Reflectivity estimation from passive seismic data

Lin Zhang

ABSTRACT

One of the key issues of passive seismic data processing is how to convert the transmission data into the corresponding reflection data. Autocorrelating the data is a simple method to do so. The effectiveness of this method, however, is seriously limited when the unknown sources are non-white. After identifying the arrival directions of the events, I use Burg's method to estimate the reflection coefficients directly from the data. This technique also provides an opportunity to remove some of the band limited effects of the sources. The algorithm works well on synthetic data, and it gives interesting results when applied to field data.

Under the acoustic assumption, a two dimensional conversion algorithm is proposed. Experiments applying this algorithm to synthetic data reveal several obstacles to applying the method in practice.

INTRODUCTION

One of the fundamental theories of passive seismic data processing is the Kunetz-Claerbout equation. Claerbout (1967) showed that, for plane waves at normal incidence, the autocorrelation of the transmission data is the reflection data for the same layered medium. The theory assumes that the unknown sources have white spectra. Scherbaum (1987) adapted Claerbout's formulation to the transmission problem for SH waves under oblique incidence. He demonstrated that microearthquake recordings can be used to calculate the impedance structure of the earth. He also addressed the problem of the limited high frequency content of the signals, and developed a deconvolution algorithm to increase the resolution of the inversion results.

In September 1988, SEP recorded a three dimensional passive seismic dataset. Claerbout et al. (1988) explained the theory and practice in designing this novel experiment. The geophone array in this experiment was designed with a sampling grid fine enough to allow conventional signal processing techniques to be applied in all three dimensions. In this paper, I present two processing techniques that I have tried on the passive seismic dataset. First, I will show the existence of events arriving near-vertically by using the horizontal slowness spectra of the stacked data. Then I will explain an inversion algorithm to estimate the reflection coefficients of the medium from the passive data. I will show the results that I obtain when applying the algorithm to synthetic and field data. Finally, I will state a 2-D extension of autocorrelation method, and show some synthetic examples.

HORIZONTAL SLOWNESS SPECTRUM

There are several local noise sources near the site where we did our passive experiment. Power lines run over our array and introduce strong 60 Hz monochromatic noise. Cars on the nearby freeway generate low frequency surface waves, especially during the day-time. Nichols et al. (1989) explain a simple method to increase the signal-noise ratio. They stack the data along the two diagonal directions of the geophone array. They show that vertical arrivals are present on the stacked sections recorded during night-time, but not on those recorded during the day-time.

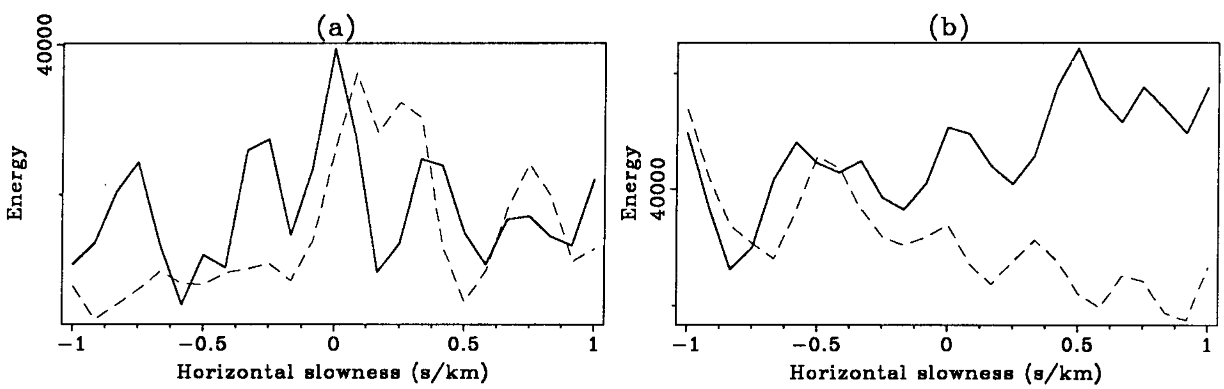

FIG. 1. The horizontal slowness spectra of: (a) the data recorded during night when traffic is quiet; (b) the data recorded during day-time. The solid line and dotted line show the results of two perpendicular directions.

We want to know how the energy of these vertical-arrival events compares with the energy of other events. We also want to know the precise apparent velocity of these steep arrivals. The apparent velocity spectrum, or equivalently, the horizontal slowness spectrum offers this information. By using the slant stack transform, we can decompose the data into plane waves of varying wave parameters. Then we sum the energy of the plane waves of the horizontal slowness p and plot the energy as a function of p. Figure 1 shows the horizontal slowness spectra of the two sections that are obtained by stacking the traces along the two diagonals of the surface array. For night-time recordings, the vertical arrivals are dominant, and the apparent velocities of these events range from 4 to 8 km/s. From the relative positions of the peaks of these curves we can approximately determine the directions of the sources. As we expected, for day-time recordings, surface noise generated by cars on the freeway is dominant.

