# Migration under velocity uncertainty

*Lin Zhang*


## ABSTRACT

For inversion purposes, wave propagation may be best described statistically, where velocities are modeled as sums of deterministic and random components. Under the assumption that the probability density functions of the velocities are known, an optimal migration operator is formulated.


## INTRODUCTION

Velocity is a sensitive parameter in the migration processing. Incorrect velocity information will cause the data either to be under-migrated or to be over-migrated, corresponding to lower or higher estimated velocity respectively. Therefore, many techniques have been developed to reduce the uncertainty in estimating the velocity.

NMO and stacking velocity analysis is one of the most widely used methods for estimation of seismic velocities from reflection data. Since the method was formulated by Dix (1955), extensive studies of the theories and applications have been made. Taner and Koehler (1969) adapted this principle to computer processing and developed the concept of velocity spectrum. In their method, to estimate the velocity, one locates the maximum energy points in the velocity spectrum. Later, Taner et al. (1970) pointed out the limitations of their method when it is used in the presence of the complex structures. Thorson and Claerbout (1985) addressed the problems of the finite data points and coherent noise caused by the overlapping events. They used the stochastic inversion method to obtain a high-resolution velocity spectrum. Their work makes the velocity picking much easier and more accurate.

The nonhyperbolic normal moveout can cause systematic errors in the velocity estimation. Brown (1969) studied the accuracy of the NMO correction in long-offset CMP gathers. His results showed that, for horizontal reflectors, a higher order approximation of $v_{rms}^2$ can be obtained if the RMS deviation of the velocity from its mean is considered.

The random timing errors, caused mainly by the inhomogeneity of the media, are a kind of convolutional noise. Their appearance in the seismic data increases the velocity uncertainty much more than does any additive noise. With the aid of standard statistical techniques, Powell (1984) examined the effect of errors in arrival-time measurement on the velocity estimation. His results indicated that the effects of the timing error greatly exceed the nonhyperbolic effects for the horizontally layered model, especially for small offsets.

Extensive studies of the velocities over a horizontally layered medium were done by Al-Chalabi (1974). He showed that the difference between RMS and stacking velocities depends on the length of the recording spread, and the heterogeneity factor, a quantity that measures the degree of velocity deviation from the uniform medium. He demonstrated that the accuracy of stacking velocities in the presence of random timing noise is independent of the heterogeneity factor. If an extra term is included in the travel time expansion, the bias of the RMS velocity estimation can be reduced, but the uncertainty of this estimation will increase. Al-Chalabi's velocity estimation method requires one to pick the hyperbolic events in the CMP gather, which is sometimes impossible in practice.

In conventional migration processing, the operator is designed under the assumption that the estimated velocity is the true velocity. In fact, all previous work (as mentioned above) indicates that no matter which methods are used, the results of the velocity estimation will eventually contain some degree of uncertainty that is sometimes large enough to damage the effectiveness of the migration. The problem that will be addressed in this paper is to find an optimal migration operator for the given statistics of velocities. I will first discuss the causes of the velocity uncertainty and illustrate the similarities between an imaging method used in statistical optics and that used in reflection seismology. Then I will construct an objective function that measures defocusing. The optimal migration operator is found when the objective function is minimized.

## VELOCITY UNCERTAINTY

If the velocities at different locations are assumed to be independent, the uncertainty of the velocities can be fully described by the probability density functions (pdf) of the velocities. To find these pdf, we first derive their analytic forms under simple assumptions, and then estimate the parameters of the functions from the data. In order to make appropriate assumptions, we need to understand where the uncertainty comes from.

### Causes of uncertainty

The causes of the velocity uncertainty can be classified into two types. The first type is related to the effects of the experiment imperfections; for example, the finite length of the recording spread, the finite bandwidth of the source signal, and the

