Fast prestack depth migration using a Kirchhoff integral method

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ABSTRACT

A Kirchhoff integral method using approximate Green's function calculations can accurately migrate data before-stack, faster than previous implementations. Traveltimes and amplitudes of the WKB Green's functions for either stratified or laterally varying media are precomputed using an efficient top-down approach and stored on disk. Two-point ray tracing is avoided. For any given surface point, only a sparse fan of rays is calculated; and extensive interpolation creates a traveltime map from that surface point to all image points. For laterally varying media, only a "few" surface locations are used for ray tracing. The traveltime maps for locations not ray-traced are found by interpolation of existing traveltime maps. These results apply to post-stack depth migration as readily as they do to prestack depth migration. Two examples demonstrate the method.

INTRODUCTION

As long as the assumption of 2-D structure holds, prestack depth migration of seismic reflection data collected along a line should be a powerful tool for both imaging and velocity analysis of data collected in areas of lateral velocity variation. The high cost of most prestack depth migration algorithms and the fact that an accurate estimate of the velocity may not be available before migration are the most serious drawbacks to the application of the method. The solution to the lack of velocity information may be in migrating the data several times while improving the estimate of the Earth's velocity each time. Because many migrations may be necessary, the cost of any given migration is a concern. In some cases it is too costly to migrate the data before stack even once. This motivation led me to search for a prestack depth migration algorithm that was accurate for laterally varying media, and appreciably faster than other algorithms for laterally varying media such as the reverse-time algorithm of a previous paper (Etgen, 1986). A conversation with Norman Bleistein led me to consider a Kirchhoff integral approach in pursuit of the above goals. This approach can be applied to Born-Kirchhoff inversion described in (Bleistein, 1986) as well.

The largest portion of the effort in migrating data in heterogeneous media with an integral method is the computation of the traveltimes and amplitudes associated with the required Green's functions. For example, consider a model with 256 surface points by 256 depth points. For a laterally variable velocity model, a traveltime and an amplitude from each subsurface point to each possible source or receiver location is required. In this example, 256^3 traveltimes and amplitudes are required to connect all surface points to the subsurface points. If two-point ray tracing is used for each combination of subsurface image point and source/receiver point, a tremendous number of rays will be required to obtain all the necessary traveltimes and amplitudes. The summation step to do the migration is much less expensive. Therefore most of the effort in speeding up the Kirchhoff prestack depth migration will be spent on efficient and accurate traveltime calculations. Post-stack depth migration suffers from the same burdensome traveltime and amplitude calculations so the methods presented here can be applied to post-stack depth migration by Kirchhoff integral as well.

The theory of prestack Kirchhoff migration is not new, so I will only include a brief review of the theory. I will describe the main points of the traveltime and amplitude calculations where the main computational effort is spent in conventional implementations. Because interpolation methods are used, some of the generality of finite-difference type algorithms is lost. I will discuss these limitations and describe when they may be important and when they are not of concern. Finally, I will show two synthetic examples that demonstrate the method's ability to correctly image data in the presence of lateral velocity variation.

KIRCHHOFF INTEGRAL PRESTACK DEPTH MIGRATION

This section will give a brief review of the integral theory of migration, which by no means should be considered complete. I will describe the results and explain their importance to prestack depth migration. For a more complete discussion see (Deregowski, 1985) and (Schneider, 1978).

The Kirchhoff integral representation of migration usually begins with a statement of Green's theorem that describes a wave field in the interior of a region given its values on the boundary. The starting point of this description will be the three dimensional representation for downward-continuation of a wave field (Schneider, 1978). For 2-D prestack migration, the wave field is only known along a line, but the amplitude is governed by three dimensional spreading, so the downward-continuation equation will be the same for both 2-D (linear) data and 3-D (areal) data. When we have 2-D data; we simply have no knowledge of the wave field away from the line. The wave field most probably is a function of the cross-line direction; so collecting data only along a line loses information about the cross-line properties of the wave field. Imaging only beneath the survey line is tantamount to assuming the subsurface is independent of the cross-line direction. The downward-continued wave field is calculated using only the information recorded. These assumptions are different from the assumptions made when migrating a stacked section. For post-stack migration of 2-D wave fields, the wave field is assumed to be independent of the out of plane direction giving a slightly different equation for downward-continuation; the wave field is assumed to be cylindrical instead of spherical as in prestack migration.

Equation (1) will downward-continue a wave field recorded at the surface of the Earth. In this equation, $\tau(r, x, z)$ is the traveltime of the WKB Green's function for a wave traveling from the given image point x, z to the receiver point r. $A(r, x, z)$ is the amplitude of this WKB Green's function, also for a wave traveling from the image point to the receiver point.

U_{dwcnt} is the downward-continued wave field as a function of x, z, t. $U_{recorded}$ is the wave field recorded on the surface. Note that this wave field extrapolation does not account for multiples so is useful in the context of migrating primary reflections only.

$$U_{dwcnt}(x,z,t) = -\frac{1}{2\pi}\frac{\partial}{\partial z}\int_{receivers} A(r;x,z)\cdot U_{recorded}\left[s,r,t+\tau(r;x,z)\right]\ dr \tag{1}$$

Once the wave field has been downward-continued to the desired image point, prestack migration can be accomplished by imaging the downward-continued wave field at the arrival time of the shot wave field. This can also be though of as downward-continuing both the shot and the receiver wave fields and imaging at time zero. There are several possible methods for implementing this imaging step. I will describe a method similar to the correlation method described in (Etgen, 1986) for wave fields downward-continued by finite-differences. Imaging is accomplished by multiplying the downward-continued shot wave field by the downward-continued receiver wave field. Equation (2) is the resulting integral showing the imaging condition applied for a given shot s_o. $A(s_o;x,z)$ is the amplitude and $\tau(s_o;x,z)$ is the traveltime from the shot location to the image point of the shot wave field which gets multiplied by the downward-continued receiver wave field given by equation (1). Equation (2) can be considered a prestack depth migration of a shot profile.

$$\begin{aligned} U_{image}(x,z) &= -\frac{1}{2\pi}\frac{\partial}{\partial z}\int_{receivers} A(s_o;x,z)\cdot A(r;x,z)\cdot \\ &\quad \cdot U_{recorded}\left[s_o,r,\tau(r;x,z)+\tau(s_o;x,z)\right]\ dr \end{aligned} \tag{2}$$

To implement this for a set of discrete shot locations, the equation becomes:

$$\begin{aligned} U_{image}(x,z) &= -\frac{1}{2\pi N}\sum_{shots}\frac{\partial}{\partial z}\int_{receivers} A(s;x,z)\cdot A(r;x,z)\cdot \\ &\quad \cdot U_{recorded}\left[s,r,\tau(s;x,z)+\tau(r;x,z)\right]\ dr \end{aligned} \tag{3}$$

Where N is a normalization factor accounting for the fold of the data. Of course we never have continuous receiver coverage so the integral over receivers is replaced by a discrete sum, giving equation (4), a prestack depth migration formula for surface seismic data.

$$\begin{aligned} U_{image}(x,z) &= -\frac{1}{2\pi N}\sum_{shots}\frac{\partial}{\partial z}\sum_{receivers} A(s;x,z)\cdot A(r;x,z)\cdot \\ &\quad \cdot U_{recorded}\left[s,r,\tau(s;x,z)+\tau(r;x,z)\right]\cdot\Delta r \end{aligned} \tag{4}$$

If the receiver spacing were unequal, then the equation can be modified to account for a spatially variable Δr. The requirements for the migration are the data, traveltimes, and amplitudes $\tau(\xi;x,z)$ and $A(\xi;x,z)$, from each source or receiver point ξ to each output image point x, z.

