Velocity analysis by iterative prestack depth migration: a proposal

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ABSTRACT

In areas of complex structure, velocity analysis and imaging are interrelated. Knowledge of the interval velocities is required to image the subsurface properly. However, the interval velocities are not usually known in advance. An estimate of the subsurface structure can improve the estimate of the interval velocities and lead to a better image of the geologic structure. This suggests an iterative approach to the imaging and velocity analysis problem in complicated structure. If the problem is posed in an optimization framework, iterative prestack depth migration can provide an interval velocity model that best images the subsurface. I propose an algorithm to do this. Prestack migration (like CMP stacking), is more affected by the low wavenumbers of the interval velocity model than high wavenumbers. Low wavenumber components of the interval velocity field control gross timing and positioning of events. When forming an image of the subsurface, it is most important to estimate the lowest wavenumber components of the interval velocity first. This will give a properly focused and positioned image. With this in mind, I propose approximating interval velocity models with a set of smooth basis functions. This results in far fewer parameters to be optimized than if the velocity were allowed to be independent at many thousands of points. Most importantly, the interval velocity model will be in terms of more easily resolved parameters.

INTRODUCTION

Prestack depth migration is a powerful tool for imaging the Earth's interior for both two and three-dimensional data. The lack of sufficient information about the interval velocities, needed to get worthwhile results from prestack depth migration processing, is a significant drawback. It may be difficult, therefore, to justify the use of prestack depth migration schemes. In areas of complicated structure, the problem is difficult; the velocity is needed to get the structure, and the structure is needed to get useful velocity estimates. An iterative method might be the solution to this problem. As the structure is unscrambled, better velocity estimates are available, boot-strapping our way out of the problem. The goal of this approach is two-fold; obtain an estimate of the interval velocities of the subsurface, and accurately image the subsurface. Putting iterative prestack depth migrations in an optimization framework is one way to accomplish these goals. Although any given migration may have been done at the wrong velocity, changing the parameters using an optimization method can give a better estimate of both the structure and the velocity.

DISCUSSION

Velocity analysis based on prestack depth migration relies upon the same principle as velocity analysis based on NMO. When the correct velocity model is used to image the data, there will be agreement among the images obtained from different offsets. Using NMO, events on a common midpoint gather will be "flat". Using prestack depth migration, the images of events on several independent, migrated shot gathers will be coherent. The events on a gather at one ground location (a true common depth point) from different migrated shot profiles will be "flat" (Al-Yahya, 1984). When the different offsets on an NMO corrected midpoint gather or images of different migrated shots agree, the stack of those different offsets or images will constructively interfere, increasing the stacked amplitude (or power) in the image. At any incorrect velocity, the images will not correspond to each other to some degree; and destructive interference will reduce the amplitude in the resulting stacked image. Semblance or stack power is a possible objective function to measure the quality of a prestack depth migration, and thus the accuracy of the velocity model.

The velocity analysis that I propose, unlike velocity analysis based on NMO will be driven by an interval velocity model rather than the root-mean-square velocity commonly used by NMO. All references to velocity are to the interval velocity of the medium. The prestack migrations used to evaluate the velocity models will be depth migrations, allowing structure independent velocity estimation that correctly handles lateral velocity variation.

Prestack depth migration is a costly process; iterative use of a prestack depth migration algorithm makes cost an even greater concern. An efficient and accurate prestack depth migration routine is necessary. The algorithm described in another paper in this report (Etgen, 1987) is ideal for velocity analysis. Most importantly, it is suited for fast prestack depth migration when the velocity field is not accurately known; and a smooth representation of the interval velocity is used. There are many opportunities to lessen the computational burden by subsampling the output space or not imaging portions of the data that are unimportant or uninteresting.

Parametrization of interval velocity models

For prestack depth migration, the interval velocities of the subsurface are required to form an image of the subsurface structure. These velocities may be difficult to specify in an efficient and useful manner. In an iterative velocity analysis and imaging procedure, if the velocity were allowed to be independent at every point in a discrete model of the subsurface, there would be many thousands of velocity parameters to estimate. Moreover, changing only one parameter or a small set of nearby parameters would not produce a noticeable effect on the migrated image. This would cause difficulty with most optimization techniques because the objective function would be insensitive to changes of the parameters.