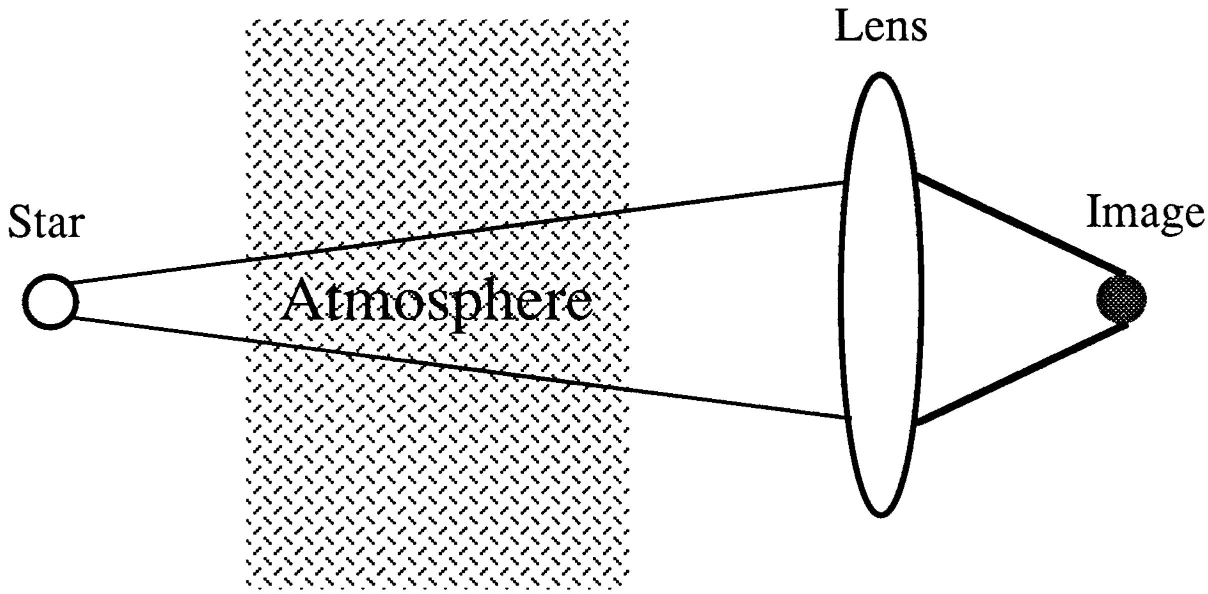


FIG. 1. The geometry of optical imaging through an inhomogeneous medium.

discrete recording position. It is known that the estimation of high velocity contains higher uncertainty than that of low velocity.

The second type of cause is more general. It includes all the effects of the imperfect media, such as the heterogeneity of the media, anisotropy of the media, irregular subsurface, and irregularities caused when 3-D geometry is represented in 2-D data. Some of the processing artifacts and experiment errors can also be included in this type; for example, imperfect static correction and irregular spread geometry, etc. All these phenomena produce either non-hyperbolic moveout or random time perturbations superimposed on the exact hyperbolic diffraction curves. We call these time shifting errors *convolution noise* because the shifting is a convolution operator.

We intend to treat the general problems of velocity uncertainty. Therefore we try to find a migration operator that is uniquely determined by the pdf of the velocities, and for which the causes of the velocity uncertainty are irrelevant.

### Seismic imaging vs. optical imaging

One of the typical problems in optical imaging is how to get a better image of a star. Figure 1 illustrates the geometry of the experiment. The star, as a source, generates light waves that propagate through the inhomogeneous medium (for example, the atmosphere). The image will be degraded because of the presence of the randomly inhomogeneous medium. The goal is to design a lens to focus the image correctly.

Consider the electromagnetic wave propagation through the inhomogeneous medium. The medium is described by the refractive index that is assumed to consist of a large deterministic component and a small random component

$$n(\vec{r}) = n_0 + n_1(\vec{r}) \tag{1}$$

with $|n_1| \ll n_0$. The wave equation for a monochromatic wave with time dependent $\exp(-j\omega t)$ is

$$\nabla^2 U + \frac{\omega^2 n^2}{c^2} U = 0. \tag{2}$$

In reflection seismology, if we accept the exploding reflector model, the situation is very similar. The major difference of the two experiments is just the relative band width of the frequency. Therefore, we can model the medium of the seismic experiments in the same way. The wave propagation velocity will play the role of the refractive index. We assume that the velocity consists of a deterministic component and a random component of zero mean,

$$v = \hat{v} + w, \tag{3}$$

where $\hat{v}$ is the deterministic component and $w$ is the random component with a pdf $f(w)$. Generally, both $\hat{v}$ and $w$ could change with the positions. Here we consider the simple case. The velocity is space invariant and its pdf, $f(v)$, is known.