Considering that some portions of data may contain more vertical arrivals than other portions, I chose a portion of data and processed it with the same algorithm. The result is shown in Figure 2a. Clearly, in this portion of data, the total energy of surface waves is much smaller than the energy of waves at near-vertical incidence.

We can separate, in the frequency domain, the energy propagating vertically and the energy propagating horizontally. Figure 2b shows the spectrum after filtering out the energy below 10 Hz. The surface waves are attenuated.

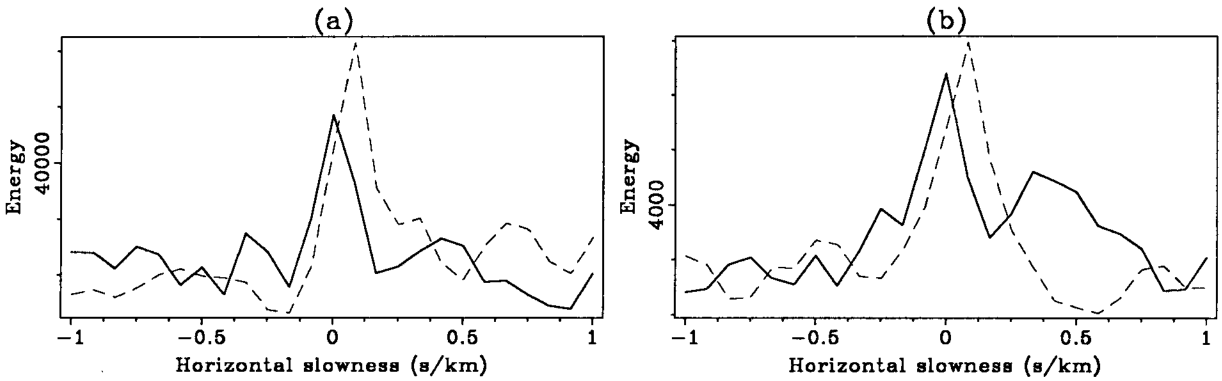

FIG. 2. The horizontal slowness spectra of: (a) a portion of data in which vertical arrivals are dominant; (b) the data after filtering out the energy below 10 Hz. The solid line and dotted line show the results of two perpendicular directions respectively.

REFLECTIVITY ESTIMATION

The discussion in the last section indicates that the energy of the plane waves that propagate near-vertically is dominant in the night-time recordings. Therefore we can focus the array on these plane waves and obtain the reflection data with the Kunetz-Claerbout equation. This method, however, does not work well for our data because the unknown source wavelet in our data is nonwhite. Strong periodic signals appear in the autocorrelation function of the data. In this section, I discuss an algorithm in which Burg's prediction method is used to estimate the reflection coefficients of the medium directly from the data. This approach also provides us with an opportunity to remove the band limited effects of the unknown sources by a deconvolution technique.

Model

To solve an inversion problem or a deconvolution problem, we usually constrain the solutions by defining an appropriate model. Let $X(\omega)$ be the spectrum of the recorded plane waves at near-vertical incidence. We consider a simple convolutional model in which $X(\omega)$ can be written as the product of the unknown source spectrum $S(\omega)$ and the transmission function $T(\omega)$ of the layered medium.

$$X(\omega) = T(\omega)S(\omega). \tag{1}$$

The source is a band-limited random sequence. We assume that the band-limited effects can be described by a transfer function $H(\omega)$. Therefore,

$$S(\omega) = H(\omega)W(\omega), \tag{2}$$

where $W(\omega)$ is the spectrum of a white sequence. We allow $H(\omega)$ to have a mixed phase.

The transmission function $T(\omega)$ has a nice form. As derived in FGPD,

$$T(\omega) = \frac{1}{A(\omega)} \tag{3}$$

where $A(\omega)$ is a minimum phase polynomial related to the reflection coefficients of the layered medium. In equation (3), we omit the transmission delay because we do not have the reference time anyway.

The goal of our inversion is to extract $A(\omega)$ from the recorded data. The algorithm we will present next works in two steps: removing $W(\omega)$ and then removing $H(\omega)$.

The prediction method

By using Burg's prediction algorithm we can find a minimum phase filter $F(\omega)$. When the recorded data $X(\omega)$ are fed into this filter, the output will be a white sequence with a spectrum $W(\omega)$. Therefore,

$$F(\omega) = \frac{1}{\hat{H}(\omega)T(\omega)} = \frac{A(\omega)}{\hat{H}(\omega)} \tag{4}$$

where $\hat{H}(\omega)$ is a minimum phase filter that has the same spectrum as $H(\omega)$. The length of the filter $F(\omega)$ is determined according to the depth of the subsurface layers we want to investigate.