EFFICIENT TRAVEL-TIME AND AMPLITUDE CALCULATIONS

Why is the conventional approach so slow

Given a seismic survey with nx ground locations and $nx \cdot nz$ desired image points, a naive, direct implementation of equation (4) above would require $nx \cdot nx \cdot nz$ rays to be traced with a two-point ray tracing scheme in laterally varying media. On SEP's Convex I can trace approximately 20 rays per second for a 400 step ray with the Runge-Kutta method described in the next section. If $nx = nz = 256$ then the ray tracing for a heterogeneous model would take many hours. Most importantly, if rays are traced from the image points to the surface points, it is difficult to re-use information from one z level to the next (Gray, 1986). Because of this, at each z level rays have to be traced from that z level all the way up to the surface. Gray (1986) points out that this causes the cost of traveltime calculations to exceed the cost of Kirchhoff summation. If the media were stratified $nx \cdot nz$ rays (one surface point to all image points), would be required to find the appropriate traveltimes and amplitudes because traveltime is only a function of offset. The total effort required would be much less than the $v(x, z)$ case, but still expensive. For most migrations with $v(z)$ only, an rms approximation to the velocity model would be used. However, in some cases, such as velocity analysis, it might be desirable to retain the connection with interval velocities. The main point is that conventional traveltime calculations for complicated models consume far more effort than the sums used to do the migration. The cost of calculating traveltime and amplitudes can be made less than the summation cost in virtually all cases. Most importantly, little or no accuracy is lost in those cases.

Interpolation of traveltimes and amplitudes from a sparse set of rays

In this section I will describe a method to calculate the large number of traveltimes and amplitudes needed for Kirchhoff integral migration, accurately and efficiently. The goal of the ray tracing and interpolation method is to obtain traveltime and amplitude "maps" for each surface point that is a source or a receiver. These maps will be stored and used in the summation step of the migration. Only the case where velocity varies both laterally and vertically will be discussed as the vertically stratified case is contained in the more general case. At the end of the section I will discuss some cases where this method may not work effectively.

To construct the WKB approximate Green's function for a given surface point, (traveltimes and amplitudes), the method of characteristics is used to solve the eikonal equation of the scalar wave equation. Characteristic curves or rays are traced in the given velocity model using a fourth-order Runge-Kutta method for the ray equations. The calculations can be reorganized so that much of the work of the numerical solution of the ray equations can be done simultaneously for many different rays. This allows the Runge-Kutta routine to be vectorized. The first step in constructing the traveltime and amplitude maps involves shooting a fan of rays from a selected surface point. A sparse fan of rays (usually 64 or 128 rays) is shot from the surface into the velocity model. This top-down approach allows the rays at a given depth to be determined from the rays at the depth just above it by simply extending the rays downward from the level above (Gray, 1986). Figure 1 shows a fan of rays starting at a point on the surface of the model of the first example. This fan of rays is the basis for finding traveltimes and amplitudes of the WKB approximate Green's function at an even grid of points covered by the ray fan.

After the ray tracing for a given surface point, the traveltimes and ray positions are not on an even grid either in x or z. The rays are interpolated onto an even grid in z, with x position and traveltime a function of z. The final even grid in x and z is then interpolated from the ray coverage. The traveltime to a given x, z position is determined using the three rays. At a given z level, the ray with the shortest traveltime is located. Call this ray the

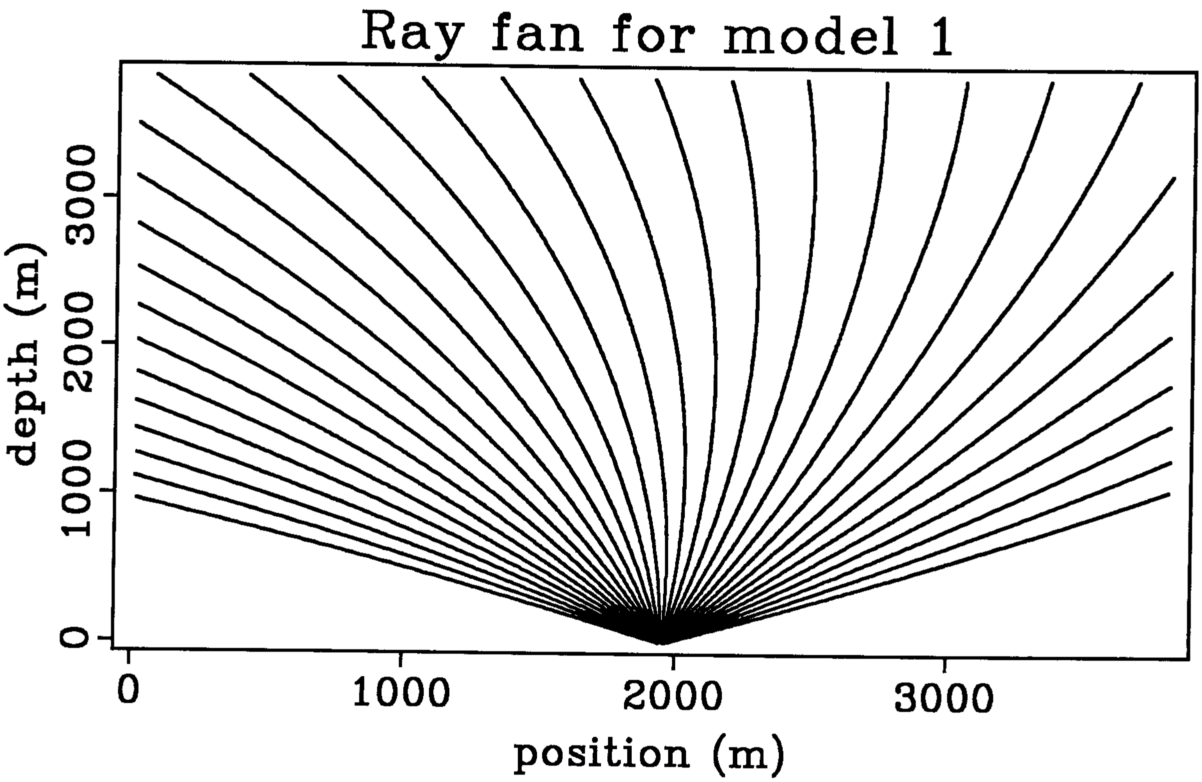

FIG. 1. Fan of rays used to compute traveltimes and amplitudes from the surface point x=1 950 meters to the subsurface for the first model. Note that the surface is at the bottom with depth increasing upwards. Only 32 of the 64 actual rays are shown.

central ray. The traveltimes for each x location on that z level are found using a local hyperbolic interpolation. The shape of the hyperbola is determined by the two rays that immediately surround the given point and the x position of the central ray, x_o. Equation (5) gives the interpolation equation used to obtain the necessary traveltimes.

$$\tau(\xi; x, z)^2 = \tau_o^2 + \frac{(x - x_o)^2}{V_e^2} \tag{5}$$

The position of the point is given by x, z, the location of the central ray is x_o, z. The parameters τ_o, zero-offset traveltime, and V_e, "effective velocity", are determined by solving a system of two linear equations in two unknowns involving the position of the central ray, and the positions and traveltimes of the rays that immediately surround the point. The "zero-offset" traveltime τ_o is not necessarily the same as the traveltime of the central ray. Also, there are no restrictions on V_e^2, it does not have be related to any physical quantity; it doesn't in fact have to be positive. The only requirement is that the interpolation hyperbola must go through the two known offset, traveltime points. The surface location ξ can be either a shot or a receiver. In essence, the parameters describe the hyperbola that goes through the two rays and whose apex is at the location of the central ray. Figure 2 is a slice of the traveltime map at a depth of 2 520 meters. The traveltime values are on an even grid in x, z. The locations for which traveltime equals zero were outside of the ray fan.

