# Datum shift and velocity estimation

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**ABSTRACT**

I derive an equation for a family of hyperbolas with various datums, each member of the family having a different asymptote velocity but the same rms velocity. Non-hyperbolic traveltime is readily mimicked by such datum-shifted hyperbolas. Datum adjustment was installed in the *Overlay* program. Dragging the mouse up and down changes the datum of the overlay hyperbola (after NMO it is a difference of two hyperbolas). Two of the Yilmaz field profiles gave datums of 120-220ms. An Alberta profile was found to be a textbook quality example.

## INTRODUCTION

Seismic waves on land often pass through a soil layer which has a very low velocity—often as low as the air velocity. Since rays bend near to the vertical in this near-surface layer, the effect on deeper events is mainly a constant delay. Mathematically, the deeper events may have a hyperbolic traveltime, but because the hyperbolas do not have asymptotes passing through the time origin, they are not hyperbolas in the seismological sense. This phenomena is the most common violation of the "straight-ray approximation" that underlies a Dix velocity analysis. Here we investigate such cases.

## ANALYSIS

The moveout equation is

$$t^2 \quad = \quad \tau^2 + x^2 s \tag{1}$$

where $t$ is traveltime, $\tau$ is vertical (or NMO) time, $x$ is offset, and $s$ is *sloth,* i.e. inverse velocity squared. We can change the datum (the time origin) by an amount $a$ with the equation:

$$(t-a)^2 \quad = \quad (\tau - a)^2 + x^2 s \tag{2}$$

Equation (2) is a hyperbola whose asymptotes generally do not go through the origin, so, seismologically, (2) isn't regarded as hyperbolic. Equation (2) has two parameters, $s$ and

$a$, that can be searched to best match a field gather. Although datum $a$ and velocity $1/\sqrt{s}$ are two independent ***physical*** parameters, they are not independent ***mathematically***, that is, any adjustment to the datum will require a readjustment of the velocity if equation (2) is to continue to match the field gather. So, let us recognize this fact by replacing $s$ in (2) by $s(a)$

$$(t-a)^2 \quad = \quad (\tau - a)^2 \; + \; x^2\, s(a) \tag{3}$$

### The problem

We need to decompose $s(a)$ into two parts, one part that represents the material velocity (say sloth $s_0$), and another part that represents the velocity variation associated with datum shift.

## CONSTANT VELOCITY

A choice of $s(a)$ that does the job is

$$s(a) \quad = \quad s_0\, \frac{\tau - a}{\tau} \tag{4}$$

Substitute (4) into (3).

$$(t-a)^2 \quad = \quad (\tau - a)^2 \; + \; x^2\, s_0\, \frac{\tau - a}{\tau} \tag{5}$$

This is the proposed equation for NMO with datum correction. Plots are in figure 1.

### Verification

First, if $x = 0$, then (5) says $t = \tau$ for any $a$. Next, if $a = 0$, then (5) reduces to the usual NMO equation (1). Finally, let us compute the apex curvature of (5) and see that it is independent of $a$. Differentiating (5) by $x$ at constant $\tau$ gives

$$(t-a)\, \frac{dt}{dx} \quad = \quad x\, s_0\, \frac{\tau - a}{\tau} \tag{6}$$

$$\frac{dt}{dx} \quad = \quad x\, \frac{s_0}{\tau}\frac{\tau - a}{t-a} \tag{7}$$

Differentiating again and evaluating at $x = 0$ and $t = \tau$ gives

$$\left.\frac{d^2t}{dx^2}\right|_{x=0} \quad = \quad \frac{s_0}{\tau} \tag{8}$$

which says that the apex curvature is independent of the datum $a$. Incidentally, the so-called rms velocity or "apparent" velocity is given from (8) by solving for $s_0^{-2}$.

Equation (5) an *exact* hyperbola in $(t, x)$-space. To see this, inspect (5) for constant $\tau$ and $a$. Although (5) is not the familiar conic, and at $t = const$ it does not give the familiar circle in $(x, z)$-space, there won't be any surprises in the travel-time curves, even far from the apex, because (5) is an exact hyperbola.

Equation (5) creates no difficulties for an NMO program because when the velocity is depth variable, it is easily solved for $t$ in terms of $\tau$, $a$, and $s_0(\tau)$.

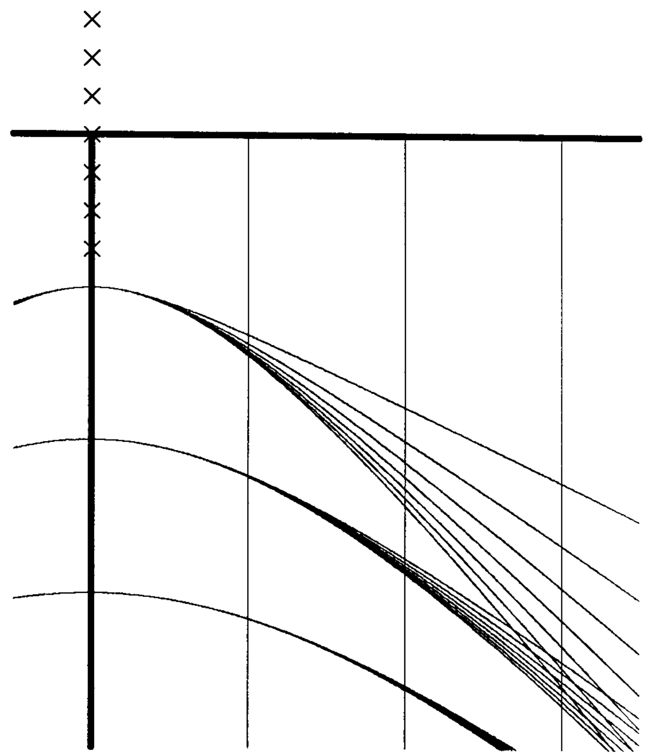

FIG. 1. Equation (5) in $(t, x)$-space scanned over seven datums and three depths. Each curve is a hyperbola. Each hyperbola has a different asymptotic velocity. But each hyperbola has the same value of rms velocity— a measure of curvature at zero offset.