The prediction filter $F(\omega)$ is a product of two minimum phase, causal filters. To separate them, we must make further assumptions. Let the length of $\hat{H}(\omega)$ be L, which is assumed to be much shorter than the length of the prediction filter $F(\omega)$. We also assume the interfaces in the layered medium are well separated so that the autocorrelation function of $A(\omega)$ has zero values at the first L nonzero lags. Under these conditions, we can use Burg's algorithm to estimate the filter $\hat{H}(\omega)$ from $F(\omega)$. When we feed $F(\omega)$ into the estimated filter, the output will be $A(\omega)$ from which we can easily find the reflection coefficients.

We can also do the deconvolution step with the method proposed by Scherbaum and Stoll (1985). In their method, the first L samples of $F(\omega)$ are considered to be $\hat{H}(\omega)$. The filter length L is increased until the deconvolved result is minimum phase, which is a necessary condition for a sequence to be a transmission sequence. The minimum phase property can be checked by Jury's test.

Synthetic example

The following synthetic example illustrates how the proposed algorithm works. The synthetic model consists of three flat layers. The reflection coefficient sequence is shown in Figure 3e. From equation (3), we can generate a sequence that can be considered as a plane-wave-decomposed component of transmission data. The random source $W(\omega)$ has a Gaussian distribution with zero mean and variance $\sigma^2 = 10$. The bandpass effects are represented by a transfer function, $H(Z) = (1 - 2.5Z + Z^2)/(1 + 0.2)$, that has mixed phase. Finally, equation (1) gives the synthetic data $X(\omega)$, as shown in Figure 3a. Figure 3b, 3c and 3d show the results of the reflectivity estimation. Compared with the reflectivity sequence of the model, the estimated reflection coefficients are well resolved. The small amplitude fluctuation in the estimation is caused by the effect of finite data length.

Field-data example

I tested the algorithm on the passive data recorded by SEP at Stanford. The data recorded by a two dimensional array is stacked to obtain the component of the vertical arrival. The stacked trace contains 8000 samples with the sample interval being 4 ms. The length of the prediction filter is chosen to be 3.2 seconds and the length of the deconvolution filter is 80 ms. Figure 4 shows the results obtained by averaging the estimation from 14 independent recordings. We can identify several impulses in the reflectivity sequence. The interpretation of this result, however, is difficult. We can exclude the possibility that these impulses are caused by the monochromatic noise because the period of the impulses are is longer than the period of the monochromatic noise.

2-D CORRELATION PROCESSING

The Kunetz-Claerbout equation tells us that for plane waves at normal incidence, the autocorrelation of the transmission data is the reflection data of the same layered medium. This theory can be extended to subcritically incident, non-plane waves if we assume an acoustic medium.

Theory

Claerbout (1988) suggests that, for oblique incidence, the cross-correlation will play the same game as does the autocorrelation for normal incidence. In this section, I will show this fact by using the method of plane wave decomposition. I will use the