The proposed interpolation equation is complicated, and other simpler methods can be used. Gray (1986) proposes using linear interpolation, but for parts of the traveltime curve

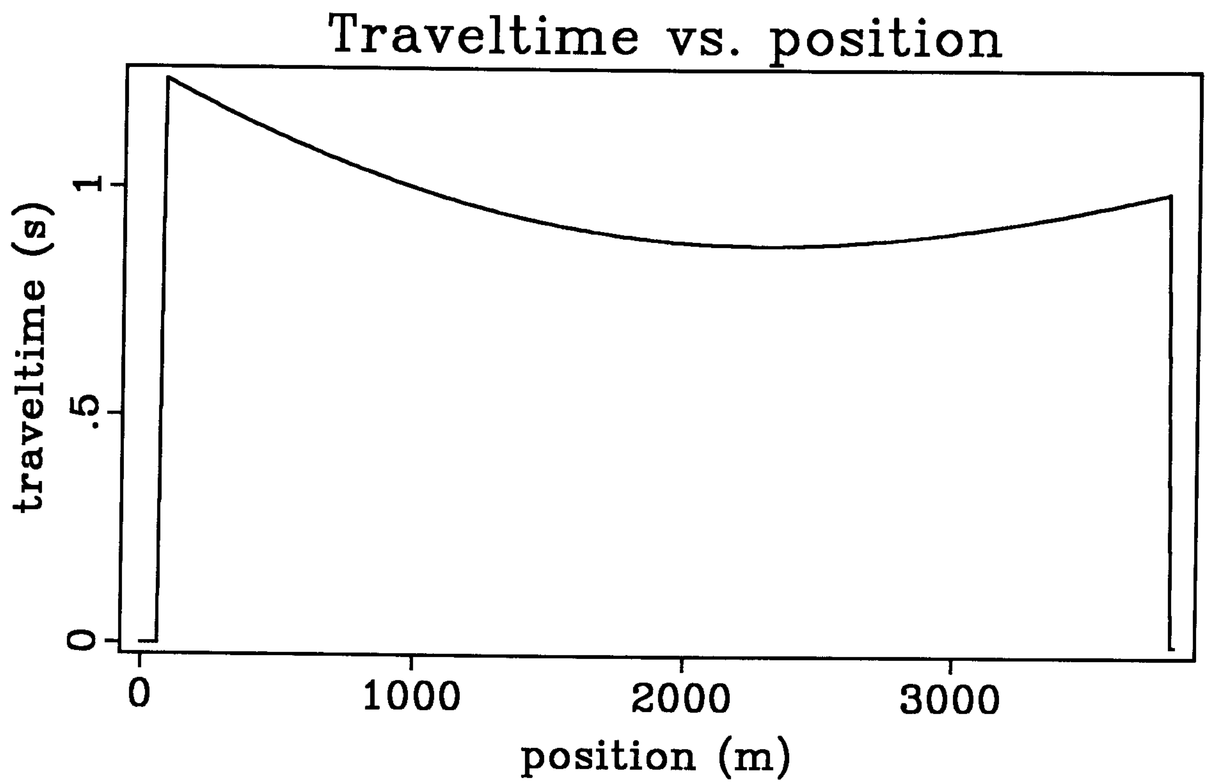

FIG. 2. Traveltime vs. position at 2 520 meters depth for the model of the first example. The starting surface location for the rays is 1 950 meters. Note the shift in the apex of the traveltime curve away from the starting point of the rays.

where there is significant curvature, such as near the central ray, the linear approximation may be poor and a hyperbolic one more accurate.

The amplitude at the point is determined by correcting the spherical divergence factor for a constant velocity medium by a term that accounts for ray divergence because of velocity variation. A term that approximates source or geophone directivity can be added as well. Equation (6) shows an amplitude term for a given point that takes into account ray spreading and a cosine weighted source or geophone directivity function. J_c is the spreading Jacobian for constant velocity and J is the spreading Jacobian calculated from the ray-traced model. In practice, the ray spreading Jacobian J is only computed approximately. The take-off angle of the ray at the surface is given by Θ_t .The surface location ξ is either a shot or a receiver. Figure 3 shows the amplitude of the WKB approximate Green's function for the first model at a depth of 2 520 meters as a function of position.

$$A(\xi, x, z) = \frac{1}{\tau(\xi, x, z)} \cdot [\frac{J_c}{J}]^{\frac{1}{2}} \cdot \cos(\Theta_t) \tag{6}$$

This approach is used to determine traveltimes and amplitudes from the given surface point to all points in the subsurface that fall within the ray coverage. What remains is to calculate the traveltime and amplitude maps for all surface source/receiver points.

Even with this approach to traveltime calculations, there would still be significant effort spent to calculate a traveltime and amplitude map for each possible source or receiver location. Also, the required disk storage for a traveltime and amplitude map for each

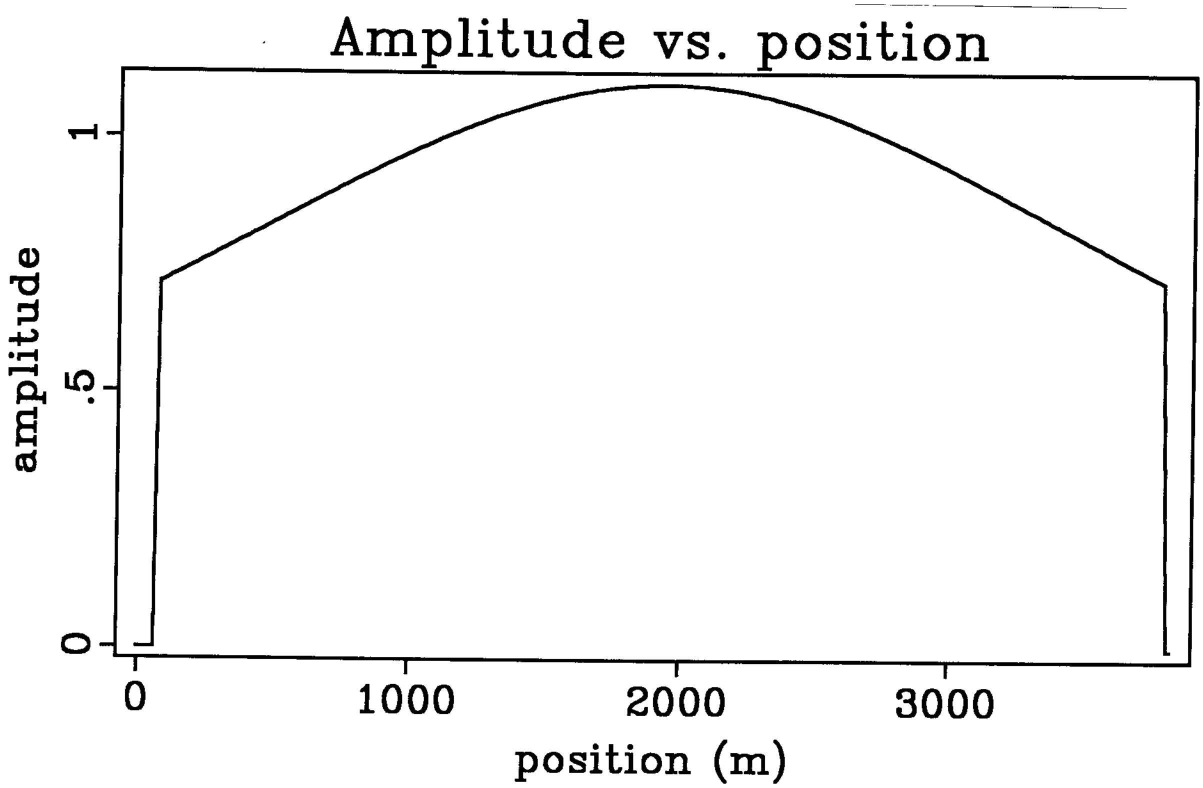

FIG. 3. Amplitude vs. position for a depth of 2 520 meters in the first model. Starting location for the rays was 1 950 meters.

source/receiver would be great. If the lateral velocity variation does not contain high spatial-frequencies, then further interpolation is possible. Traveltime and amplitude maps are generated by ray tracing for only a small set of reference surface locations. A traveltime map for a surface location that was not ray-traced can be obtained by interpolation of two surrounding reference traveltime maps. To obtain traveltimes from a given surface point that was not ray-traced to a given offset from that point, the traveltimes at that same offset on two surrounding reference traveltime maps are interpolated. This stage of interpolation is done from the set of reference traveltime and amplitude maps during the Kirchhoff summation step of the migration. The same approach is used for the amplitude maps. Presently only linear interpolation of those known, reference traveltime and amplitude values, is used. For the examples shown, this appears to be adequate.

