Stochastic normal moveout correction

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ABSTRACT

Using velocity uncertainty in data processing has the advantage over "precise" velocity that it includes additional useful information and improved results can be expected. Including uncertainty in velocity analysis attenuates some aliased high-frequency energy, and cleans up the display. Estimating velocity uncertainty is related to velocity analysis.

INTRODUCTION

Performing normal moveout correction requires knowledge of velocities. The usual approach is determination of exact velocities. There are various reasons for which knowledge of exact velocities is impossible. Let us list some of them:

- Due to lateral variation of velocities exact velocities in fact do not exist in a CMP gather.
- Measurement errors may cause lateral variations of velocities.
- Determination of velocities from seismic data is subjected to errors.

In this paper we are not interested in reasons which cause velocity uncertainty. We take as a fact that there are numerous reasons which cause velocities variations in a CMP gather. We are interested in how the range of velocities variations can be estimated and how to process seismic data for given ranges of velocities. We will name it a processing with uncertain velocities.

It will be shown in the following that in spite of this "uncertain" approach better results can be expected than in the deterministic approach.

STOCHASTIC NORMAL MOVEOUT CORRECTION

Notation

We will try to stick close to the notation as used in Claerbout, 1986. In the following we will work with a CMP gather that will usually be referred to as a (t, x)-space, t denoting time and x distance coordinates. Velocity analysis transforms (t, x)-space into a velocity analysis panel which will be referred to as a (τ, m)-space, τ denoting time and m sloth coordinates.

Sloth m is defined by the equation

$$m = \frac{1}{v^2}, \tag{1}$$

where v means a velocity. Using Muir's regular sampling in the sloth domain instead of in the velocity or slowness domains has the advantage of more even distribution of hyperbolas in the (t, x)-space (Fowler,1987). It can be seen on Figure 2c.

A probabilistic density function (PDF) will sometimes be referred to because of stochastic approach to this subject. I will refer to it in connection with sloth only.

Representation of sloth as a random variable

The stochastic approach requires that velocity or sloth will be considered as random variables. These random variables are characterized by PDF's, so that instead of a set of velocities in the deterministic approach we have now a set of PDF's.

One of the tasks is to determine the PDF's. For my purposes I choose to work with a simple function. If the sloth m is known with uncertainty Δm, i.e. the sloth m lies in the interval $(m - \Delta m, m + \Delta m)$, then we will approximate the Gaussian by a triangular function with a base of length $2\Delta m$ (Figure 1a).

Definition of SMOC

I want to develop a concept of stochastic normal moveout correction (SMOC) for the case that we do not know sloth exactly, but we know its PDF.

The hyperbola in (t, x)-space is never a perfect hyperbola. There are various reasons for this fact, all of which may be included into velocity uncertainty. Thus uncertain velocity is for our purposes the reason of the non-perfect hyperbolic arrivals. Let us suppose that the sloth m is known with the uncertainty Δm. Then our non-perfect hyperbola lies between hyperbolas with sloths $m - \Delta m$ and $m + \Delta m$ (Figure 2a).

Let us denote $p(m)$ the probability that an event of the non-perfect hyperbola is an event with the sloth m, i.e. it lies on a hyperbola corresponding to the sloth m. The hyperbolas for m satisfying inequality