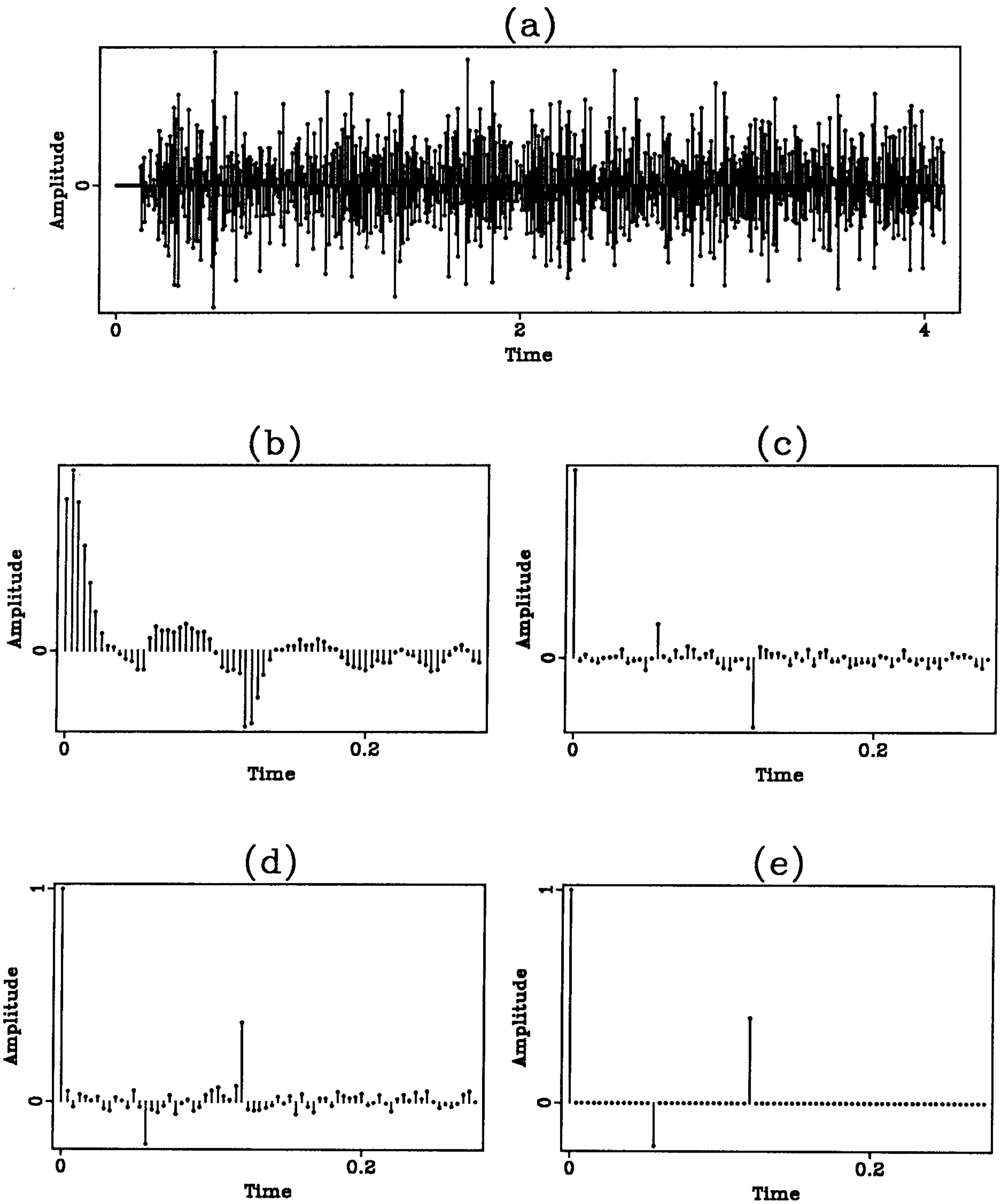

FIG. 3. A synthetic example: (a) transmission data; (b) prediction filter; (c) prediction filter after deconvolution; (d) estimated reflection coefficients; (e) true reflection coefficients.

