Elastic inversion of field data

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ABSTRACT

I previously proposed a nonlinear inversion algorithm based on the full two-way elastic wave equation that takes into account all the elastic wave events and obtains a maximum probability compressional and shear wave velocity model. Theoretically, the algorithm can obtain both the reflectivities and interval velocities simultaneously (high- and low-wavenumber components of the velocity) so it is like a simultaneous migration and reflection tomography. Reflection amplitudes resolve the higher vertical wavenumbers of the P- and S-wave velocity models while the reflection hyperbola shapes resolve the lower wavenumbers. I tested the elastic inversion algorithm using a two-component field shot gather containing primary P-P reflections and mode converted P-S reflections. High vertical wavenumbers are best resolved so the result looks like a prestack elastic migration. Its meaning is not simply reflectivity; it is P- and S-wave velocity perturbation. Also, confirming the theory, some lower vertical wavenumbers appear in the solution.

INTRODUCTION

There are many complicated wave-like events contained in seismic data that are inadequately handled by conventional seismic processing. These include mode converted waves, refracted waves, Rayleigh waves, evanescent waves, multiple reflections etc. All these events can be modeled well by the elastic wave equation. Each of the different events provides useful information about the subsurface compressional and shear wave velocities. Therefore, it is useful to base an inversion algorithm on the elastic wave equation. The model corresponding to the best matching wavefield to the observed seismic amplitudes is the maximum probability inverse solution. Assuming Gaussian errors in the data leads to a least squares formulation for this maximum probability solution. The theoretical formulation was done by Tarantola (1984) and Mora (1987a). The assumption of Gaussian noise is not bad considering that most coherent noise in seismic data is wave-like and can be modeled by the elastic wave equation. Therefore many events that conventional processing methods may treat as noise are accounted for by the inversion theory and hence provide useful information. Some examples of inversions of synthetic data are given by Mora (1987a,1987b). His results indicated that for ideal synthetic data (contaminated with Gaussian noise) that this inversion method could resolve very well between P- and S-wave velocity using two-component seismic data (vertical and radial geophones and vertical shots). However, it has always been questionable whether this method could be applied to real seismic data with all their complexity.

What would constitute a successful inversion? The answer to this question would critically though naively be, inversion results that compare with well logs. The reason it is naive is that it is overambitious to expect such quantitative answers considering that there is a null space in the seismic inversion problem. In any case, the inversion result is two-dimensional while well logs are only one-dimensional. My own answer is an inversion that resolves two different images for P- and S-wave velocity constitutes a success. Perhaps, the inversion can at least resolve between the three different elastic parameters and indicate lateral variations in a qualitative sense. This paper is a discussion of my first two-dimensional elastic inversion result using real data. To my knowledge, it is the first two-dimensional nonlinear elastic inversion of real seismic data using the full-elastic wave equation and back-propagation approach.

THEORY

Mora (1987a) presented an elastic inversion algorithm based on the least squares principle. It was an iterative algorithm to obtain the P- and S-wave velocities and densities directly from some seismic wavefield (see Tarantola (1984) for a derivation in terms of the Lamé parameters and density). The method matched the observed data to a synthetic wavefield computed by doing elastic finite differences. It was based on conjugate gradient iterations to minimize the sum of squared error between the observed and synthetic data. The P- and S-wave velocity model corresponding to the best-fit synthetic wavefield was the most probable velocity and density model. The only requirement of the method was the direction of steepest descent on the least squares functional surface which indicates the way to update the velocity model. This was obtained by two finite difference simulations of the elastic wave equation using

$$\delta\gamma(\mathbf{r}) \;=\; \sum_{s} \int dt \;\left[\Omega^{\gamma}_{ijk} u_j(s,\mathbf{r},t)\right]\left[\Omega^{\gamma}_{ijk} \psi_j(s,\mathbf{r},t)\right] \qquad (1)$$