$$M - \Delta m \leq m \leq M + \Delta m \tag{2}$$

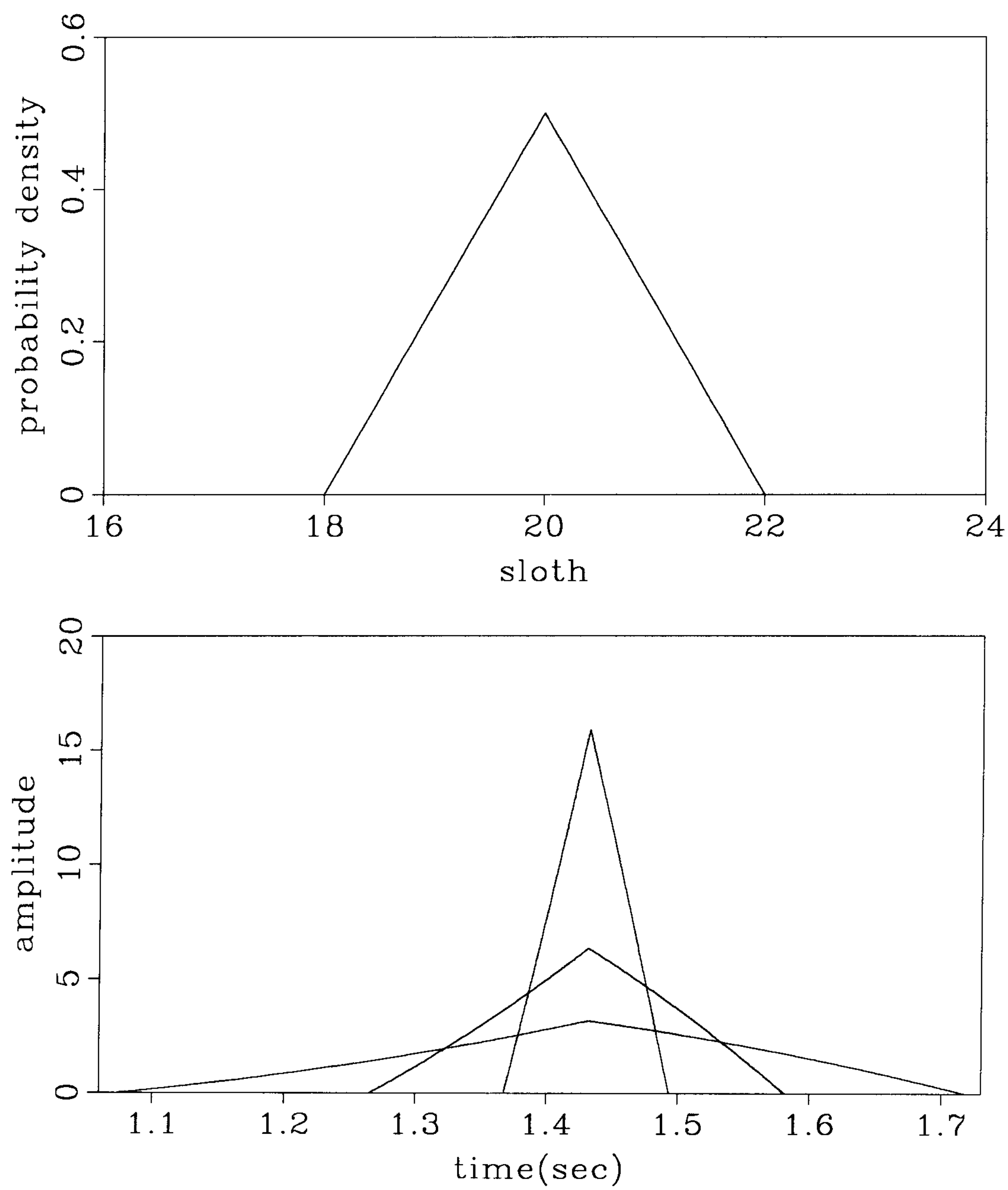

FIG. 1. (a) A triangular PDF used as an approximation to a Gaussian. From the properties of PDF it follows that the area of the triangle is 1. The mean sloth m is in this case $20 \times 10^{-8} sec^2/m^2$ and uncertainty Δm is $2 \times 10^{-8} sec^2/m^2$. (b) The shape of a stochastic filter for $\Delta m = 2, 5, 10 \times 10^{-8} sec^2/m^2$ is shown. The hyperbola is defined by $\tau = 0.5 sec$ and $m = 20 \times 10^{-8} sec^2/m^2$, offset was chosen $3000m$. The width of the filter increases with increasing Δm. For low Δm the shape is undistinguishable from an equilateral triangle. For higher Δm the sides of the triangle do not differ much from a straight line , but the triangle is no more equilateral. The triangle for $\Delta m = 2$ differs from an equilateral triangle by 0.01%.

fill the area between two margin hyperbolas (Figure 2a).

We can imagine the following process of performing NMO under uncertain velocity:

$$image_M = \sum_{m=M-\Delta m}^{M+\Delta m} p(m)\, NMO_m\, data, \tag{3}$$

that is we make an averaged sum of NMOed data for sloths m in the region of uncertainty. NMO_m means NMO with sloth m. We shall call this process a stochastic normal moveout correction (SMOC).

Equation (3) is equivalent to the following scheme:

for all x {
    for all τ {
        for all $m = M - \Delta m, M + \Delta m$ {
            $image(\tau, M) = p(m)\, data(\sqrt{\tau^2 + x^2 m}, x) + image(\tau, M)$
        }
    }
}

This algorithm requires interpolation of values between sampled data. A better approach is to do without interpolation. It will be shown in the following that this scheme can be replaced by weighted averaging over the sampled data.

Stochastic filtering

Let us call the averaging over the time domain stochastic filtering. SMOC can now be considered as a superposition of stochastic filtering SF and normal moveout NMO:

$$\mathbf{SMOC} = \mathbf{NMO\ SF} \tag{4}$$

The length of the stochastic filter (Figure 2b) is determined by the intersection of the margin hyperbolas with the corresponding trace. Let us determine, how the length of the filter changes with time and offset. The equations of the hyperbolas corresponding to sloths $m - \Delta m$ and $m + \Delta m$ are

$${t_1}^2 = \tau^2 + x^2(m - \Delta m) \tag{5}$$

$${t_2}^2 = \tau^2 + x^2(m + \Delta m) \tag{6}$$

By subtracting the two equations we get

$${t_2}^2 - {t_1}^2 = 2x^2 \Delta m \tag{7}$$

and if we denote $t = (t_2 + t_1)/2$, we get for the length of the filter

$$\Delta t = t_2 - t_1 = \frac{x^2 \Delta m}{t}. \tag{8}$$

Hence the length of the filter decreases linearly with time and increases with offset.