The main assumption made, is that the traveltime and amplitude maps are single-valued functions of position. This does not totally prevent handling caustics, but only one branch of the traveltime curve can be taken in the implementation I have now. It is possible to overcome this limitation by allowing multi-valued traveltime and amplitude maps or accumulating the Kirchhoff sums during the ray tracing as in (Gray, 1986). This was not done because in practice the lateral velocity variation is usually not known well enough to justify using multivalued Green's functions.

Velocity field restrictions

Laterally smooth velocity functions are always used to do ray tracing and interpolation. If velocity functions containing high lateral wavenumbers were allowed, the interpolations, both between rays, and between different traveltime maps, would be far more difficult, and even the ray tracing itself would be poorly behaved. Sharp jumps in the velocity model would result in many crossed rays and many small caustics, which would be difficult to handle properly. The main justification of smoothing the velocity field is that rarely do we know the high spatial-frequency components of the real velocity field even if they do exist. Almost surely, the velocity of the Earth does have rapid lateral variations on small scales, but they are rarely seen reliably on surface seismic data. Even if caustics were observed, the high spatial-frequency components of the velocity field would still most likely remain unknown. Using an inaccurate velocity function that contains high lateral wavenumbers will not add to the clarity of the migrated image and will make velocity analysis more difficult. It may be best to migrate using a smooth velocity field that will still contain most of the traveltime and amplitude information necessary for a good migration. Later it might be possible to extract information on the high spatial-frequency components once a better estimate of the structure is available.

There are some examples of velocity fields with known significant content at high lateral wavenumbers, such as an area with a rapidly variable sea-bottom. In these cases, there may be justification in handling caustics more carefully as well as doing more careful ray tracing.

EXAMPLES

A first test of the method involved a model shot profile taken over a flat reflector. The velocity was vertically constant with a lateral velocity gradient above the reflector. The data was obtained with a finite-difference modeling program. Figure 4 is the modeled shot profile. The profile consisted of 256 traces at a 15 meter trace spacing; the shot location was receiver number 128. Figure 5 is the result of migrating the profile with a laterally invariant velocity taken from beneath the shot location. Notice that the imaged reflector is dipping while in the original model the reflector was flat. Figure 6 is the result of migrating the profile using the fast Green's function calculations for laterally varying media. The reflector is correctly imaged; it is flat. The traveltime and amplitude maps for the migration were interpolated from a set of eight reference traveltime and amplitude maps. The spacing between reference maps was 30 receiver locations.

To further test the method, a full synthetic survey was generated using a finite-difference modeling program. The original model is 256 points in z by 384 points in x. The spatial sampling interval is 20 meters in both x and z. Figure 7 is a filtered version of the velocity model to show the reflectors. Figure 8 shows a vertical slice through the velocity model at 3 840 meters that shows how the velocity changes at each reflector. Twenty-five shot profiles were modeled using a finite-difference modeling program. The shot interval was 10 receiver locations. The first shot location was at 1 200 meters. Each profile consisted of 121 traces, shot split-spread. The time-length of each trace is 2.7 seconds. Figure 10 is a sample shot profile the shot point was at 3 200 meters. Before the Green's function calculations, the velocity model was smoothed both laterally and vertically with a 45 point smoothing filter on the interval slownesses in each direction. Figure 9 is a vertical slice through the smoothed velocity model at the same location as Figure 8. To compute the Green's functions for all the surface points in the survey ten reference surface locations were ray-traced. The spacing between the ray-traced reference locations was thirty receiver locations. Approximately 5 reference locations fall within the spread of a given shot profile. The ray tracing and

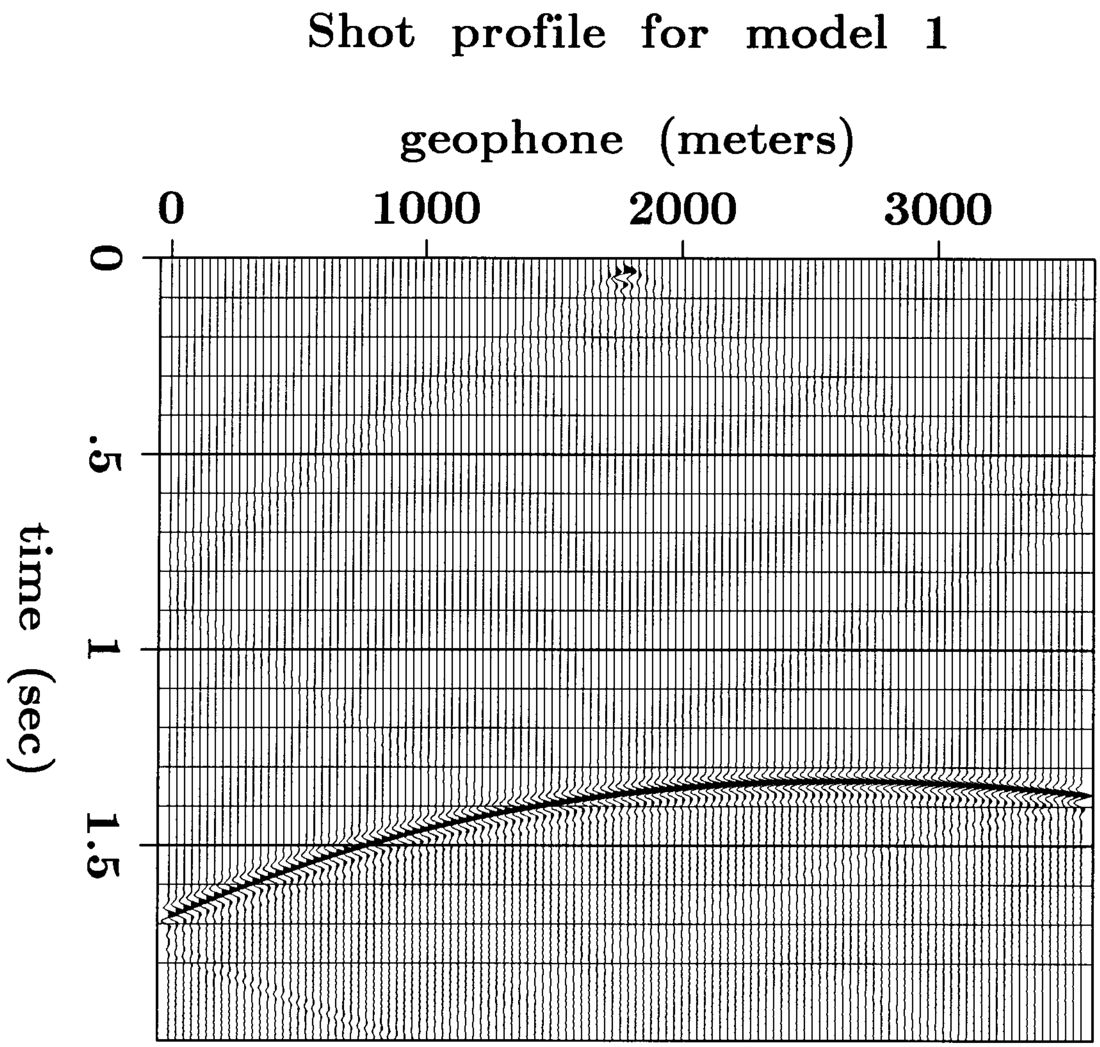

FIG. 4. Model shot profile for model 1 which has a flat reflector beneath a lateral velocity gradient.