where γ is the kind of model parameter (P-wave velocity, S-wave velocity or density), s is the shot number, and u_j and ψ_j are two wavefields that can be calculated by finite differences. Ω^{γ}_{ijk} is a model parameter unraveling operator to obtain the physical parameters, P-wave velocity, S-wave velocity or density. u_j is the j-th component of the displacement of a synthetic wavefield computed using the velocity model at the current iteration. ψ_j is a wavefield computed by using the residual between the wavefield u_j and the observed wavefield as a forcing function in the elastic wave equation and doing a wave propagation backwards in time. The time integral corresponds to a correlation between the shot wavefield and the back-propagated residuals. This is comparable to the concept of performing migration by correlation of downgoing waves u_j with upgoing waves ψ_j. The main difference is that the two wavefields in the inversion formula (equation (1)) have both upgoing and downgoing components because they are both computed by the full two-way elastic wave equation. Other differences between equation (1) and migration is that it is applied iteratively and that the operator Ω^{γ}_{ijk} converts to the physical properties, P-wave velocity, S-wave velocity and density.

FIELD DATA EXAMPLE

A two-component shot gather from a vertical vibrator source is shown in Figure 1. The data were dip-filtered to decrease the level of noise (Figure 2). A velocity model obtained from a conventional velocity analysis was used as an initial guess in the iterative inversion. Sharp discontinuities in interval velocity were present in this model. The dip-filtered synthetic data obtained by modeling using the initial model is shown in Figure 3. Many of the events including the deep P-S mode converted reflection can be seen on the synthetic data. Note that no significant Rayleigh waves can be seen in the data so they were not included in the elastic finite difference modeling. The first iteration of an inversion of the dip-filtered shot gather was performed and the result is shown in Figure 4. Notice that the S-wave velocity is not resolved directly below the shot because there were no mode converted waves there (the source was vertical component). An interesting feature in the P-wave velocity result is the smear to the right of the shot coming down in a semi-circular arc down to a reflector and back up to the shot. If many shots from different locations were stacked together then this would cancel except at a depth of about 1.0 km. Therefore, the inversion algorithm appears to resolving interval velocities (low wavenumbers) as well as P- and S-wave reflectivities (high wavenumbers).

Because only one iteration of the inversion was done the results are comparable to an elastic migration except in one respect. The full wave equation was used in the calculations and this enables the inversion algorithm to resolve interval velocities (Mora, 1987c). Non-flat events in the images are due to correlations between P- and S-waves at non-geologic positions and are comparable to migration-smile artifacts

The result is good considering that only one shot gather was used in the inversion and a two-dimensional P- and S-wave velocity image was obtained in the region around the shot. More complete images could be obtained by adding the results of many shots (i.e. including several shots in the shot sum $\sum_s$). Synthetic studies indicate that the circular migration-smile-like artifacts would tend to disappear after summing several shots and after a few iterations.