The interval $(m-\Delta m, m+\Delta m)$ in the sloth domain is mapped onto the interval (t_1, t_2) in the time domain. The shape of the stochastic filter is determined by the shape of PDF. Since the mapping from the sloth domain into the time domain is not linear, the shape of the stochastic filter is generally different from the shape of PDF. However, if we consider Δm small enough, the shape does not change much. Hence, if PDF is a triangle, the stochastic filter is a triangle, too (Figure 1b).

Proof of equivalence

We wish to prove that SMOC is equivalent to the superposition of stochastic filtering and NMO (equation (4)). It is enough to prove that averaging between marginal hyperbolas can be transformed into filtering.

In the following a transformation from the sloth axis into the time axis will be needed. Since transformations are easier to handle in continuous than in discrete domain, continuous domain will be used.

It is easier to think in terms of velocity analysis. Velocity analysis is a transformation of (t, x)-space into (τ, m)-space. The operator of transformation for the purpose of this proof is summation along a hyperbola $t^2 = \tau^2 + x^2$ in (t, x)-space. Let us denote the values in (τ, m)-space $V(\tau, m)$. Equation (3) defining SMOC can be rewritten in terms of velocity analysis as follows:

$$V'(\tau, M) = \int_{M-\Delta m}^{M+\Delta m} f(M-m)V(\tau, m)dm. \tag{9}$$

This equation represents a convolution of values in (τ, m)-space with a symmetric filter $f(m)$, values of which are equal to the values of PDF of sloth. Values $V'(\tau, m)$ represent velocity analysis panel, where SMOC was used instead of NMO.

Each value $V(\tau, m)$ can be expressed as an integral over a hyperbola $t^2 = \tau^2 + x^2 m$:

$$V(\tau, m) = \int A(\sqrt{\tau^2 + x^2 m}, x)dx, \tag{10}$$

where $A(t, x)$ are amplitudes in (t, x)-space.

Substituting (10) into (9) we get

$$V'(\tau, M) = \int_{M-\Delta m}^{M+\Delta m} f(M-m) \int A(\sqrt{\tau^2 + x^2 m}, x)dxdm. \tag{11}$$

After substitution $t = \sqrt{\tau^2 + x^2 m}$ and rearranging the order of integration we have

$$V'(\tau, M) = \int \int_{t_1}^{t_2} f(M - \frac{t^2 - \tau^2}{x^2})A(t, x)\frac{2t}{x^2}dtdx, \tag{12}$$

where t_1, t_2 correspond to the intersections of a trace in (t, x)-space with the margin hyperbolas (Figure 2a).