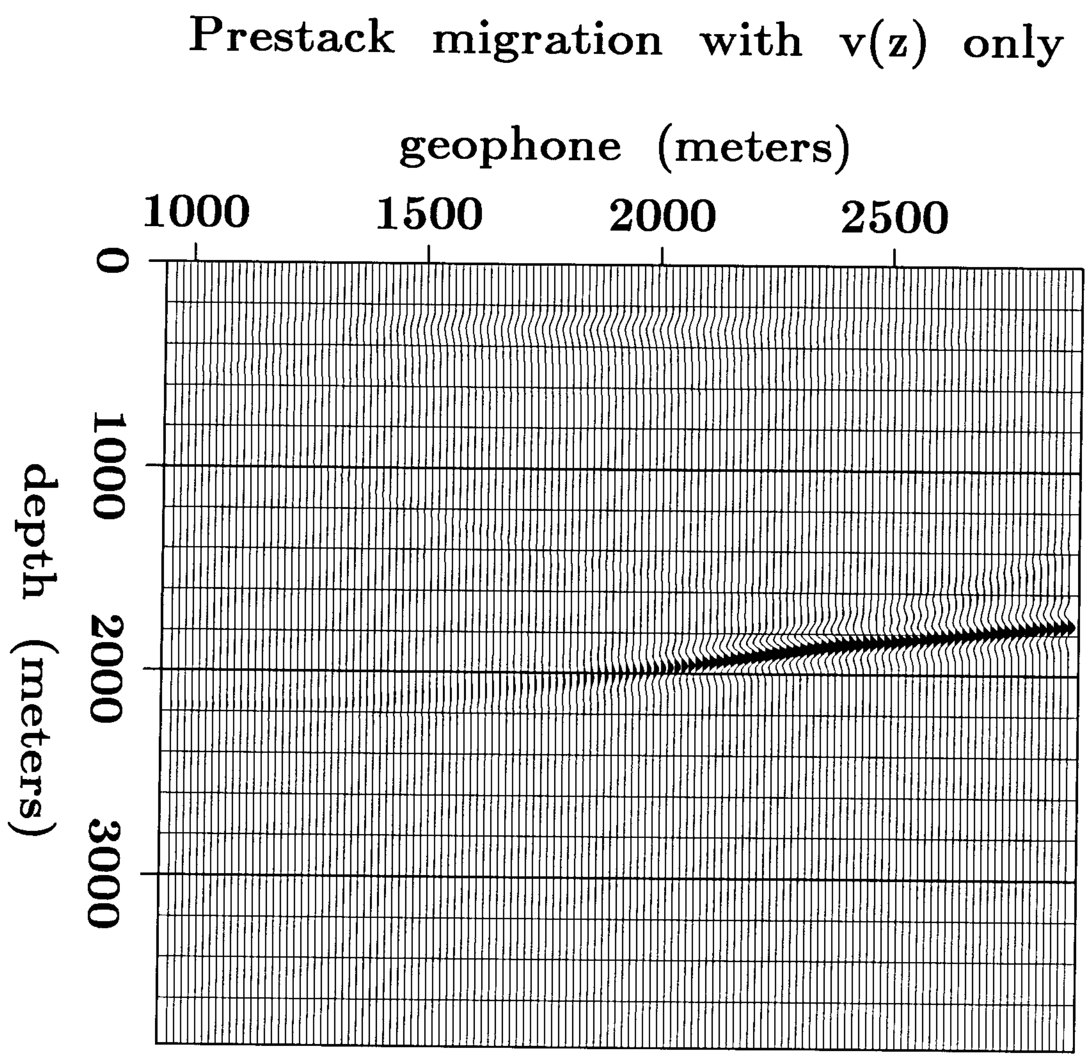

FIG. 5. Prestack migration of the profile of Fig 4 using $v(z)$ only. The reflector is not correctly imaged.

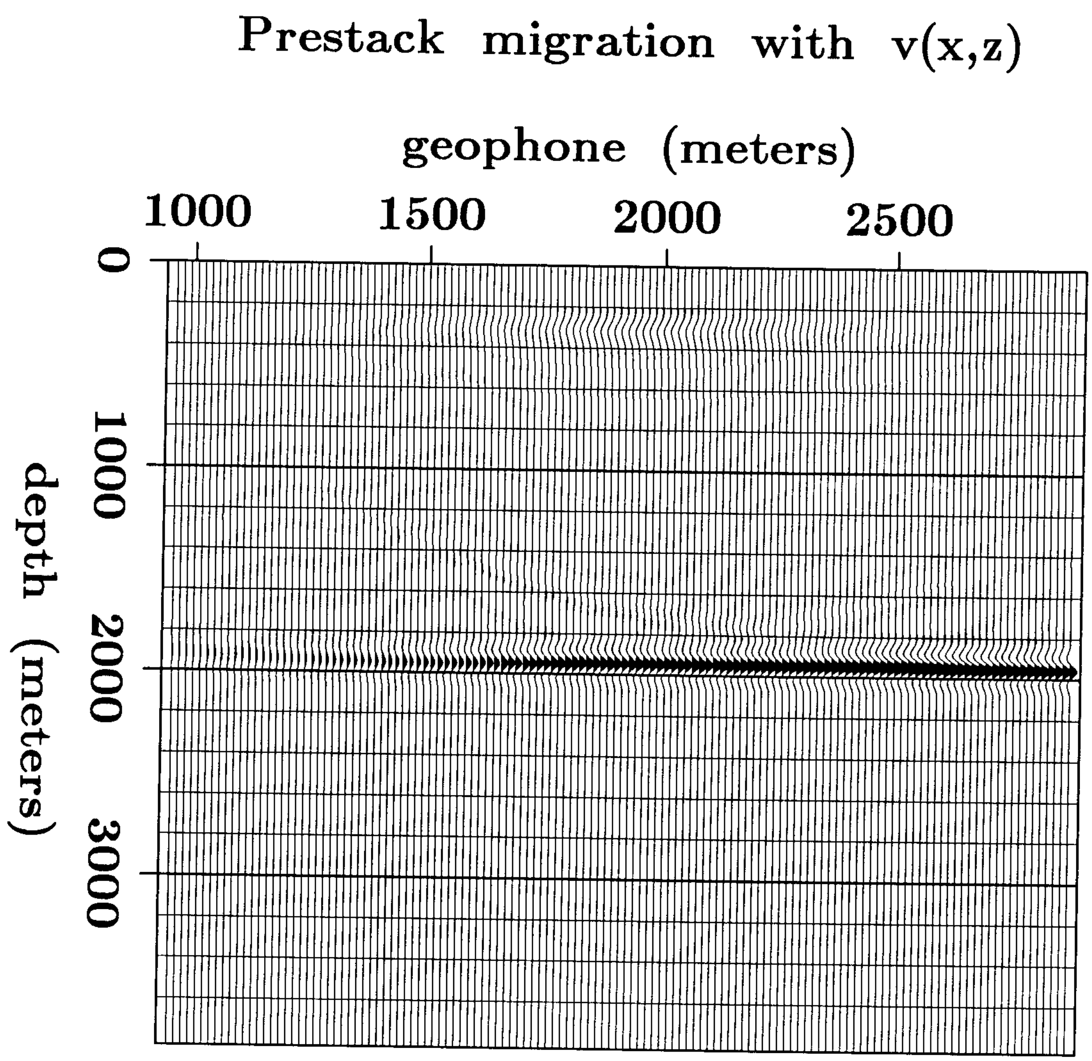

FIG. 6. Prestack depth migration of the profile in Fig 4. The lateral velocity variation is handled correctly by the interpolation method presented. The reflector is correctly imaged.