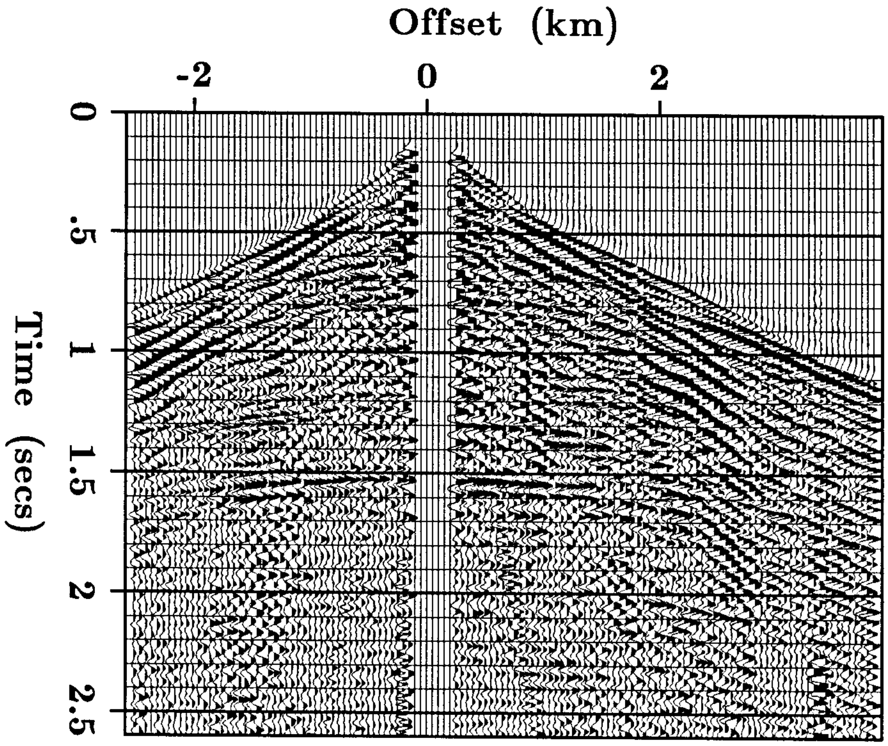

FIG. 1. (a). Vertical component shot gather from a vertical vibrator source (supplied by C.G.G.).

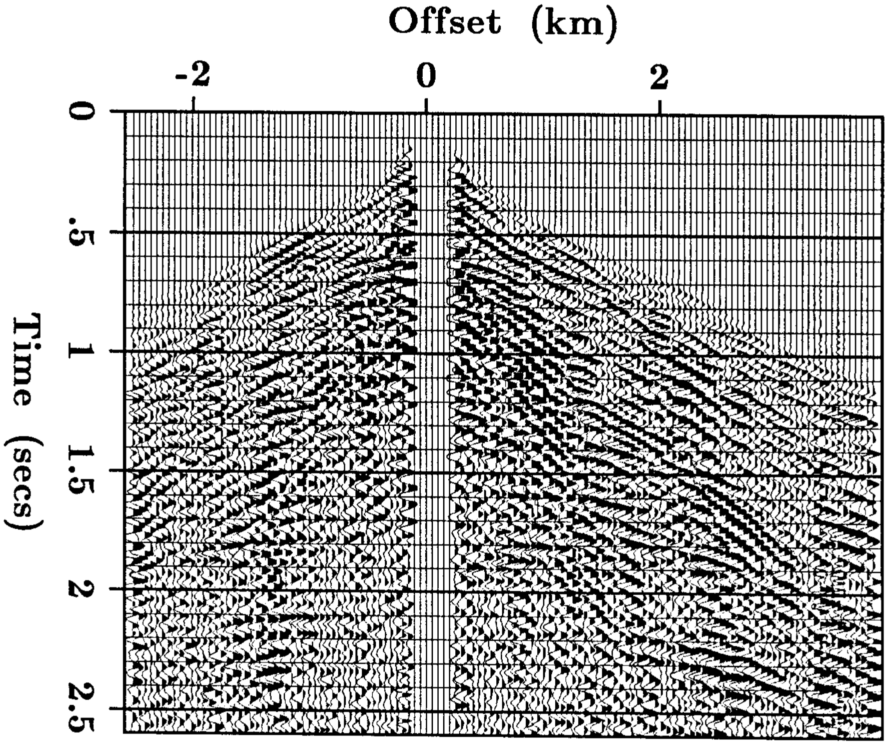

FIG. 1. (b). Horizontal component shot gather from a vertical vibrator source (supplied by CGG).