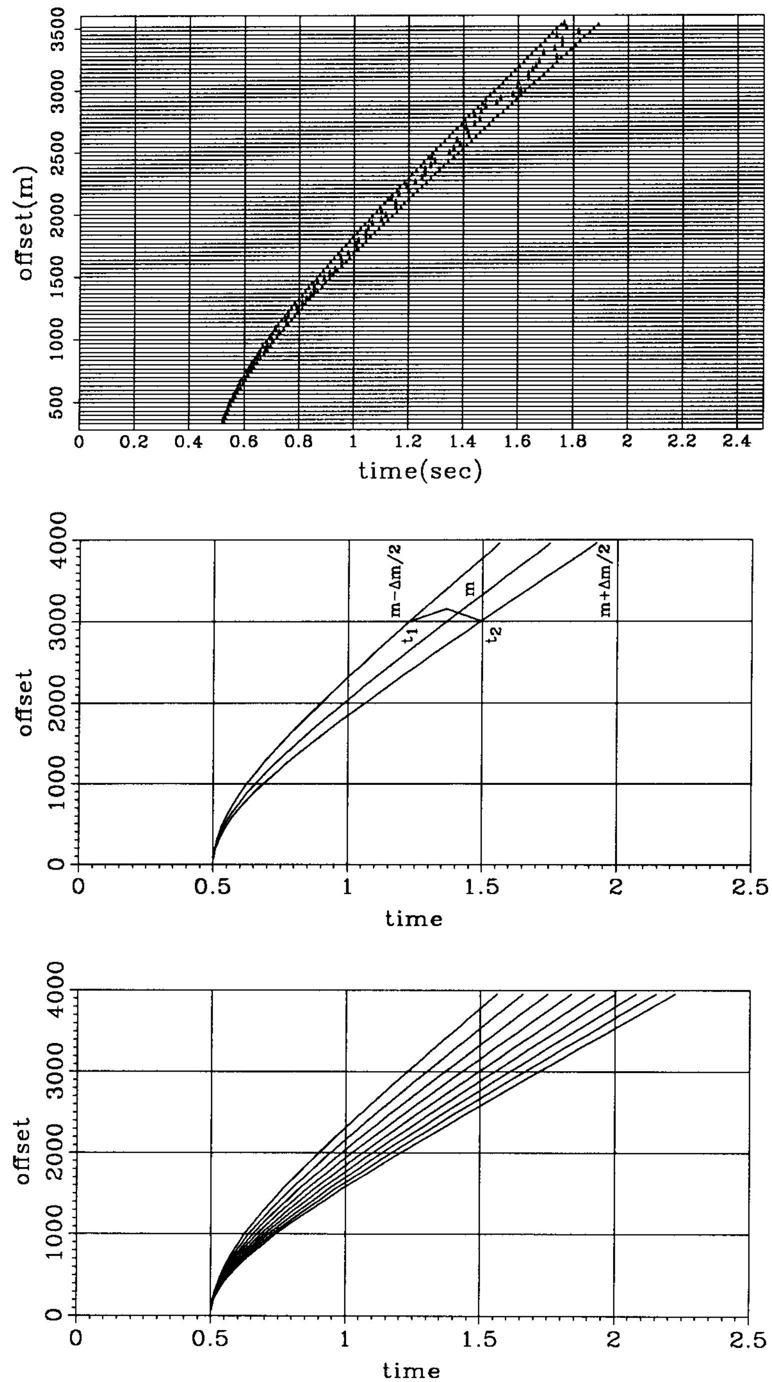

FIG. 2. (a) Arrivals with uncertain velocity lie between two hyperbolas with with sloths $m - \Delta m$, $m + \Delta m$. (b) Stochastic filter corresponding to the PDF of sloth. From the figure we can see that the length of the stochastic filter decreases with time and increases with offset. (c) SMOC can done by performing NMO for each sloth corresponding to a hyperbola between two margin hyperbolas and then summing weighted results. From the figure we can see that even sampling in the sloth domain leads to a reasonably uniform distribution of hyperbolas in the (t, x)-space.

The importance of equation (12) is that it transforms filtering over the sloth domain into filtering over the time domain. Scheme for performing SMOC was not efficient enough because intersections of hyperbolas with traces are not uniformly sampled. Equation (12) enables us to make use of the fact that traces are uniformly sampled.

Now let us have a look at the weighting factor in the equation (12). If Δm is small enough and hence t_1 and t_2 differ little, the factor can be considered constant. The inverse of this factor $x^2/2t$ is proportional to the length of the stochastic filter (equation (8)).

The shape of the filter f changes due to nonlinear transformation. For small Δm the change is not significant and therefore the shape of the stochastic filter may be taken the same as the shape of PDF of sloth. Scaling factor $2t/x^2$ causes the area of the stochastic filter to be equal to one.

Testing SMOC

The result of applying SMOC on a synthetic CMP gather is shown on Figure 3. The uncertain velocity results in stretching the wavelets on the traces. We can see that the same amount of sloth uncertainty results in different stretches of wavelets. It is a consequence of equation (8).

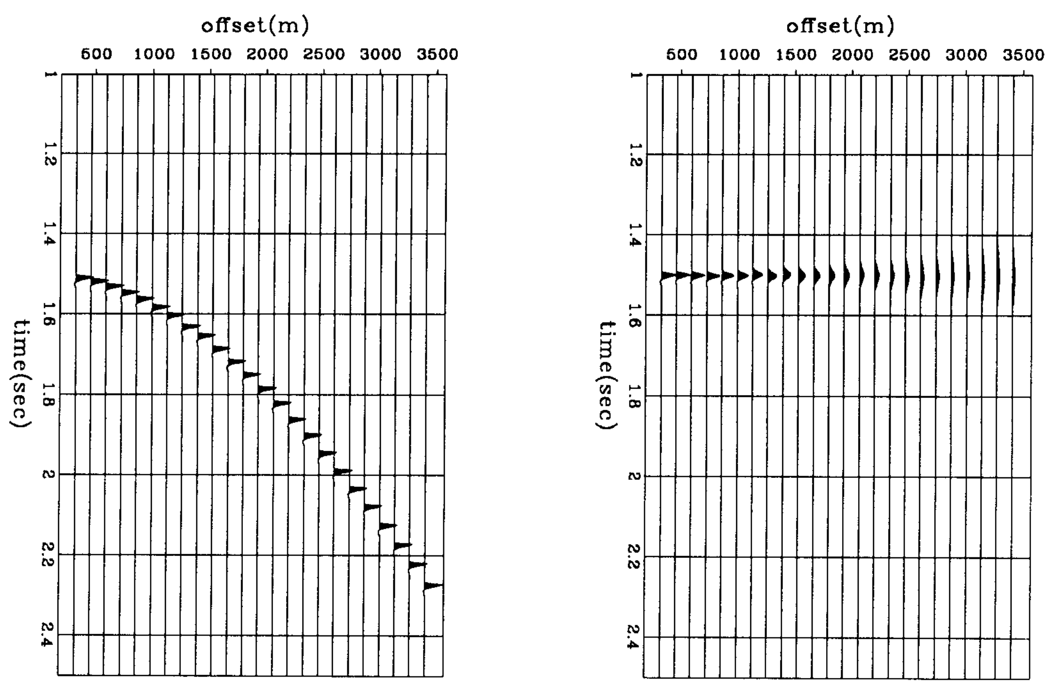

FIG. 3. (a) Synthetic CMP gather. The velocity of the medium was supposed constant and equal to $2000m/sec$ (sloth equal to $25 \times 10^{-8} sec^2/m^2$). (b) SMOC applied. Uncertainty of sloth was chosen $2 \times 10^{-8} sec^2/m^2$.