A solution to this problem is to specify the velocity model using a set of basis functions. For example, a velocity function consisting of a linear ramp can be obtained from two parameters (the velocities at the top and bottom of the model). In one dimension, piecewise

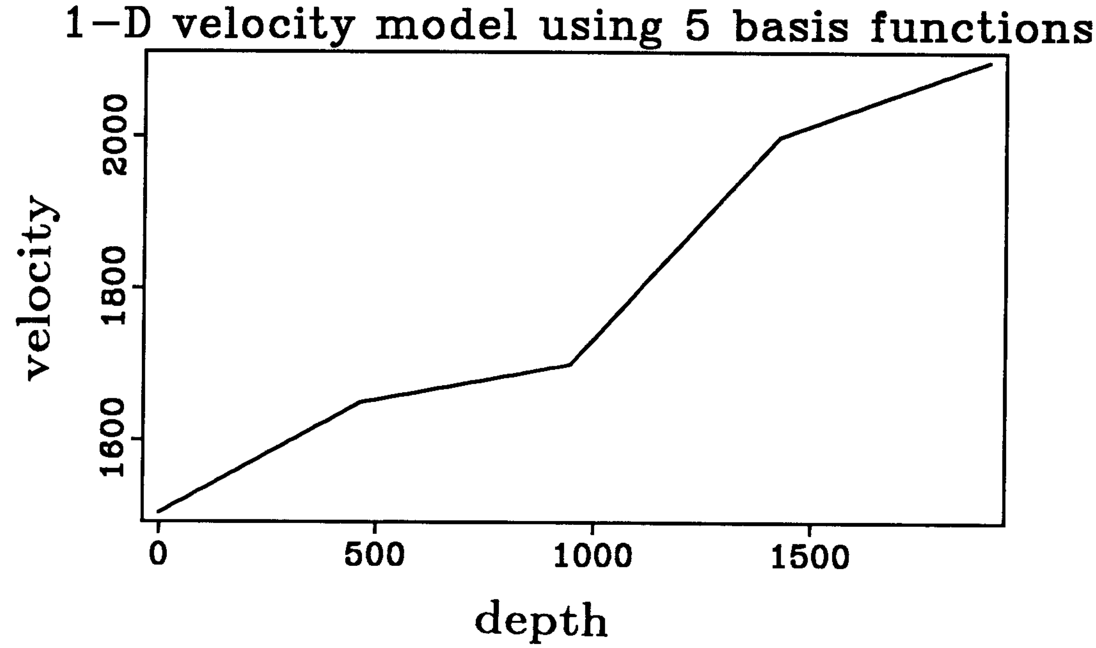

FIG. 1. A velocity model in one dimension using piecewise linear basis functions. 5 velocity basis functions are used.

linear, continuous basis functions, or cubic splines allow many unique velocity fields to be specified using a minimum number of parameters. This extends readily to two and three dimensions. Using basis functions also expedites the optimization in two ways. First, there are many fewer parameters to optimize over. Second, by changing one parameter or a few parameters, the resulting effect on the migrated image is more readily observed. This has the advantage of making resolution of poorly determined parts of the model less difficult. Lumping many poorly determined parameters together in a small number of basis functions will result in parameters that are easier to resolve.

Figure 1 shows a one-dimensional velocity model comprised of a set of piecewise linear basis functions. Figure 2 shows a two-dimensional velocity model comprised of a set of two-dimensional plates. These plates result from interpolating linearly in both x and z from a sparse grid of points. In the example, 20 points were used to generate the velocity model. In practice, piecewise continuous basis functions may not be smooth enough, cubic splines may be more effective smooth basis functions.

There is another reason for limiting our representation of the velocity model to a set of smooth basis functions. Migration can be insensitive to the effect of neglecting high wavenumber components in the velocity model. It is the lowest wavenumber components that control the overall positioning and focusing that most affect the quality of the image. This is because migration, post-stack or prestack, involves a downward continuation or a back-propagation of the wave field through a velocity model. In a rough sense, this back-propagation is affected by the velocity field in two ways. There is a traveltime and an amplitude of the direct arrival of the wave field traveling from one point to another. There are also scattering or reflection effects on the wave field. The first effect is caused by the "average" properties of the medium. The second is caused by high frequencies of the