## MIGRATION WITH AN UNCERTAIN VELOCITY

In a homogeneous medium, the wave phenomenon is described by a second order partial differential equation

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} = \frac{1}{\hat{v}^2}\frac{\partial^2 P}{\partial t^2}, \tag{4}$$

where $\hat{v}$ is a constant. When we consider the presence of velocity uncertainty, we can use the velocity $v$, defined in equation (3), in place of $\hat{v}$.

### Modeling

Because the migration operator we try to find will be a linear operator, it will not change with the input data. Without loss of the generality, we assume that the earth model contains a single diffractor.

$$P(x, z) = \delta(x - x_0, z - z_0). \tag{5}$$

Then, in the Fourier domain, the modeled zero-offset data can be expressed as

$$P(\omega, k_x, v) = \int_{\hat{k}_z} P(k_x, \hat{k}_z)\delta(\omega - v\sqrt{k_x^2 + \hat{k}_z^2})d\hat{k}_z. \tag{6}$$

Remember that $v$ is a random variable instead of a constant. Now if we apply the conventional zero-offset migration operator associated with the mean velocity, the resulting image is

$$\hat{P}(k_x, k_z) = \int_{\hat{\omega}} P(\hat{\omega}, k_x, v)\delta(k_z - \sqrt{\frac{\hat{\omega}^2}{\hat{v}^2} - k_x^2})d\hat{\omega}. \tag{7}$$

Generally, $\hat{P}(k_x, k_z)$ is different from $P(k_x, k_z)$. We can replace the $\delta$ function in equation (7) by a more general migration operator $M(\omega, k_x, k_z)$ to be determined.

$$\hat{P}(k_x, k_z) = \int_{\hat{\omega}} P(\hat{\omega}, k_x, v) M(\hat{\omega}, k_x, k_z) d\hat{\omega}. \tag{8}$$

$M$ is automatically constrained to be a linear operator.

## Objective function

Our goal is to find the optimal operator $M(\omega, k_x, k_z)$, which minimizes the discrepancy between $\hat{P}(k_x, k_z)$ and $P(k_x, k_z)$. The objective function can be defined as

$$J = E\left[\int_x \int_z [P(x,z) - \hat{P}(x,z)]^2 dx dz\right]. \tag{9}$$

The expectation is taken over the velocity variable,

$$E[Y] = \int_v f(v) Y dv. \tag{10}$$

Applying Parseval's theory, we can find the Fourier domain representation of equation (9):

$$J = E\left[\int_{k_x} \int_{k_z} \left|P(k_x, k_z) - \int_\omega M(\omega, k_x, k_z) P(\omega, k_x, v) d\omega\right|^2 dk_x dk_z\right]. \tag{11}$$

## Optimization

The optimal migration operator is defined to be that which minimizes the objective function. Because of physical considerations, we know that such a minimizer exists. Because the objective function $J$ defined in equation (11) is continuous, the minimizer will be a function for which J is stationary. In appendix A, I show that the optimal migration operator, $M(\omega, k_x, k_z)$ satisfies a linear integral equation,

$$\int_{\hat{\omega}} B_P(\omega, \hat{\omega}, k_x) M(\hat{\omega}, k_x, k_z) d\hat{\omega} = \overline{P}(\omega, k_x) P(k_x, k_z), \tag{12}$$

where

$$B_P(\omega, \hat{\omega}, k_x) = E[P^*(\omega, k_x, v) P(\hat{\omega}, k_x, v)],$$

and

$$\overline{P}(\omega, k_x) = E[P^*(\omega, k_x, v)].$$

The expectation is taken over the velocity, $v$, a random variable. In appendix B, I show that $M(\omega, k_x, k_z)$ does not depend on $x_0$. This is expected because I assume that the statistics of the velocity does not change laterally.

Equation (12) is similar to the normal equation of a common least-squares problem. In fact, if we discretize the continuous functions, then for each $k_x$ and $k_z$, equation (12) becomes an algebraic linear equation

$$B_P M = P, \tag{13}$$

where $B_P$ is a complex square matrix, and $M$ and $P$ are complex vectors. In principle, the optimal migration operator can be found by solving equation (13) numerically. As a check of our derivation, we let the $f(v)$ be a $\delta$ function. Indeed, the conventional migration operator satisfies equation (12).