The result of uncertainty of velocity is that SMOC maps a spike onto a broader wavelet which is in agreement with the general expectation of how a stochastic mapping should work. The picture after SMOC is not so sharp, it is blurred.

Stochastic filtering in (t^2, x^2)-space

It seems that from the theoretical point of view, (t^2, x^2)-space is more convenient than (t, x)-space. Hyperbolas in (t, x)-space become straight lines in (t^2, x^2)-space. The length of the stochastic filter is

$$\Delta t^2 = 2x^2 \Delta m, \tag{13}$$

i.e. the length does not depend on time squared. The mapping from the sloth domain to the time squared domain is linear and hence the shape of the stochastic filter is exactly triangular, not only approximately as in the (t, x)-space. There may be, however, problems with a change of the wavelet shape in (t^2, x^2)-space.

The data is measured in the time domain, and for this reason we will prefer the (t, x)-space in our calculations.

Stochastic modeling

Similarly to SMOC, the equation for modeling can be written as follows:

$$image_M = \sum_{M-\Delta}^{M+\Delta} p(m)\, NMO_m^{-1}\, data. \tag{14}$$

The following scheme can be written for modeling:

for all x {
    for all t {
        $\tau = \sqrt{t^2 - x^2 M}$
        for all $m = M - \Delta m, M + \Delta m$ {
            $image(\sqrt{\tau^2 + x^2 m}, x) = image(\sqrt{\tau^2 + x^2 m}, x) + p(m)\, data(\tau, x)$
        }
    }
}

Here again as in SMOC most of the square roots are not necessary to compute. What the algorithm does is that it takes a point and spreads it around in accordance with sloth PDF.

The results of modeling are on Figure 4. By applying SMOC and modeling a smoother wavelet is obtained.

VELOCITY ANALYSIS

We will try to reformulate what we know about NMO and SMOC in terms of velocity analysis.

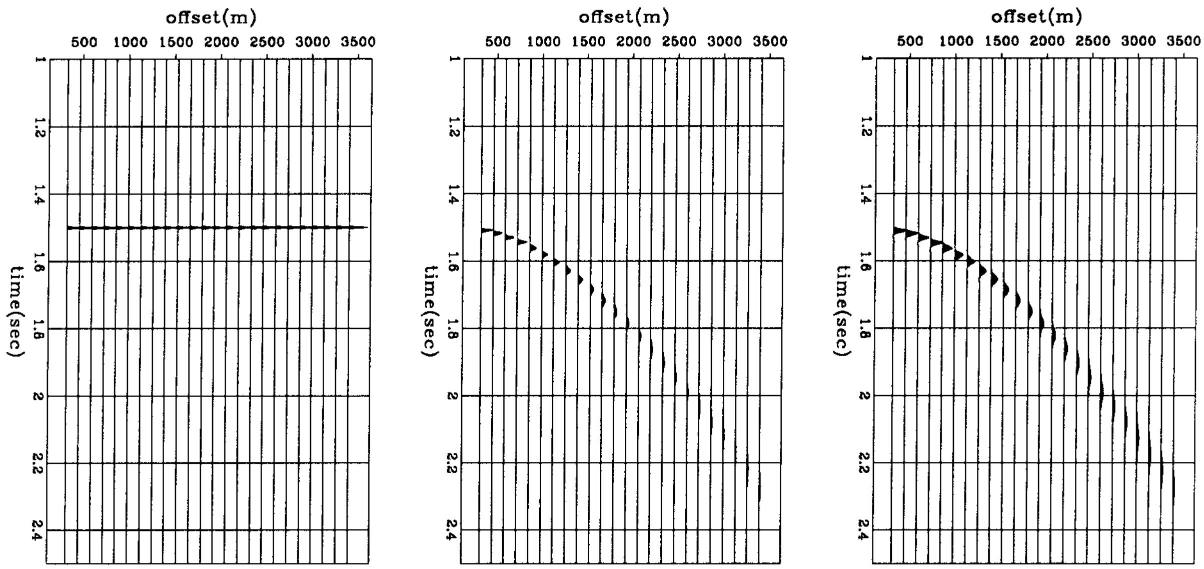

FIG. 4. (a) Input for modeling. (b) Modeling applied. The velocity of the medium is equal to $2000 m/sec$, uncertainty of sloth is $2 \times 10^{-8} sec^2/m^2$. (c) The result of applying SMOC and modeling on a synthetic gather from Figure 3a.