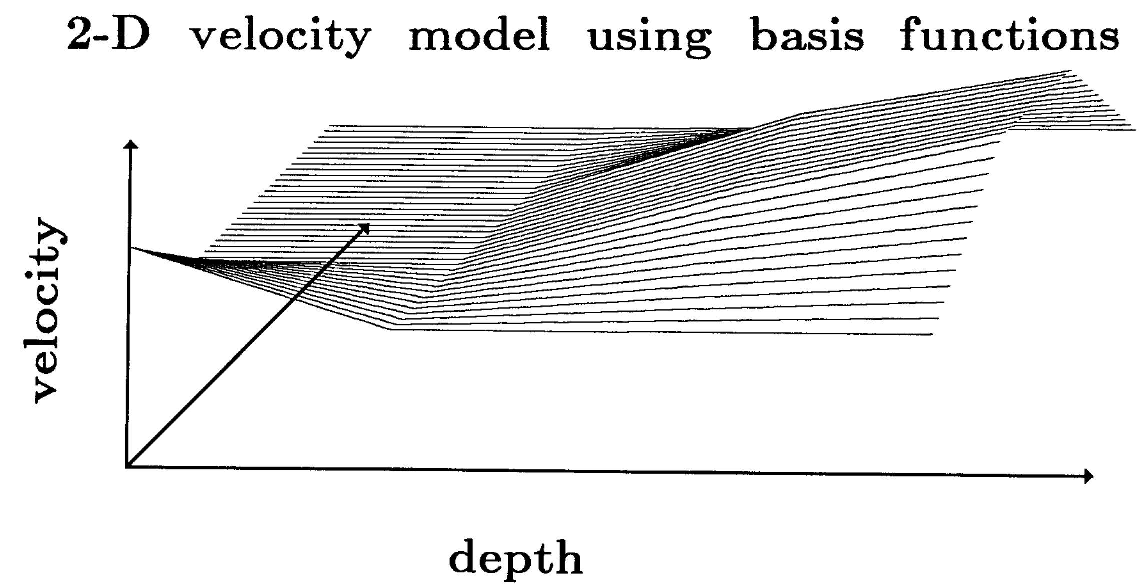

FIG. 2. A velocity model in two dimensions using piecewise linear basis functions in two directions.

medium, especially discontinuities (Stolt and Benson, 1986). Ray theory provides another way to see this point. The traveltime of energy along a ray is an integral of the interval slowness (inverse velocity) of the medium along the ray. The integral tends to smooth out the effect of rapidly varying components of the integrand, namely the slowness. A suite of prestack time migrations used for velocity analysis relies on this fact. It finds, in a sense, the best d.c. component of the velocity field, or the best constant velocity replacement medium that images the data at a given location. Prestack migration desires to propagate the recorded wave fields back to their reflection points; additional scattering caused by the velocity model does not add information to the image. A smooth velocity model is therefore desired.

From an optimization standpoint, it is natural to estimate the well determined parts of the model first, leaving those with less effect on the objective function to be estimated only after the more important components have been resolved. With prestack migration, once the lowest wavenumber components have been resolved, progressively higher wavenumber components can be estimated. It is then possible to parametrize the interval velocity using more basis functions, allowing for higher wavenumber components as the optimization proceeds. For example, the optimization may begin with velocity as a function of depth only, later allowing lateral variation after estimating the depth dependence of the velocity field.

Optimization

The proposed velocity analysis technique will require a method to maximize a nonlinear function of several variables. I have not presented a linear theory such as in Fowler's (1985) or Toldi's (1984) work; it would be difficult to find derivatives of the objective function by analytical (or approximate analytical) means for general laterally varying velocity fields. It is possible to form a gradient by approximating the needed derivatives with finite-difference calculations. This will allow use of appropriate algorithms, such as the conjugate-gradient method, for nonlinear problems. Another possible optimization algorithm known as Powell's method can be used. This method does not require gradient information, but instead builds up conjugate directions iteratively by successive line searches. I intend to test the above methods and others.

Choice of the proper objective function to maximize is also important to velocity analysis by prestack depth migration. Semblance is not the only objective function that will respond to changes in the prestack migrated image due to changes in the interval velocities used for depth migration. An entropy measure, which will detect over or under migration in the final section is another possible objective function. Even another approach to velocity analysis is outlined by van Trier in this report (Van Trier, 1987).