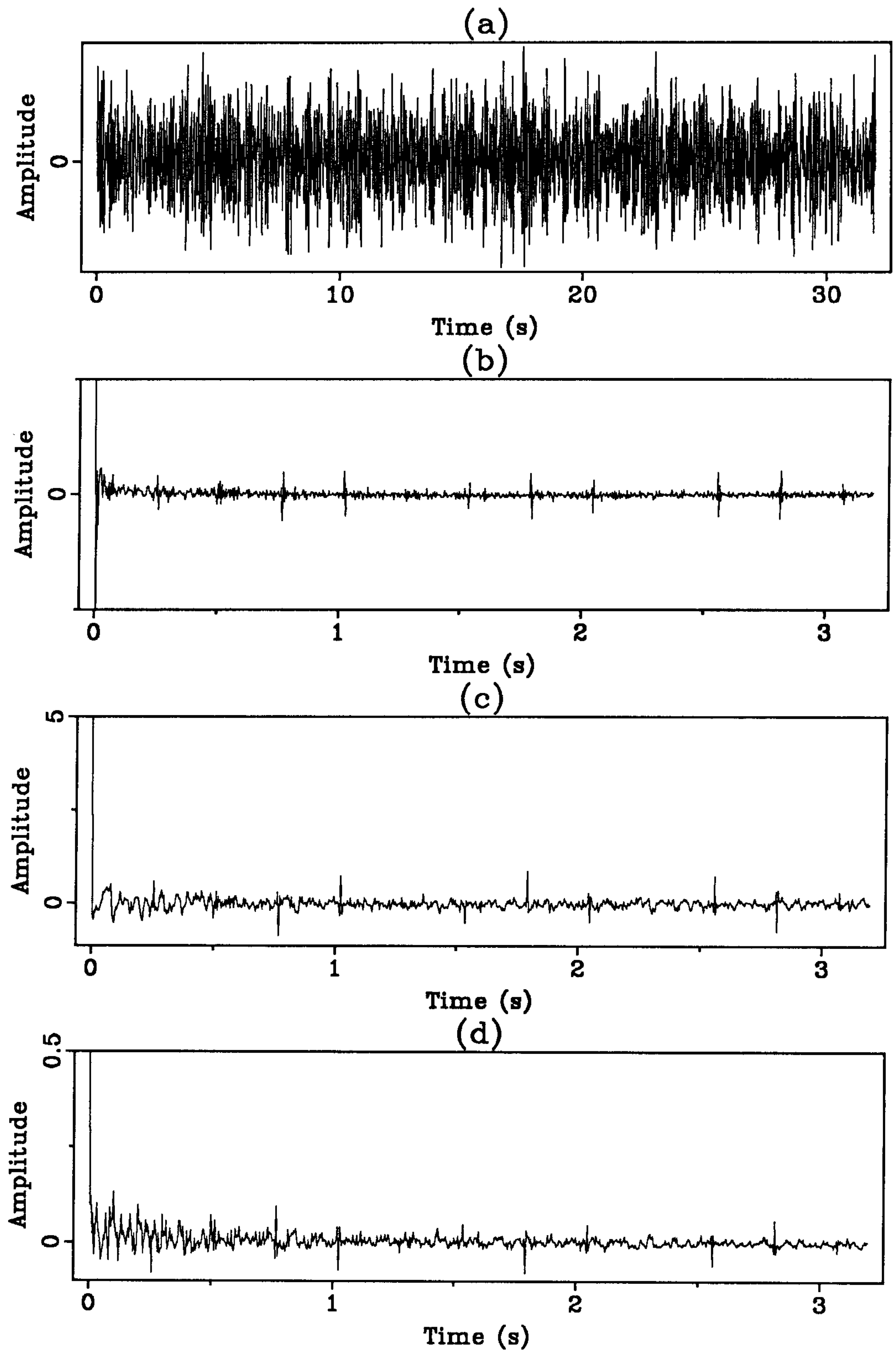

FIG. 4. A field data example: (a) vertical transmission data; (b) prediction filter; (c) prediction filter after deconvolution; (d) estimated reflection coefficients.

following notation: lower case letters denote the data in time and space domain; upper case letters denote the data in the Fourier domain; the "hatted" letters represent the data in slant stack domain.

Let $u(x,t)$ denote the data recorded by a 1-D geophone array. Applying the slant stack transform, we can decompose the wave field into plane waves of varying wave parameter p,

$$\hat{U}(p,\omega) = |\omega| U(\omega p, \omega). \tag{5}$$

It is easy to show that for each p we can use the Kunetz-Claerbout equation.

Let $q(x,t)$ denote the converted reflection data. Then we have:

$$\begin{aligned} \hat{Q}(p,\omega) &= \hat{U}(p,\omega)\hat{U}^*(p,\omega) && (6) \\ &= |\omega|^2 U(p\omega,\omega) U^*(p\omega,\omega). && (7) \end{aligned}$$

Here $\hat{Q}(p,\omega)$ can be considered as the reflection data for the plane waves of wave parameter p. The next operation is inverse slant stack transform.

$$\begin{aligned} Q(k,\omega) &= \frac{1}{|\omega|}\hat{Q}(\frac{k}{\omega},\omega) && (8) \\ &= |\omega| U(k,\omega) U^*(k,\omega) && (9) \end{aligned}$$

or

$$q(x,t) = rho(t) * \int\int u(x',t')u(x+x',t+t')dt'dx'. \tag{10}$$

Thus $q(x,t)$ is the two dimensional autocorrelation function of $u(x,t)$. Let us consider the discrete form of equation (10) and ignore the $rho(t)$ filter that has zero phase.

$$q(x,t) = \sum_{x'}\sum_{t'} u(x',t')u(x+x',t+t'). \tag{11}$$

Define $r_{x',x}(t)$ to be the cross-correlation function between trace $u(x',t)$ and trace $u(x,t)$,

$$r_{x',x}(t) = \sum_{t'} u(x',t')u(x,t+t'). \tag{12}$$

$r_{x',x}(t)$ can be considered as a shot gather with shot position x' and offset $x - x'$. Thus

$$q(x,t) = \sum_{x'} r_{x',x+x'}(t) \tag{13}$$

is the stacked shot gather. Since the earth model is assumed to be laterally invariant the shot position is irrelevant. We can also sort $r_{x',x}(t)$ into other types of gathers as we do in conventional reflection data processing. For examples, for fixed x, $r_{x',x+x'}(t)$ is a constant offset gather and $r_{x-x',x+x'}$ is a common midpoint gather.

Synthetic experiment

Figure 5 displays a model from which we generate the synthetic data. The model consists of two layers. The interface between two layers has a strong reflectivity.

The geophone array on the surface records the wave field generated by plane waves coming from various directions. The amplitudes of the incident plane waves, as a function of time, are band-limited random sequences of zero mean and unit variance.