Velocity analysis is a transformation of (t,x)-space into (τ,m)-space. The operator of transformation can be for example stacking along a hyperbola and placing the sum at zero-offset time. It can be considered as a superposition of NMO and stack along the x-axis.

We can try a modification of velocity analysis, in which NMO is replaced by SMOC:

$$V(\tau,m) = \mathbf{STACK\ SMOC}\ A(t,x). \qquad (15)$$

Practical implementation is based on the fact that SMOC is a superposition of stochastic filtering and NMO (equation (4)).

The relation between the two kinds of velocity analysis is

$$\mathbf{STACK\ SMOC} = \mathbf{f\ STACK\ NMO}, \qquad (16)$$

where f is an operator of filtering with a symmetric filter with coefficients equal to sloth PDF over the sloth space for all τ's of (τ,m)-space. Filter f acts as a low-pass filter. It removes high frequencies from (τ,m)-space that are due to aliasing (Figure 5; discussion of aliasing see Schultz, 1976). The gather was not muted, because we wanted to see the effect of SMOC under difficult conditions.

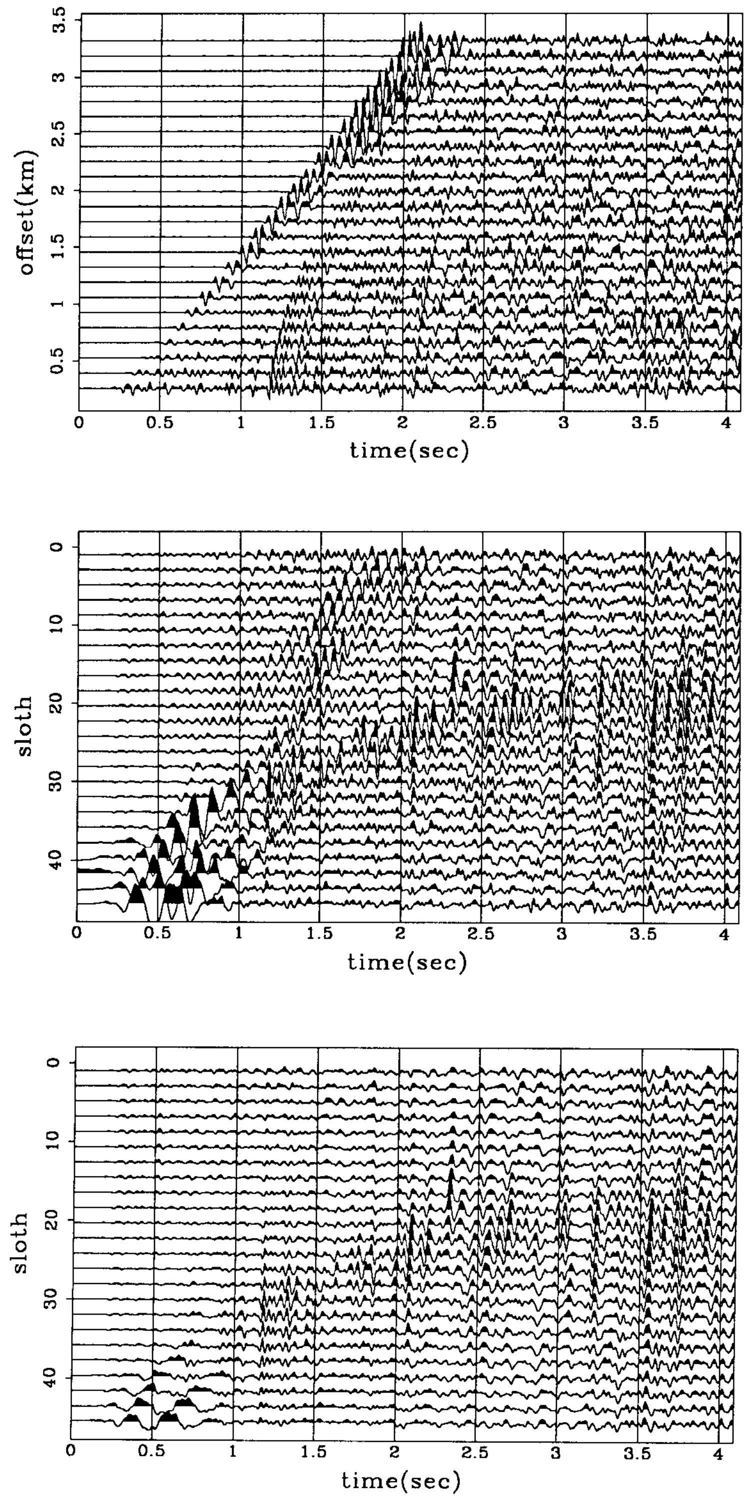

FIG. 5. (a) CMP gather from a marine air-gun survey. Data courtesy of Western Geophysical. (b) Conventional velocity stack. (c) Velocity stack using SMOC. The picture is much cleaner. Length of the filter is equal to twice the sampling interval in the sloth domain. Sampling in the sloth domain is $1.93237 \times 10^{-8} sec^2/m^2$.