EXAMPLE

A simple model was used to test a preliminary version of the proposed algorithm. Figure 3 shows a velocity model, a function of depth only, that was used as input to a finite-difference modeling program. Figure 4 shows a model shot profile. For velocity analysis the velocity was parametrized into two basis functions, restricting the interval velocity model for migration to a linear ramp. Ten iterations of the iterative prestack depth migration algorithm using a Fletcher-Reeves conjugate-gradient optimization method were performed on the data. To calculate the required gradients, derivatives were approximated by finite-difference calculations. The migrations were performed using the fast prestack depth migration previously mentioned. Ten shot profiles consisting of 121 traces each were input to the prestack migration velocity analysis. To compute the objective function, ten surface locations were imaged at all depths and the total semblance was computed. The starting guess for the interval velocity was a constant 1 500 m/s at all depths. Figure 5 shows the progress of the velocity analysis algorithm during the ten iterations. The straight heavy line is the smooth estimate of the interval velocities after ten iterations. The other heavy line shows the true interval velocities used to create the data. Figure 6 shows the migrated section obtained using the result of the tenth iteration of the velocity analysis algorithm. For comparison, the migrated section using the starting guess velocity, 1 500 m/s is shown in Figure 7. The plot parameters are the same in Figures 6 and 7. The algorithm has chosen the linear-ramp velocity model that best migrates the data, even though this required some error to be made in migrating the events. It did not choose the migration velocity for one event to the exclusion of the other events. This is encouraging, because the objective function seems to be equally sensitive to each of the events. To improve the result, more parameters should be introduced now. The total cpu-time for the velocity analysis using ten iterations was about 2 minutes; so cost is not a major concern until many parameters are used, and many iterations run.

CONCLUSIONS

Prestack depth migration in two and three dimensions provides the necessary positioning and focusing of events to measure the correctness of a given velocity model. Interval velocities are not known in advance, so an iterative approach to imaging and velocity anal-