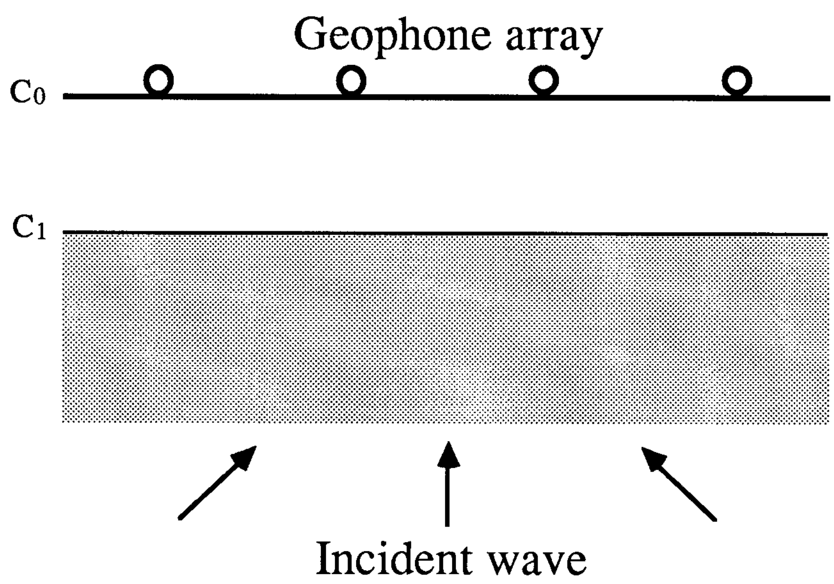

FIG. 5. A model of a layered medium. It is used to generate the synthetic passive data.

Figure 6 shows the synthetic transmission data and the converted reflection data, a shot gather. In the transmission data, we do not see any coherent signal because of the randomness of the incident waves. After the correlation processing, we can identify several events with hyperbolic moveout, corresponding to the first reflection and a sequence of multiples. It can be shown that the velocity associated with the hyperbolic moveout is the RMS velocity of the medium. The geometry of an equivalent reflection experiment that could generate converted reflection data is shown in Figure 7.

Problems in application

I tested the 2-D autocorrelation method on the stacked passive data. The results are not acceptable, as shown in Figure 8. One reason for the failure is because of the non-uniform distribution of the arrival energy. Figure 9 shows a synthetic example in which the plane waves coming from one direction are much stronger than those from other directions. In the converted section, the strong events with linear moveout interfere with the events having hyperbolic moveout. We can imagine that if the arrival energy varies dramatically with direction the hyperbolic moveout will be completely destroyed.