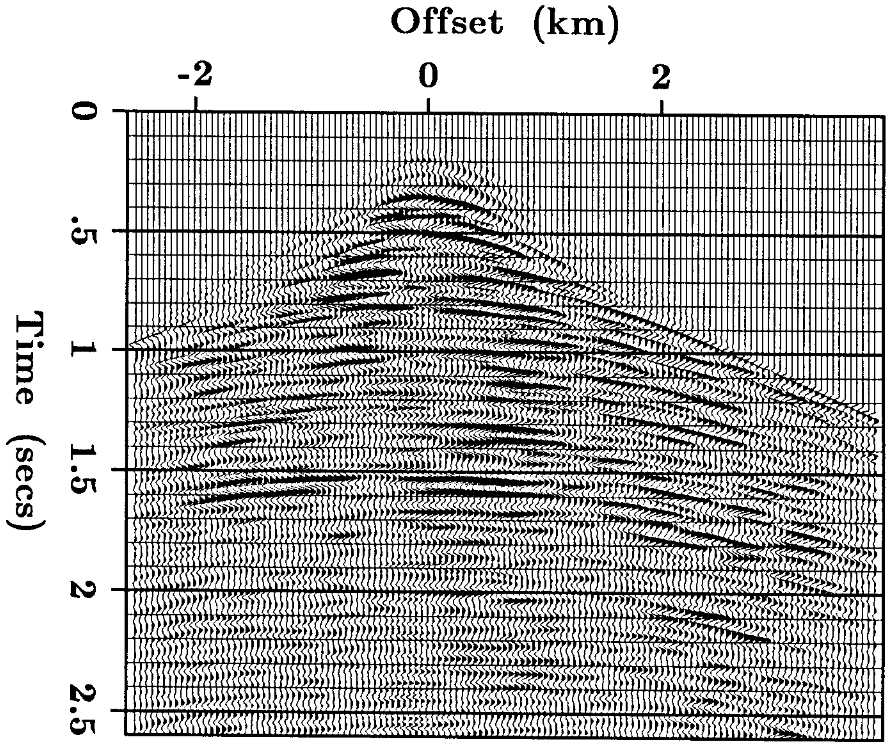

FIG. 2. (a). Dip-filtered vertical component shot gather from a vertical vibrator source.

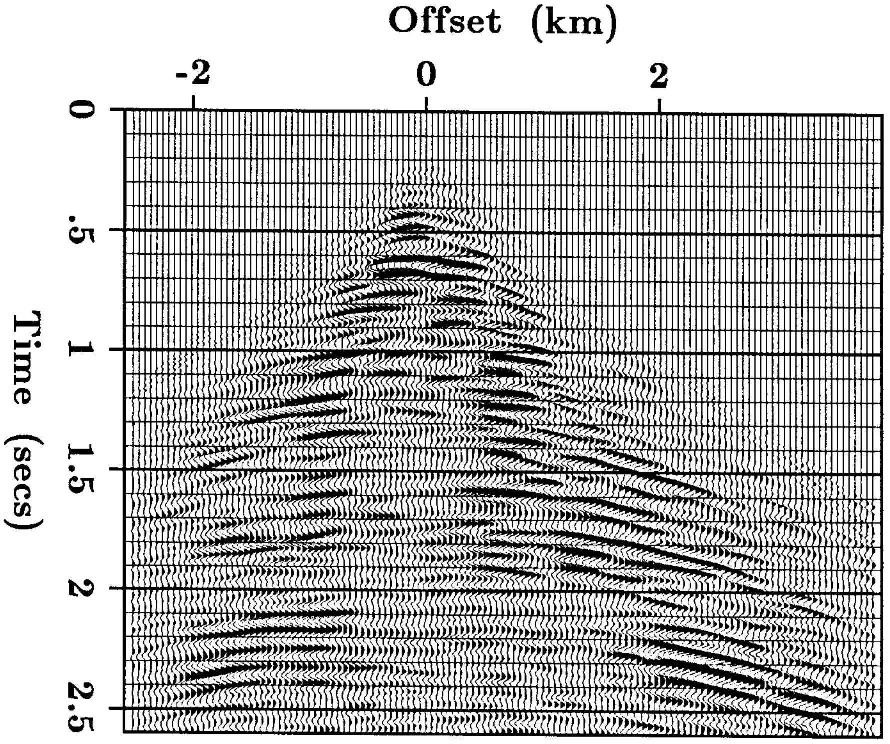

FIG. 2. (b). Dip-filtered horizontal component shot gather from a vertical vibrator source.