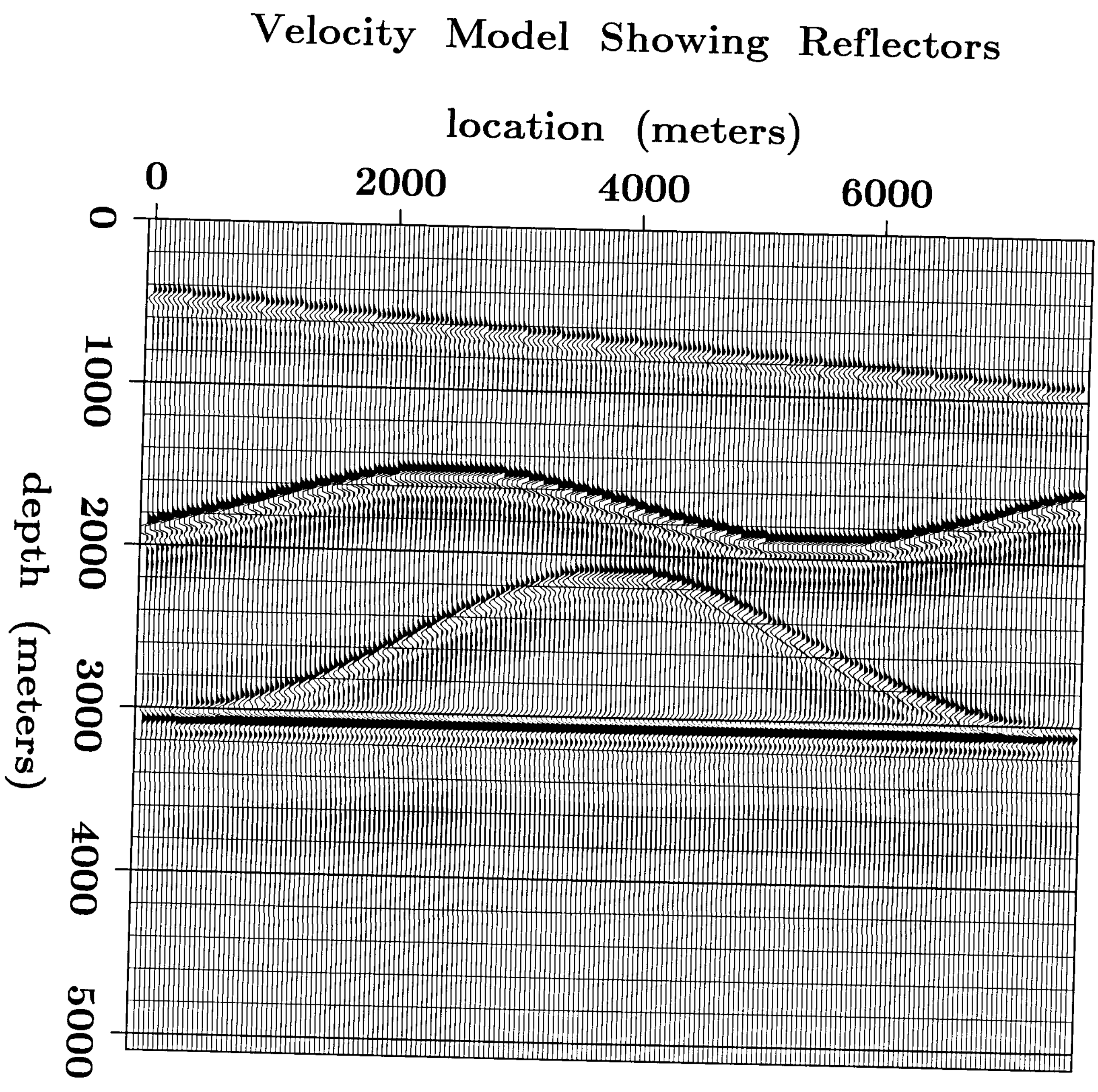

FIG. 7. Filtered velocity model showing location of reflectors in second model.

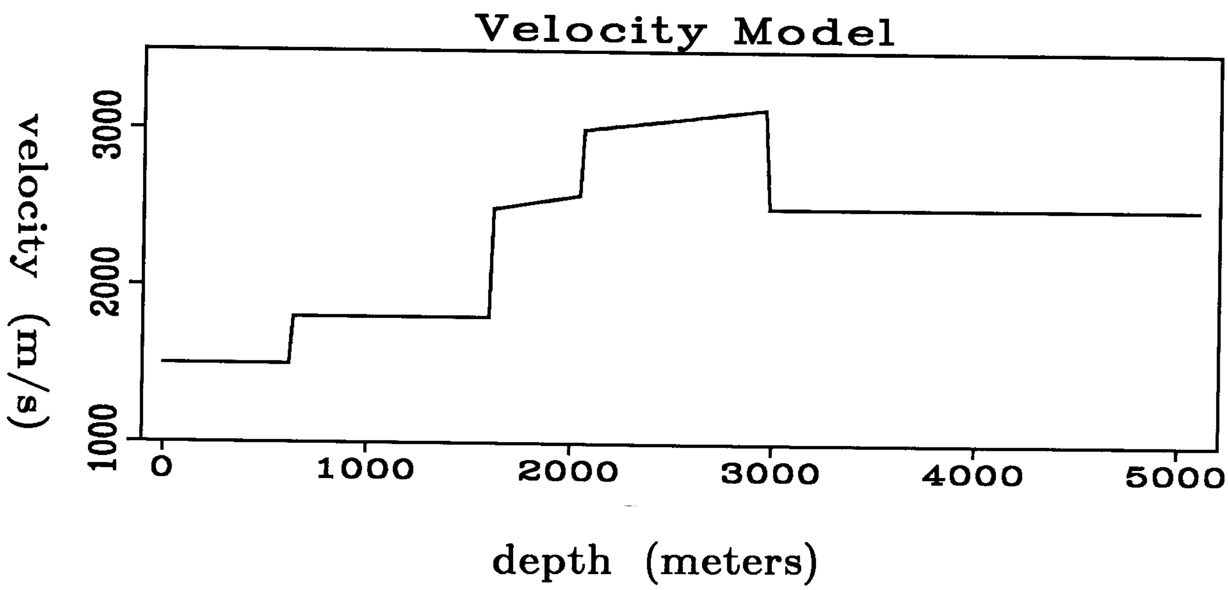

FIG. 8. Vertical slice through second velocity model at 3 840 meters showing velocity jumps at reflectors.

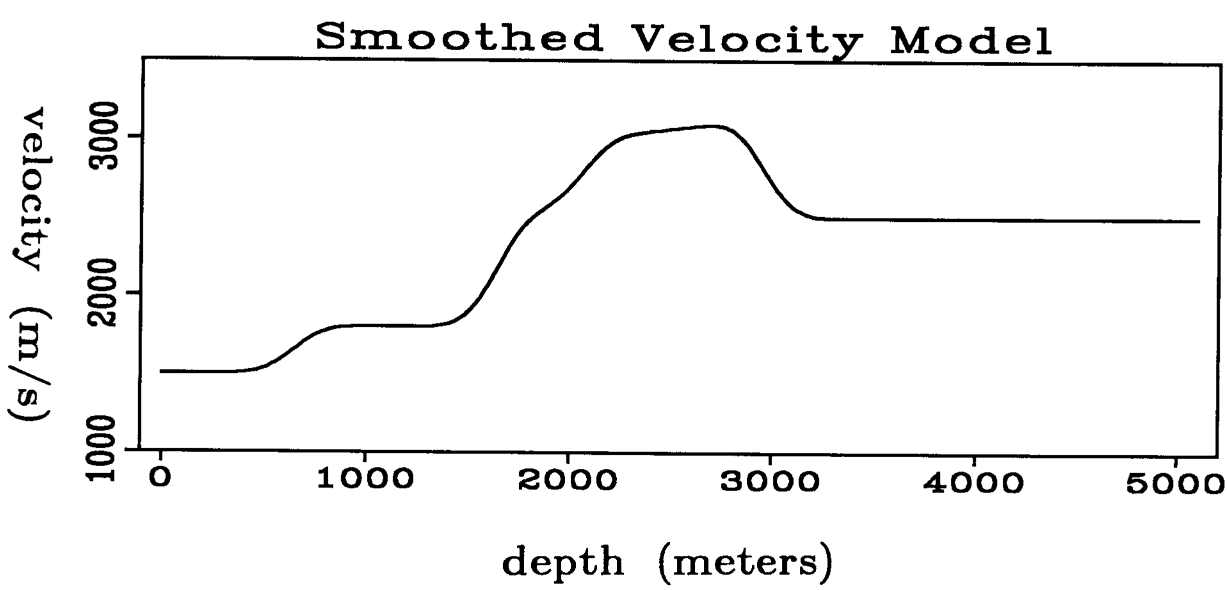

FIG. 9. Vertical slice through smoothed version of second velocity model at 3 840 meters.