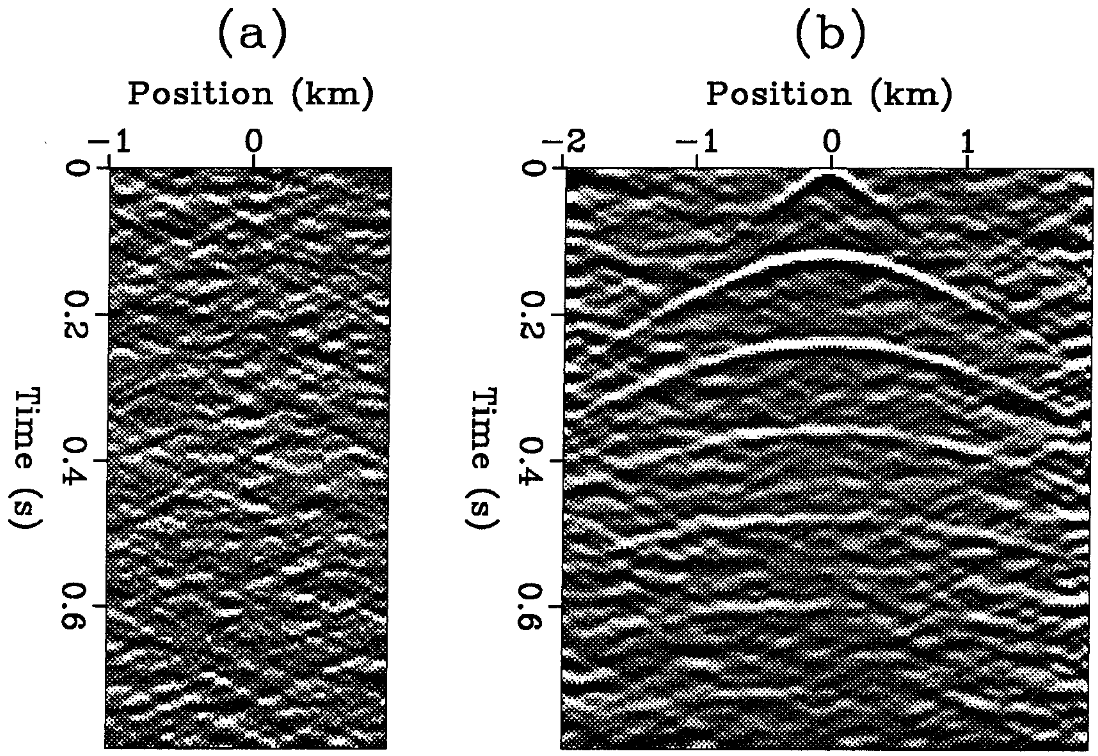

FIG. 6. A synthetic example: (a) transmission data; (b) converted reflection data.

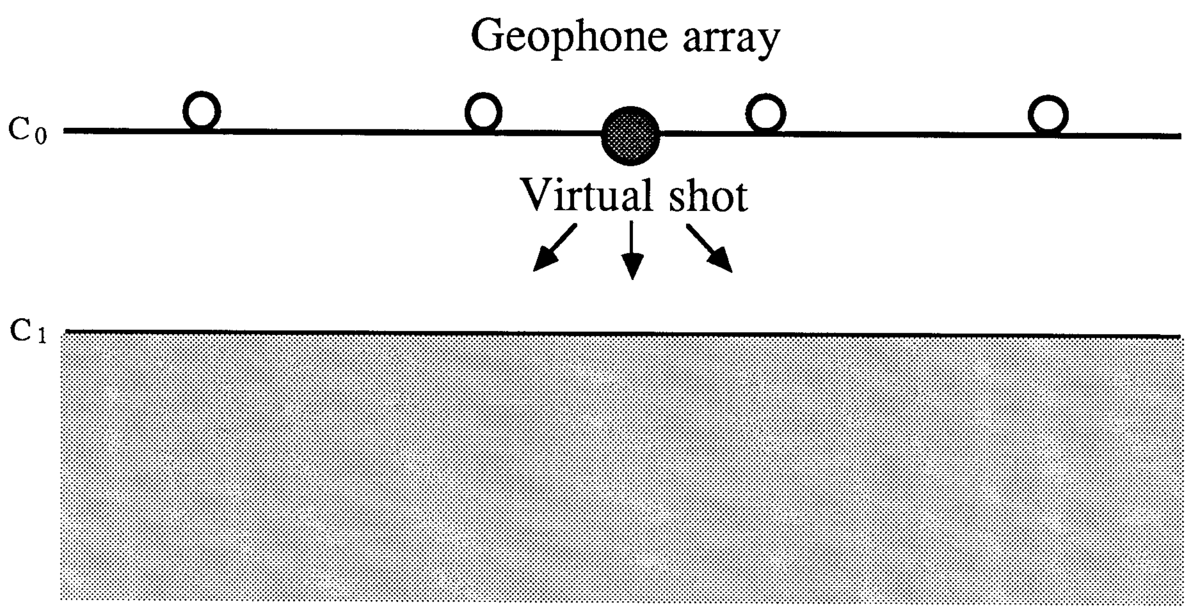

FIG. 7. The geometry of an equivalent reflection experiment that could generate the converted reflection data.