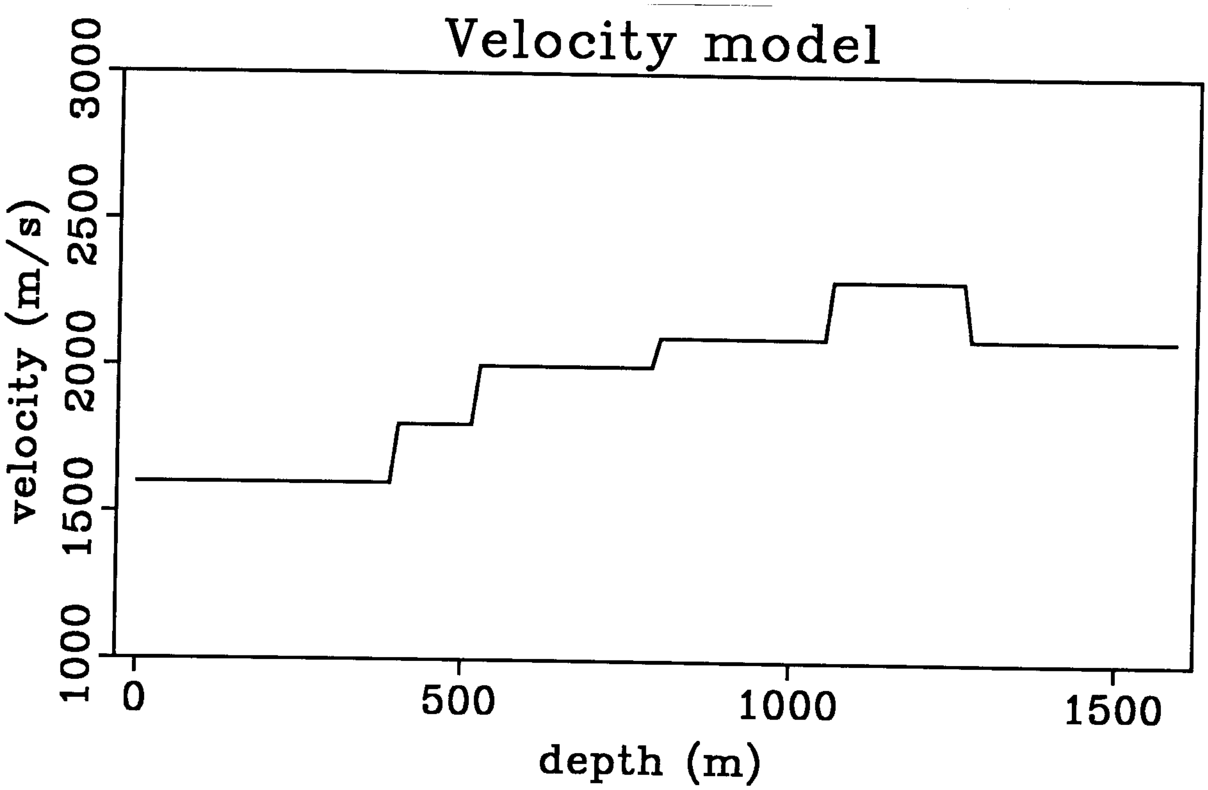

FIG. 3. Velocity model used to generate synthetic survey

ysis is proposed. A nonlinear optimization method, in combination with a prestack depth migration method can iteratively migrate the data and estimate the interval velocities of the subsurface. Because depth migrations are used, and all references to velocity involve the interval velocity, the method is capable of handling vertical as well as lateral velocity variation. The goal of the method is two-fold. First, find the optimal prestack depth migrated image of the data. Second, discover the interval velocity model that gives this optimal prestack depth migration.

Migration relies most on knowledge of the low wavenumber components of the interval velocity model, so smooth basis functions are used to parametrize the model. An example demonstrates that the method behaves correctly in a simple case. The cost of the velocity analysis is controlled by the desired accuracy of the estimate of the interval velocities. The results of any velocity analysis using only a few parameters can be used as a starting guess for velocity analysis with more parameters. The next step in my research will be to allow more basis functions, and allow lateral velocity variation. Some work will be necessary to find the most appropriate optimization method and the most appropriate set of basis functions. It remains to be seen if the method can provide meaningful interval velocity estimates for data from areas of complex structure. It is also possible to use the method in a "layer stripping" fashion. This needs to be tested as well.

ACKNOWLEDGMENTS

I would like to thank Paul Fowler for many interesting discussions on migration, prestack