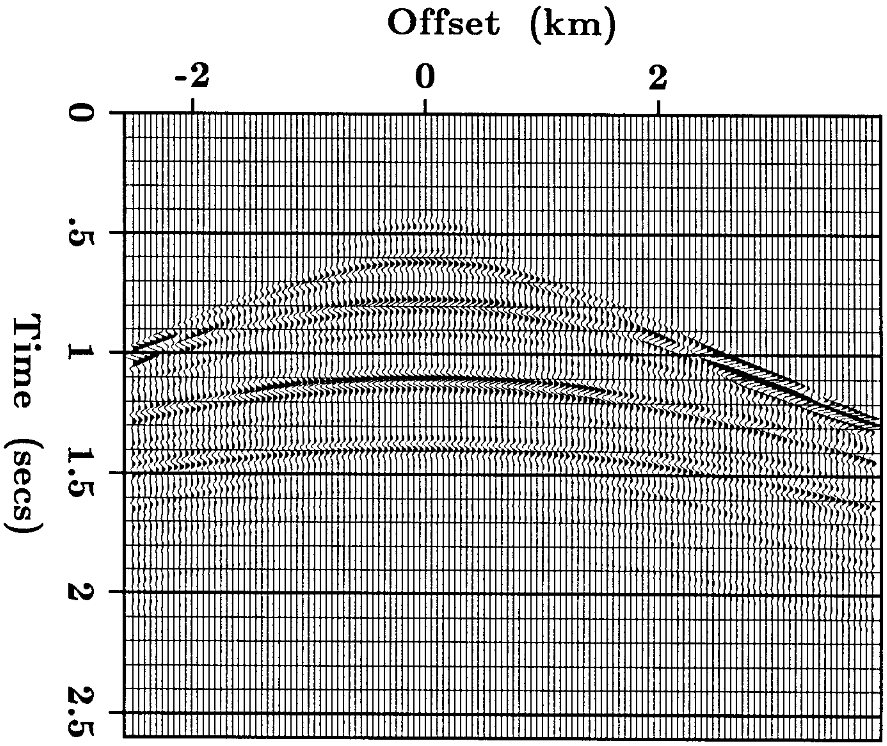

FIG. 3. (a). Dip-filtered synthetic vertical component shot gather from a vertical vibrator source.

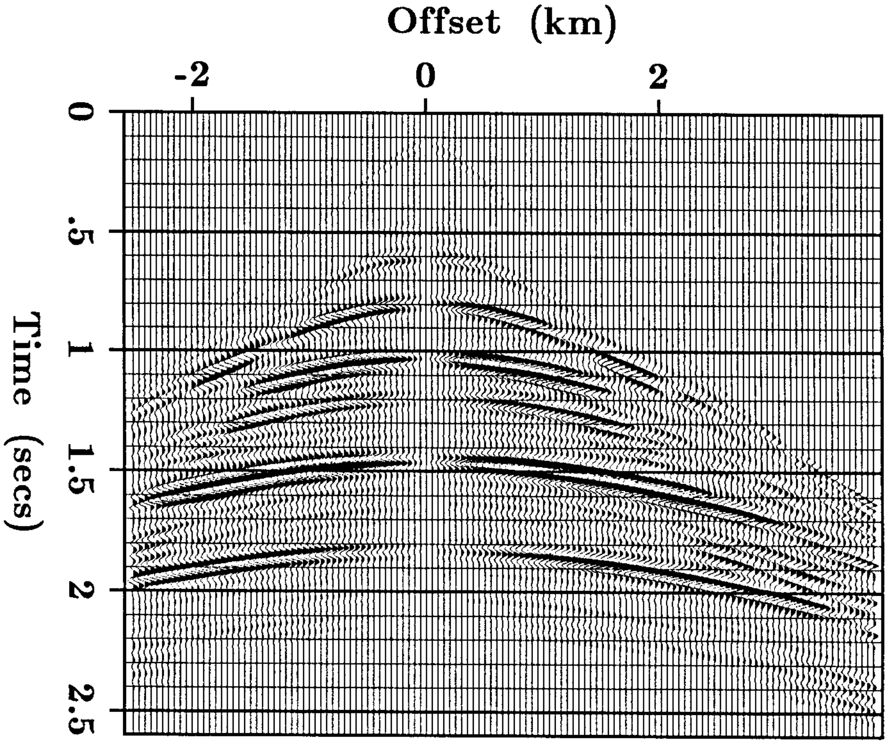

FIG. 3. (b). Dip-filtered synthetic horizontal component shot gather from a vertical vibrator source.