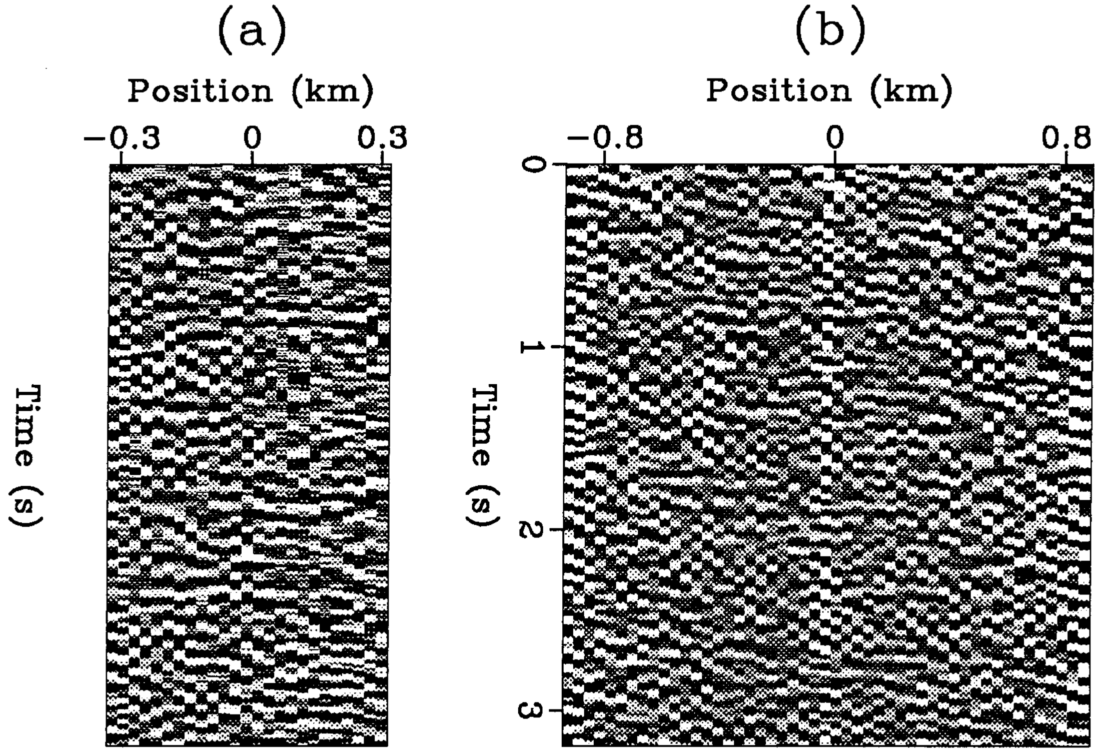

FIG. 8. A field data example: (a) transmission data; (b) converted reflection data.

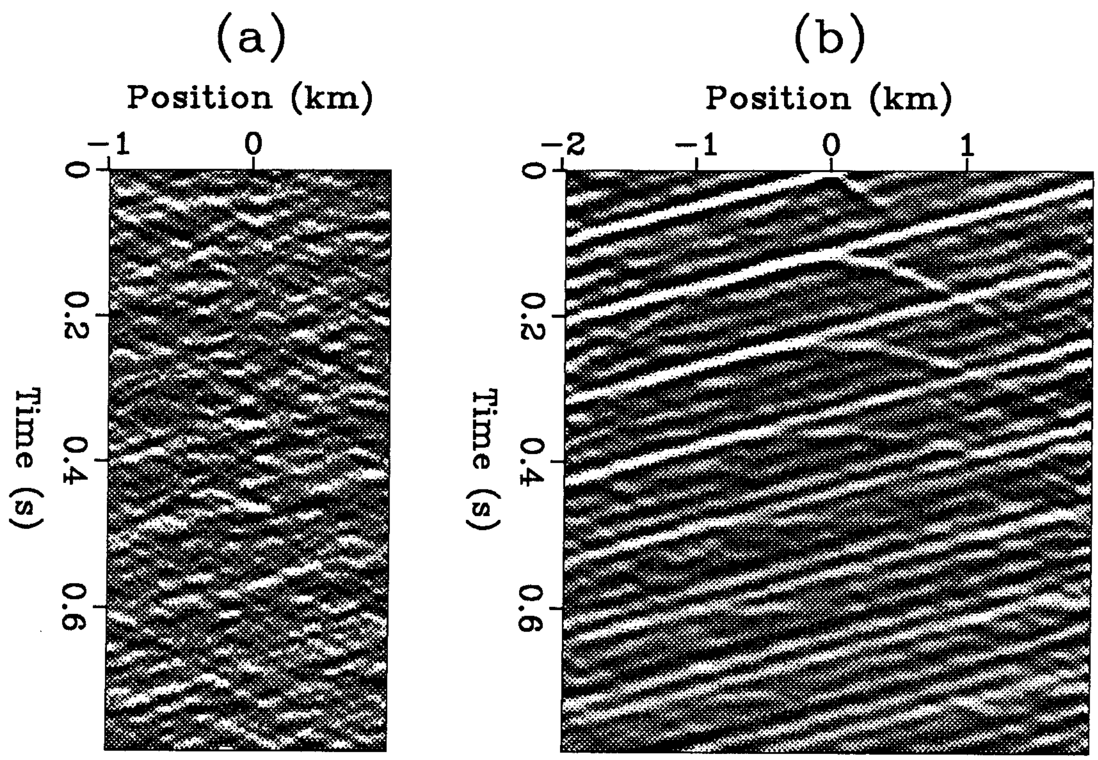

FIG. 9. A synthetic example with non-uniform energy distribution: (a) transmission data; (b) converted reflection data.

CONCLUSION

In this paper, I have discussed two issues in processing passive seismic data. The reflectivity estimation method works well for synthetic data. It gives interesting but suspicious results for field data. I am not be able to explain these results without knowing the actual subsurface structures at our experiment site. But certainly the impulses in the reflectivity sequence are not caused by monochromatic noise. The 2-D autocorrelation method is a conceptual method. It requires the energy as a function of arrival direction to be constant, which is rarely true. A possible way to get around this problem is to do trace balancing in the slant-stacked section.

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