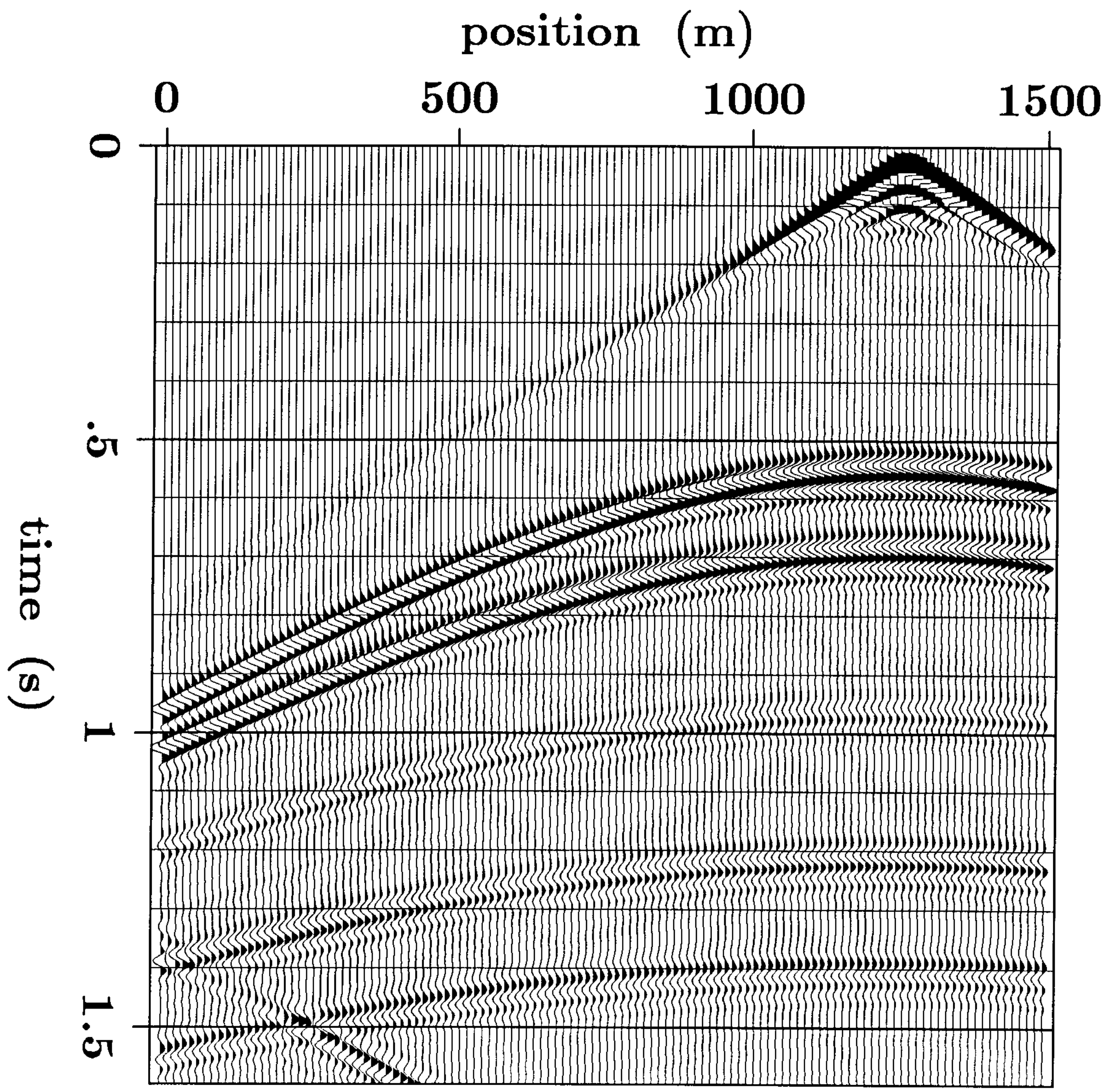

FIG. 4. Model shot profile obtained with a finite-difference modeling program. The model consists of horizontal reflectors and velocity is a function of depth only.

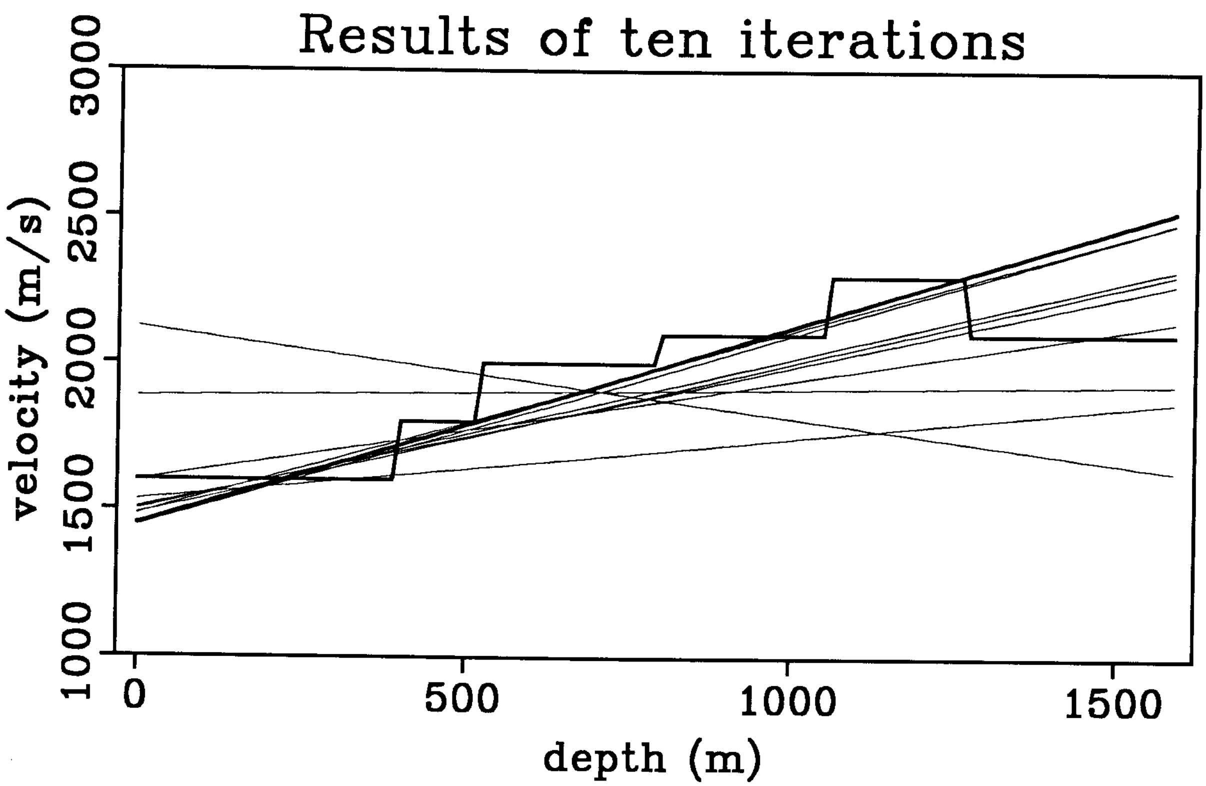

FIG. 5. The figure shows the velocity at each iteration for ten iterations. The starting guess velocity was a constant 1 500 m/s. The straight dark line is the result after ten iterations. The true velocity model is also shown with a heavy line.