### Unanswered questions

- Given a fixed $a$, what depth variable velocity is mimicked by (5)?
- On field data with the Overlay program, I felt that the asymptotes should cross after $t = 0$, but Fig 1 says that they should cross before $t = 0$ in order to model velocity increasing with depth.
- The introduction stated informally that velocity and datum perturbations are not *independent*. Perhaps the more formal statement would be that two vectors are *orthogonal*. What two vectors?

## DEPTH VARIABLE VELOCITY

We want to look at data after the best $v(\tau)$ has been used for NMO and see if a datum adjustment can improve the flattening.

### Overlays

The *Overlay* program was extended in the following way: As usual, moving a mouse moves a hyperbolic overlay. Vertical dragging changes the datum of the hyperbola (simultaneously changing the asymptote velocity to maintain a constant rms velocity). The vertical distance dragged gives $a$ directly.

The overlays can also be laid on after NMO. Let (5) define $t(a)$ for a given $x$ and $\tau$ and $s(\tau)$. For raw data the hyperbola $t(a)$ is overlaid. For moved out data, the difference between two hyperbolas, $t(a) - t(0)$ is overlaid. Examples are found in figures 6-10 of my paper "Interpretation with the overlay program" found elsewhere in this report.

### Updating the velocity profile

As the datum changes, the entire $v(\tau)$ should be changed too. (My overlay program does not yet incorporate this). The result is that $\tau {V_{rms}}^2$ should be the same both before and after the datum change, i.e.

$$Vrms^2_{new} \quad = \quad Vrms^2_{old} \, \frac{\tau \pm o1_{old}}{\tau \pm o1_{new}}$$

where $o1$ denotes the datum. There is always a question of sign convention for a datum. Our internal software (seplib) defines $o1$ as the physical coordinate of the first given data point. Notice that (4) is a special case of this equation for constant velocity and for $o1_{old} = 0$.

### A small paradox and the heart of the matter?

Isn't a nonzero datum the same as a zero-velocity layer at the surface? So a best fitting velocity-depth model could include a zero velocity layer at its top which by another name is just a datum. So what does datum analysis accomplish that is not already in interval-velocity analysis?

Hyperbolic curves constrained to pass through the origin should fit the data better when the datum is no longer constrained. Physically, freeing the datum means less reliance on the straight-ray approximation.

## FIELD DATA

Physically, datum correction arises from an effective altitude variation, more commonly from the very low velocity—often the air velocity—found in dry soils. So I excluded marine data from the study although I have seen cases where the slowness of the water layer causes an observable datum-like effect. (In a bit a research not promising enough to include in this report, I compared ordinary NMO to wave-equation NMO, i.e. downward continuation by phase shift. Small timing differences appeared only for reflectors just beneath the water bottom, thus justifying the straight ray approximation for all but such reflectors). I did a quick scan of the Yilmaz forty field profiles and some other miscellaneous on-line data. About twenty data sets were land profiles without obvious botches. Three seemed promising because of strong events about one second.

### Alberta land wz25

This split-spread profile from Alberta is a text-book case for datuming. Everyone who looked at adjustable hyperbolas displayed on top the profile could see that position and velocity adjustments alone were inadequate. (See figures 6-10 of my paper "Interpretation with the overlay program" found elsewhere in this report). Given an overlay that allowed datum adjustment, most people selected a datum of 100-250ms after $t = 0$. On more careful study I determined that the datum should lie in the range 160-200ms. The most likely value is 180. Larger values than 200 were excluded not by the event curvature but because such a large datum correction would imply velocity *decreasing* with depth.

An alternate explanation for nonhyperbolicity is always laterally variable geophone statics. And there are such statics problems on this profile. Ordinarily, the key to sorting out datum nonhyperbolicity from geophone statics is to find several events at several depths. This is not easy on this profile because of the relative positioning of strong noises and good reflectors. The value of 180ms determined for this profile is the shot static plus some averaged geophone static.

But pegleg reflections provide confirmation of the 180ms datum shift of the weathered layer. Peglegs off a primary at 500ms are unambiguously identified by their velocity. A plot of the profile moved out at the same velocity, but double the datum time, i.e. 360ms, shows the pegleg flattened.

### Denmark wz35

This split spread profile seemed promising, with some outstanding strong, early arrivals, but no datum effect was obvious. Perhaps Denmark is all water saturated.

### North Africa wz10

There is a strong shallow primary at 1.1sec. I estimate o1=-.220.

## CONCLUSIONS AND RECOMMENDATIONS

Nonhyperbolicity is readily observable in early arrivals. Velocity analyses and stacks should routinely include a datum correction. It can significantly improve the focus of shallow events.

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March 9, 1987

Charles H. Sword, Jr.
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Dear Mr. Sword,

I'm pleased to inform you that your presentation at the recent AGU meeting in San Francsico titled "Modeling of global surface waves by a finite-element method" was selected for the Best Student Paper Award by the Seismology Section. We would like to make a short announcement of this to be published in EOS. For this purpose, could you send me a photo of yourself (black and white is preferable), and a short paragraph describing your background, interests, and future plans?

With congratulations and best wishes for the future.

Yours truly,

Stewart W. Smith
President
AGU Seismology Section

cc:M. Compton, AGU
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