## CONCLUSION

I have presented a method of finding the optimal migration operator for a given velocity pdf. It is shown that the conventional migration operator is optimal only when the velocity uncertainty disappears. In general cases, the optimal operator can be found when linear integral equations are solved. I need to develop an algorithm to solve the equations efficiently. In practice, this may turn out to be finding a suitable representation of $B_P$ in equation (13). The future work will also include testing the method with synthetic data and field data.

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## APPENDIX A

To derive equation (12), we apply the standard variational calculus method and construct a perturbed function

$$M(\omega, k_x, k_z, \alpha) = M(\omega, k_x, k_z) + \alpha\eta(\omega, k_x, k_z), \tag{A.1}$$

where $\alpha$ is a scalar and $\eta(\omega, k_x, k_z)$ is an arbitrary function. Define $J(\alpha)$ to be the objective function with the perturbation.

$$J(\alpha) = E\left[\int_{k_z}\int_{k_x}\left|P(k_x, k_z) - \int_\omega M(\omega, k_x, k_z, \alpha)P(\omega, k_x, v)d\omega\right|^2 dk_x dk_z\right]. \tag{A.2}$$

If $J(\alpha)$ is stationary for the function $M(\omega, k_x, k_z)$, then

$$\left.\frac{\partial J(\alpha)}{\partial \alpha}\right|_{\alpha=0} = 0. \tag{A.3}$$

Take the partial derivative of $J(\alpha)$ with respect to $\alpha$.

$$\left.\frac{\partial J(\alpha)}{\partial \alpha}\right|_{\alpha=0} = -\int_{k_z}\int_{k_x}\int_\omega \{\eta^*(\omega, k_x, k_z)M(\omega, k_x, k_z) + \eta(\omega, k_x, k_z)M^*(\omega, k_x, k_z)\}d\omega dk_x dk_z, \tag{A.4}$$

where

$$M(\omega, k_x, k_z) = P(k_x, k_z)E[P^*(\omega, k_x, v)] - \int_{\hat\omega} M(\hat\omega, k_x, k_z)E[P^*(\omega, k_x, v)P(\hat\omega, k_x, v)]d\hat\omega.$$

Because $\eta(\omega, k_x, k_z)$ is arbitrary complex function, we can always choose it in such a way that both expressions

$$\eta^*(\omega, k_x, k_z)M(\omega, k_x, k_z) \quad \text{and}$$
$$\eta(\omega, k_x, k_z)M^*(\omega, k_x, k_z)$$

are always real and greater or equal to zero. Then equation (A.3) requires these two expressions to be zero. Actually two expressions are a conjugate pair. It will be sufficient for one of them to be zero. Because $\eta^*(\omega, k_x, k_z)$ is not always zero $M(\omega, k_x, k_z)$ has to be zero, which leads to equation (12).

## APPENDIX B

From equation (5), we have

$$P(k_x, k_z) = e^{-i(k_x x_0 + k_z z_0)}. \tag{B.1}$$

$(x_0, z_0)$ is the location of the diffractor. Equation 6 can be written as

$$P(\omega, k_x, v) = \int_{\hat{k}_z} e^{-i(k_x x_0 + \hat{k}_z z_0)} \delta(\omega - v\sqrt{k_x^2 + \hat{k}_z^2}) d\hat{k}_z. \qquad (B.2)$$

Substitute equation B.2 into equation (12).

$$B_P(\omega, \hat{\omega}, k_x) = \int_{\hat{k}_z} \int_{\tilde{k}_z} e^{i(\hat{k}_z - \tilde{k}_z) z_0} \int_v f(v) \delta(\omega - v\sqrt{k_x^2 + \hat{k}_z^2}) \delta(\hat{\omega} - v\sqrt{k_x^2 + \tilde{k}_z^2}) dv d\tilde{k}_z d\hat{k}_z$$

and

$$\overline{P}(\omega, k_x) P(k_x, k_z) = \int_{\hat{k}_z} e^{i(\hat{k}_z - k_z) z_0} \int_v f(v) \delta(\omega - v\sqrt{k_x^2 + \hat{k}_z^2}) dv d\hat{k}_z. \qquad (B.3)$$

Therefore, all known functions in equation (12) are independent of $x_0$. Thus, the unknown operator $M(\omega, k_x, k_z)$ is independent of $x_0$.