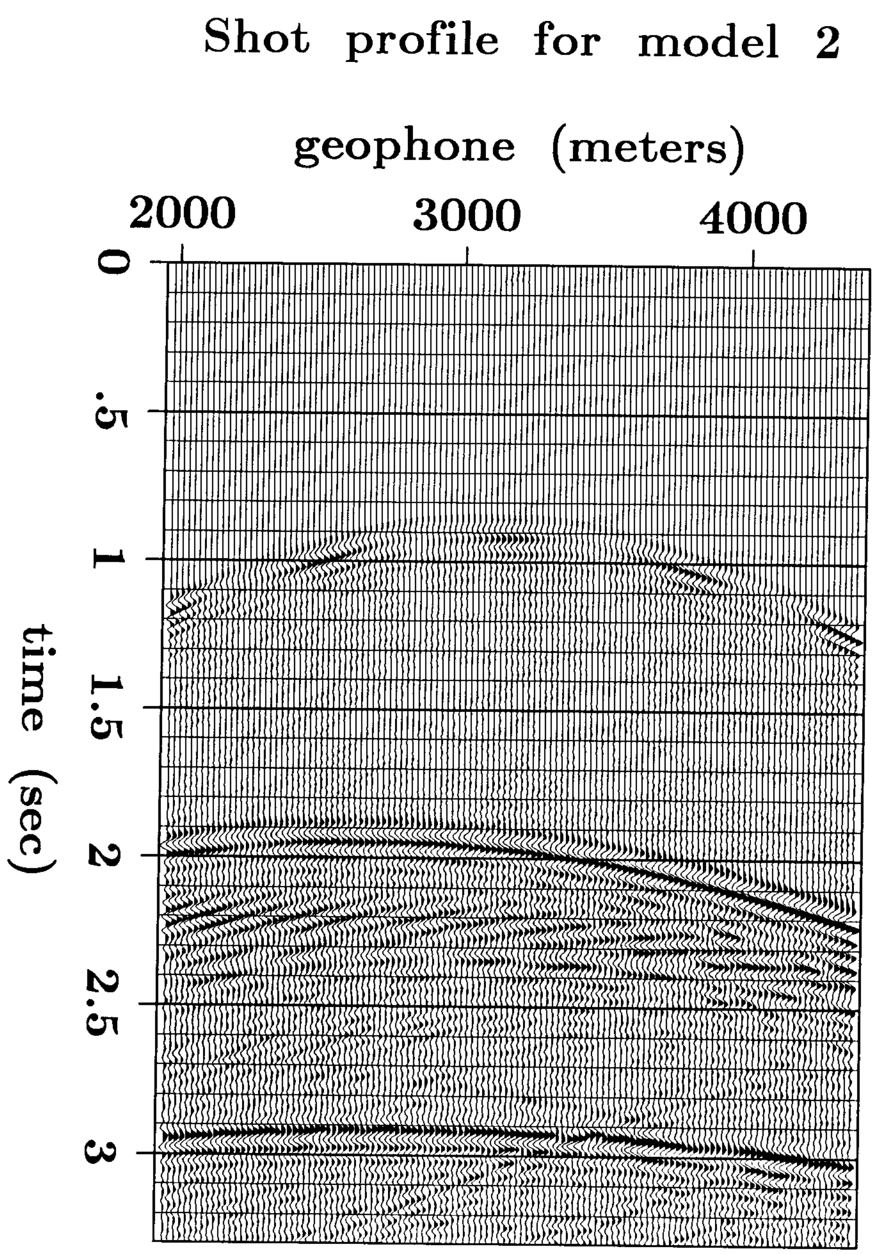

FIG. 10. Sample model shot profile from second velocity model. The shot point was at 3 200 meters.

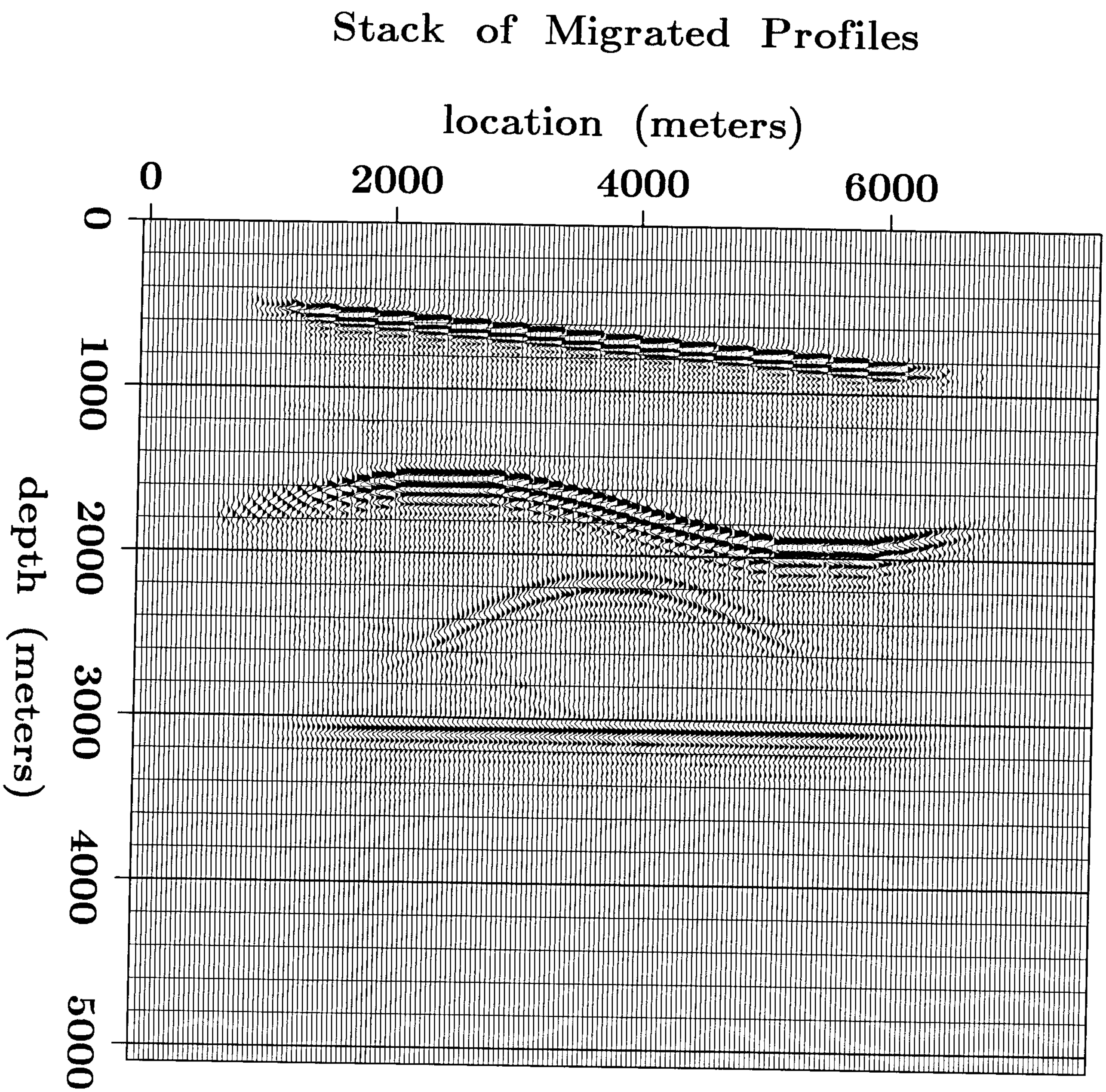

FIG. 11. Prestack depth migrated and stacked section of 25 model profiles.

interpolation calculations took 98 cpu-secs to make the reference traveltime and amplitude maps.