ESTIMATION OF UNCERTAINTY

Theoretical example

Let us consider a theoretical example of velocity analysis on a data very densely sampled in t and x domains (Figure 6). The wavelet is a spike.

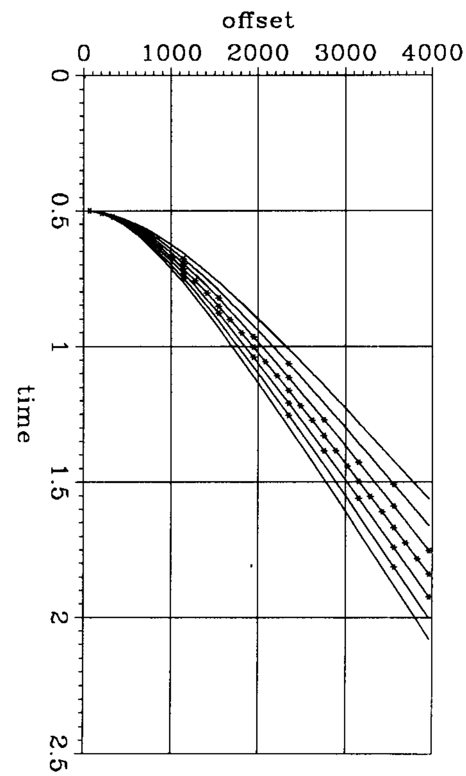

FIG. 6. This figure represents arrivals centered around the central hyperbola. Because of uncertainty in velocity there are other arrivals on hyperbolas near the central hyperbola. Number of arrivals on each hyperbola is proportional to the sloth PDF.

Let us suppose that we know the sloth PDF $p(m)$. For simplicity let us assume that sloths m can acquire values from a discrete set of values. Let the number of traces be N. Then the number of spikes on a hyperbola with sloth m is equal to $p(m)N$.

Let us transform this (t, x)-space into (τ, m)-space by using the operator of summation. The sum along a hyperbola with sloth m is equal to the number of the spikes on the hyperbola, i.e. $p(m)N$. After averaging by the number of traces we get for the amplitude $V(\tau, m)$

$$V(\tau, m) = p(m), \tag{17}$$

i.e. the shape of a curve in (τ, m)-space for τ corresponding to the hyperbola is sloth PDF. For uncertainty equal to zero the shape of PDF is a spike. Therefore the transformation from (t, x)-space into (τ, m)-space should give a spike. The result is on Figure 7. The shape differs from a spike and this is due to the interpolation in the used algorithm. Hence Figure 7 shows us a limit of resolution. The resolution depends on the sampling in the sloth domain.

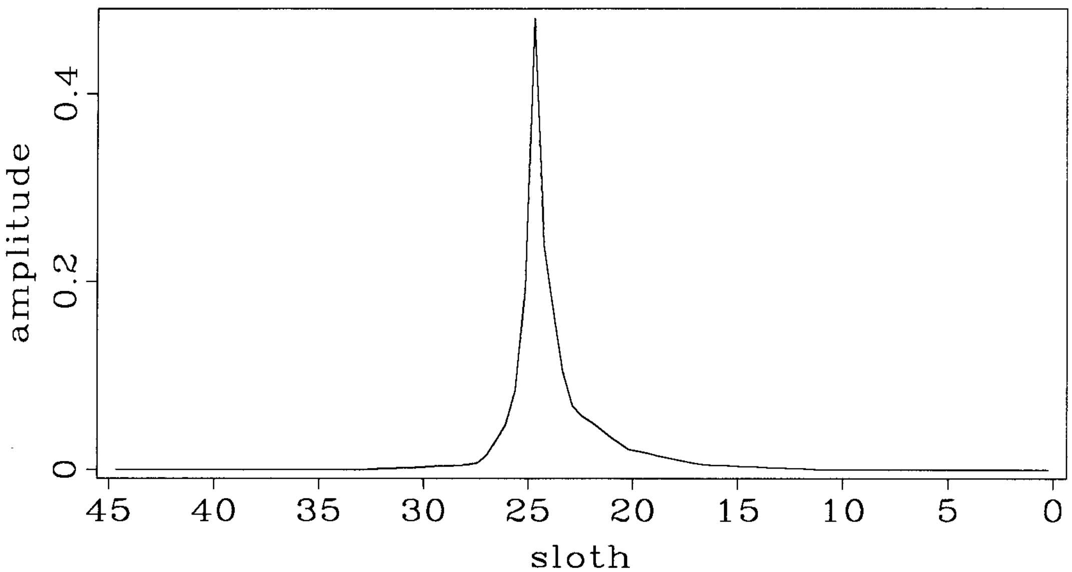

FIG. 7. Dependence of stacking amplitude on sloth. The wavelet is a spike. Ideally this curve should be a PDF corresponding to certainty (a spike). The velocity was taken $2000 m/sec$ and τ is equal to $1.5 sec$.

