit the dependence of our variables on midpoint y. Index each entire reflected raypath $\underline{x}^y, h(r)$ as a function of midpoint y and offset h. Integrate traveltimes through the best model $m(\underline{x})$ so far, and use equation (23) to find reference values of μ^y and τ^y for each midpoint. Let $\hat{\mu}^y$ and $\hat{\tau}^y$ be the measured values for each midpoint. Now let us use equations (9) and (25) to define an objective function that minimizes errors in these stacking parameters:

$$\begin{aligned} &min_{\delta m(\underline{x})} \ \textstyle\sum_y \{ \ \hat{\mu}^y - \mu^y - \frac{1}{2} \ \sum_h H_1(h) \int \delta m[\underline{x}^{y,h}(r)] dr \ \}^2 + \\ &\beta \ \textstyle\sum_y \{ \ \hat{\tau}^y - \tau^y - \frac{1}{2} \ \sum_h H_2(h) \int \delta m[\underline{x}^{y,h}(r)] dr \ \}^2. \end{aligned} \tag{26}$$

The weighting parameter β should equal the variance of expected errors in μ divided by the variance of expected errors in τ. Experiments showed that the second term contributed little to inversion. Nevertheless, both terms should be included, if only to prevent the model from creating drastic errors in zero-offset time.

CONCLUSIONS

We have examined three separate and substantial issues: the conventional inversion of traveltime information for transmission velocities along known ray paths; the estimation of reflection geometries from surface ray parameters; and a generalization of these procedures when data are taken from stacking velocity analyses.

Emphasis was placed on those algorithmic features that simplified and increased the robustness of the inversion. Simplifying physical constraints were kept to a minimum.

We can expect the resolving power of tomographic backprojections to be greatest when rays strike velocity anomalies from several angles, and when rays reflect both above and below the anomaly. Because most reflected raypaths pass vertically through the medium, horizontal resolution must inevitably be much better than the vertical.

Traveltime information does not preserve information about rapid spatial changes in transmission velocities. Even if raypaths are known with accuracy, the density and distribution of raypaths limits their ability to see detailed structure. Traveltimes represent integrated and averaged information along these raypaths–in fact, over the Fresnel zone near the raypath. Many raypaths share common regions of integration. When two anomalies are close to each other, a raypath is unlikely to be influenced by only one of them. Models of the data are thus nonunique. Errors in estimated raypaths decrease resolution further.

For all these reasons, I prefer to parametrize velocity as a sum of smooth basis functions. The width of the basis functions should agree with the expected resolving power of the data. The tomographic optimization should be forced to estimate low spatial frequencies of the velocities first because these are most important for improving the estimated raypaths. As the inversion proceedes, and as raypaths improve, these widths can reduce.

To estimate reflection points, we need more than just reflected traveltimes between different sources and receivers. We also need spatial derivatives of time to constrain the angles of rays at the surface. With these boundary values, we can trace rays downward by finite differencing of differential ray equations. Once rays from the source and receiver have reached the vicinity of a reflection point, the rays can be adjusted until the surface measurements are satisfied as well as possible. The "shooting method" calculates the effect of perturbing a reflection point on the measured derivatives of time. The plane-wave approximation gives the corresponding effect on traveltime. Minimization of an appropriate objective function will find a reflection point that is most consistent with all surface measurements. The robustness of this estimate increases if more than one raypath can be assumed to share the same reflection point.

Finally, we can use the results of stacking velocity analysis–stacking velocity and zero-offset time–to calculate the boundary values necessary for optimizing raypaths at each offset in the unstacked data. The converted values suffer from some loss of accuracy, but we can simultaneously optimize raypaths at many offsets by giving them a common reflection point.

The tomographic perturbation of local velocities can be reformulated to reduce errors directly in the picked stacking parameters. Stacking parameters recognize relative perturbations in traveltime between high and low offsets, but lose detail when these perturbed traveltimes partially cancel each other out. On the other hand, stacking velocities are much less susceptible to noise than would be traveltimes picked for individual shot and geophone pairs. As a compromise, we might stack data over different narrow ranges of offsets.

ACKNOWLEDGMENTS

This theoretical paper is part of a larger applied research project completed during 1986 and 1987 at the Osservatorio Geofisico Sperimentale in Trieste, Italy. AGIP sponsored the research and made available excellent data (to be published at a later date). Many thanks to Fabio Rocca of the Politecnico in Milano, Antonio Carlini of AGIP, and Sergio Persoglia of the OGS for their help.

REFERENCES

Bishop, T.N., Bube, K.P., Cutler, R.T., Langan, R.T., Love, P.L., Resnick, J.R., Shuey, R.T., and Spindler, D.A., 1985, Tomographic determination of velocity and depth in laterally varying media: Geophysics, **50**, p. 903-923.

Luenberger, D.G., 1984, Linear and nonlinear programming: Addison- Wesley Publ. Co., Inc.

Loinger, E., 1983, A linear model for velocity anomalies: Geophysical Prospecting, **31**, p.98-118.

Sword, C., 1987, Tomographic determination of interval velocities from reflection seismic data: the method of controlled directional reception: Ph.D. thesis, Stanford University.

Toldi, J.L., 1985, Velocity analysis without picking: Ph.D. thesis, Stanford University