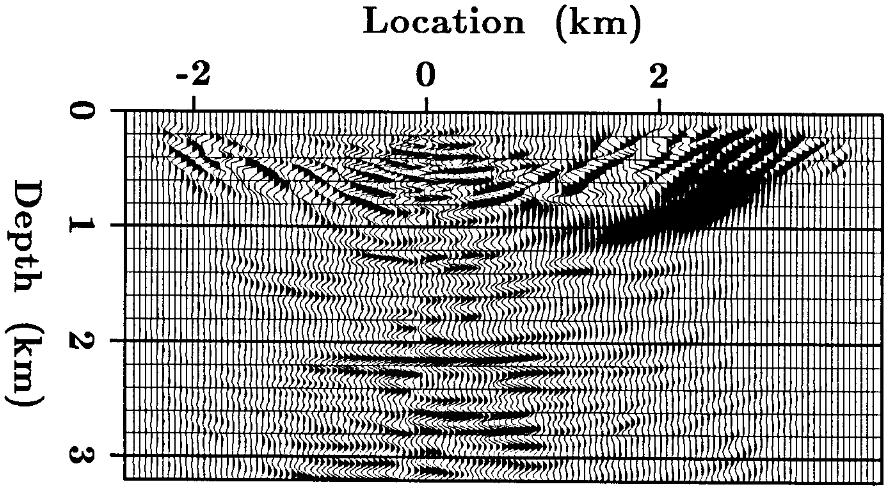

FIG. 4. (a). First iteration inversion result for compressional velocity.

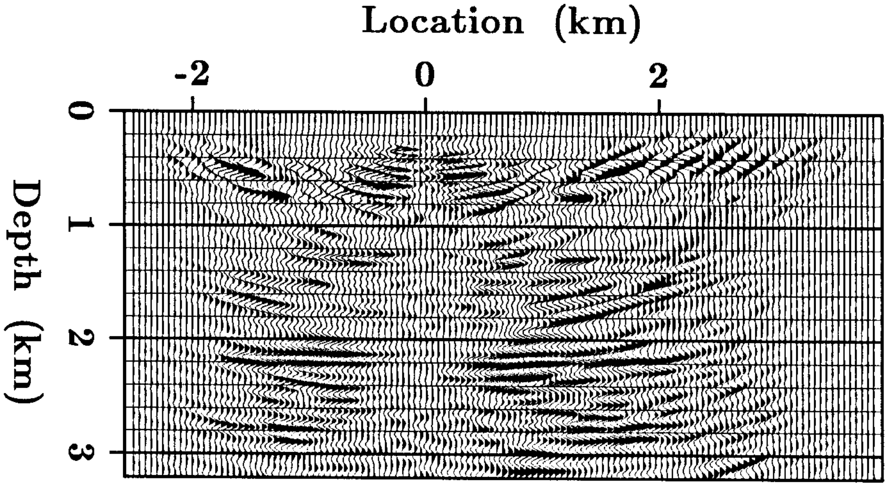

FIG. 4. (b). First iteration inversion result for shear velocity.

CONCLUSIONS

Nonlinear elastic inversion results on a single shot gather are promising. Two images were obtained, one for P-wave velocity and one for S-wave velocity. These do not correlate everywhere indicating that the algorithm can resolve different features in the two properties. These results are encouraging and we can look to the future when all seismic processing can be replaced by full elastic wavefield inversion schemes. This will probably require specialized hardware as well as software but the gains may be huge.

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