After computing the reference Green's functions, each shot profile was migrated, and all the migrated profiles were stacked to form an image. Figure 11 is the resulting stacked section after prestack depth migration. Comparing the stacked section to the input model, note that all reflectors are positioned correctly, especially the lowest one which should be flat. Therefore, the lateral velocity variation was correctly handled by the efficient Green's function calculations. Also, the smoothing applied to the interval velocity model apparently had little effect on the quality of the migration; all the reflectors are positioned in depth correctly. As mentioned before, the Green's function calculations took 98 cpu-secs; the migrations themselves took 30 cpu-secs per profile, or 12.5 minutes total. By comparison, the finite-difference modeling required about 7 minutes for each profile. Therefore, migration by reverse-time finite-difference propagation would require almost 10 minutes per profile. In this simple case, the Kirchhoff method is greater than 25 times faster than finite-difference reverse-time prestack migration. In fact, because of the high cost, I did not attempt to migrate the data with the finite-difference method.

CONCLUSIONS

Prestack depth migration can be accomplished accurately and efficiently using a Kirchhoff integral method. Expensive two-point ray tracing is not necessary to find the traveltimes and amplitudes needed for the migration. From a given surface point only a sparse ray fan can be used to generate traveltimes and amplitudes to the entire model. Moreover, only a small subset of the surface points is needed to make traveltimes and amplitudes for all surface points. Both the ray tracing itself, and the Kirchhoff summation are vectorized.

Because velocity fields are usually only approximately known, smooth representations for the interval velocities are appropriate in most cases. Prestack depth migration also appears to be less sensitive to the high wavenumber components of the true interval velocity than the low wavenumber components. These considerations allow the interpolation methods used to be very accurate in many cases. Because Kirchhoff migration methods allow the output space to be arbitrarily sampled, the method presented might be ideal for velocity analysis.

ACKNOWLEDGMENTS

I would like to thank Norman Bleistein for suggesting Kirchhoff integral methods in my pursuit of a fast depth migration scheme. I would also like to thank Paul Fowler for many interesting discussions on migration, depth migration and velocity fields.

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SERIES FOR THREE-DIMENSIONAL MIGRATION IN REFLECTION SEISMIC INTERPRETATION *

BY

JOHANN SATTLEGGER**

ABSTRACT

The problem of three-dimensional migration is solved for arbitrary, three-dimensional, continuously differentiable velocity functions $v = v(x_i)$ by means of series.

The derived formulae are applied to numerical examples.

The possibility of approximating real, discontinuous velocity distributions by continuous functions is mentioned and illustrated by an example.

The application of electronic computers, which is necessary due to the complexity of the formulas, is discussed.

ZUSAMMENFASSUNG

Das Problem der dreidimensionalen Migration wird mit Hilfe von Reihen für beliebige, dreidimensionale, stetig differenzierbare Geschwindigkeitsfunktionen $v = v(x_i)$ gelöst.

Die abgeleiteten Formeln werden auf numerische Beispiele angewendet.

Möglichkeiten zur Annäherung wahrer, unstetiger Geschwindigkeitsverteilungen mit Hilfe stetiger Funktionen werden angeführt und es wird ein Beispiel dafür gezeigt.

Die Verwendung einer elektronischen Rechenanlage, die wegen der Kompliziertheit der angegebenen Formeln notwendig ist, wird diskutiert.

1. NOTATIONS

For the following considerations, we use a right-hand coordinate system with the 1-axes pointing to the North, the 2-axes pointing to the East and the 3-axes pointing downwards, the 1-2-plane coinciding with the datum plane.

Vectors are designated by the indices i, j, k or l, that represent the three coordinates, e.g. $x_i = (x_1, x_2, x_3)$, or two coordinates in the two-dimensional case (also included in the considerations).

v is the velocity, which may be any arbitrary function of the coordinates, that is at least three times continuously differentiable; $v = v(x_i)$.

In addition, Einstein's so-called "summation convention" which is particularly suited for the analysis in this paper, has been adopted. The means is that, wherever there appears twice the same index in a product, it has to be summed over the three (or two) coordinate indices, e.g.:

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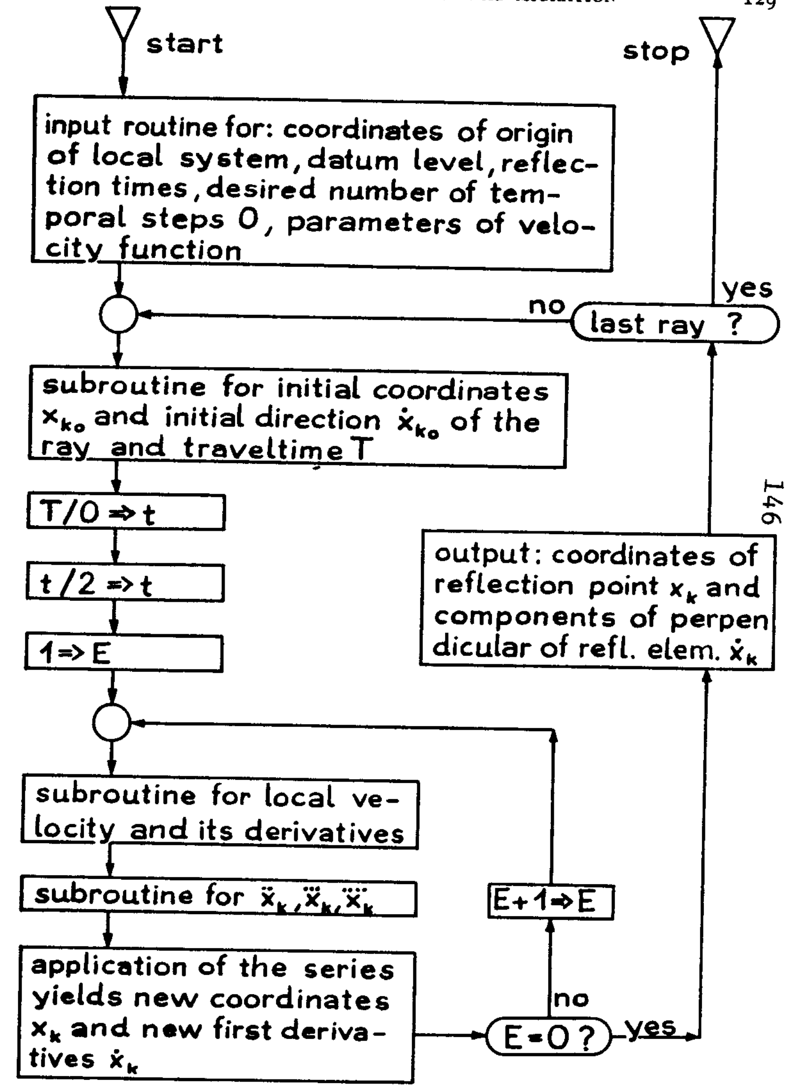

Fig. 3. Simplified computer flow diagram