Let us denote the shape of this curve $P(\tau, m)$. It represents stacking amplitudes for different sloths. The curve $P(\tau, m)$ contains information about PDF. We will keep notation $V(\tau, m)$ for summing along any hyperbola. Thus $V(\tau, m)$ generally does not contain information about PDF.

The width of the curve $P(\tau, m)$ on the Figure 7 is quite big and should be taken into account in estimating sloth uncertainty.

Relation of PDF to a curve of stacking amplitudes

Now let us consider a case of a more general wavelet than a spike. The wavelet need not be the same along the hyperbola. Let us consider again the same case as in Figure 6 with a more general wavelet. The rugged hyperbola can be considered as several hyperbolas with wavelets evenly distributed along each hyperbola in proportion to the probability $p(m)$. We can imagine the gather as a sum of several gathers, each containing only one of the hyperbolas:

$$A'(t, x) = \sum_{m=M-\Delta m}^{M+\Delta m} A_m(t, x). \tag{18}$$

After velocity analysis transformation we have

$$V'(\tau, M) = \sum_{m=M-\Delta m}^{M+\Delta m} V_m(\tau, M). \quad (19)$$

We can rewrite this equation for the shape $P(\tau, M)$:

$$P'(\tau, M) = \sum_{m=M-\Delta m}^{M+\Delta m} P_m(\tau, M) \quad (20)$$

Now we can write

$$P'_m(\tau, M) = p(m)P(\tau, M), \quad (21)$$

where $P(\tau, M)$ is obtained by velocity analysis transformation on a gather $A(t, x)$ with a perfect hyperbola with sloth M. This holds because there are $p(m)N$ wavelets evenly distributed on the hyperbola in $A_m(t, x)$. From it follows

$$P'(\tau, M) = f(m) * P(\tau, M), \quad (22)$$

where $f(m)$ is as a filter corresponding to $p(m)$.

We obtained an interesting result that the shape $P'(\tau, m)$ corresponding to a rugged hyperbola due to velocity uncertainty is equal to a convolution of the shape $P(\tau, m)$ corresponding to a perfect hyperbola with a filter, whose coefficients are equal to PDF.

In this derivation an equation (21) plays an essential role. This equation, however, does not hold exactly. The distribution of hyperbolas in the surroundings of a central hyperbola is generally different for different sloths m. As the two central hyperbolas become closer to each other, the distributions become more similar to each other. Hence the convolution is approximate and becomes more exact near the maximum value of $P(\tau, m)$, reaching a perfect precision only in the limit.

Let us return to velocity analysis. We usually work with a small amount of traces in a CMP gather. Due to velocity uncertainty the statistical properties of the stack along a hyperbola are bad. If they were good, we would receive the shape convolved with PDF. Statistical properties are improved by SMOC, because we take into account all events in the range of uncertainty. The effect of this process is a convolution of the shape $P(\tau, m)$ with PDF, which corresponds to the convolution with PDF in equation (9). It is interesting that both processes lead to the same change of $P(\tau, m)$.

Using PDF in SMOC is necessary because of insufficient amount of data. PDF computed from the real data would not correspond to the real PDF. The knowledge of sloth distribution helps improve the result.

How to get an estimate of uncertainty?

The problem is how to separate parts of the curve $P(\tau, m)$. There is a part which belongs to the shape of the wavelet in the gather. There is another part which is due to sampling and interpolation. Another part is due to uncertain velocity. And there are even other parts, but we are interesting in estimation of the width of only one part. This is the part that is due to uncertain velocity. The sloth uncertainty can be deduced from the total width of the curve and the widths of its parts.

The part due to sampling and interpolation can be estimated by performing velocity analysis on a synthetic gather corresponding to the real CMP gather in which the wavelet is a spike. If we know the shape of the wavelet, then by including this wavelet our estimate will include the influence of the shape of the wavelet.

CONCLUSIONS

A stochastic normal moveout correction corrects CMP gathers in a given range of sloths. Though the adjective stochastic invokes something less precise than deterministic, the opposite appears to be true. A correction with an uncertain velocity should handle arrivals scattered around a hyperbola in a statistical more convenient way than a deterministic approach.

Another factor is using an a priori information about sloth distribution that cannot be otherwise obtained from the data with sufficient reliability.

A problem is an estimation of the range of sloth uncertainty. This paper presents some ideas about how velocity analysis panel is related to sloth uncertainty. Obtaining reliable results is still a matter of further research.

Next step to do is to test this algorithm on real data.

ACKNOWLEDGEMENTS

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REFERENCES

Claerbout, J.F., 1986, Cable tangent stacking: SEP-**48**, 291-294.

Fowler, P., 1985, Sampling theory for velocity space dip-moveout and migration: SEP-**42**, 331-345.

Schultz, P.S., 1976, Velocity estimation by wavefront synthesis: Ph.D. thesis, Stanford University.