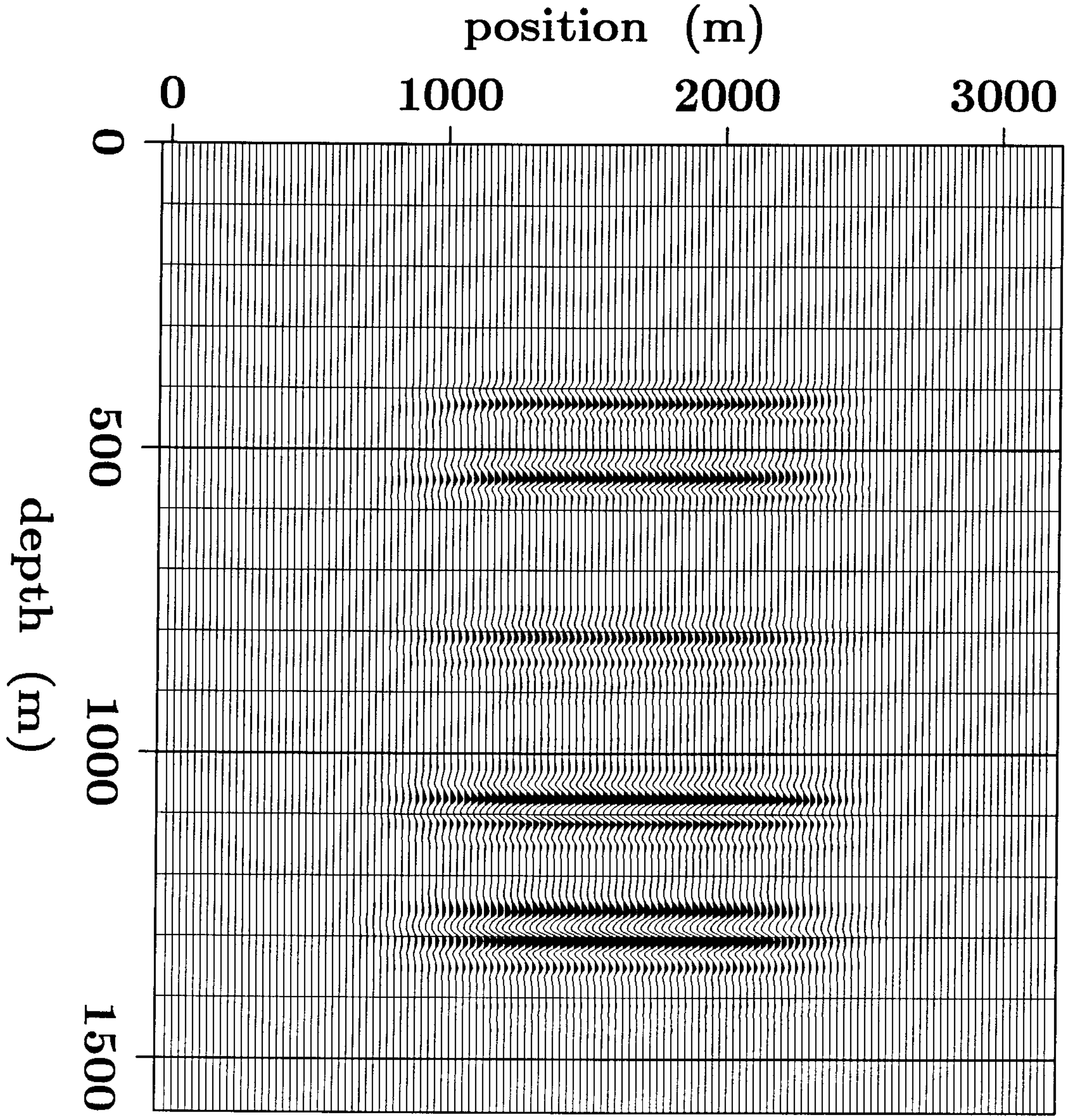

FIG. 6. Prestack depth migration of the ten profiles using the velocity obtained by the tenth iteration of the optimization algorithm.

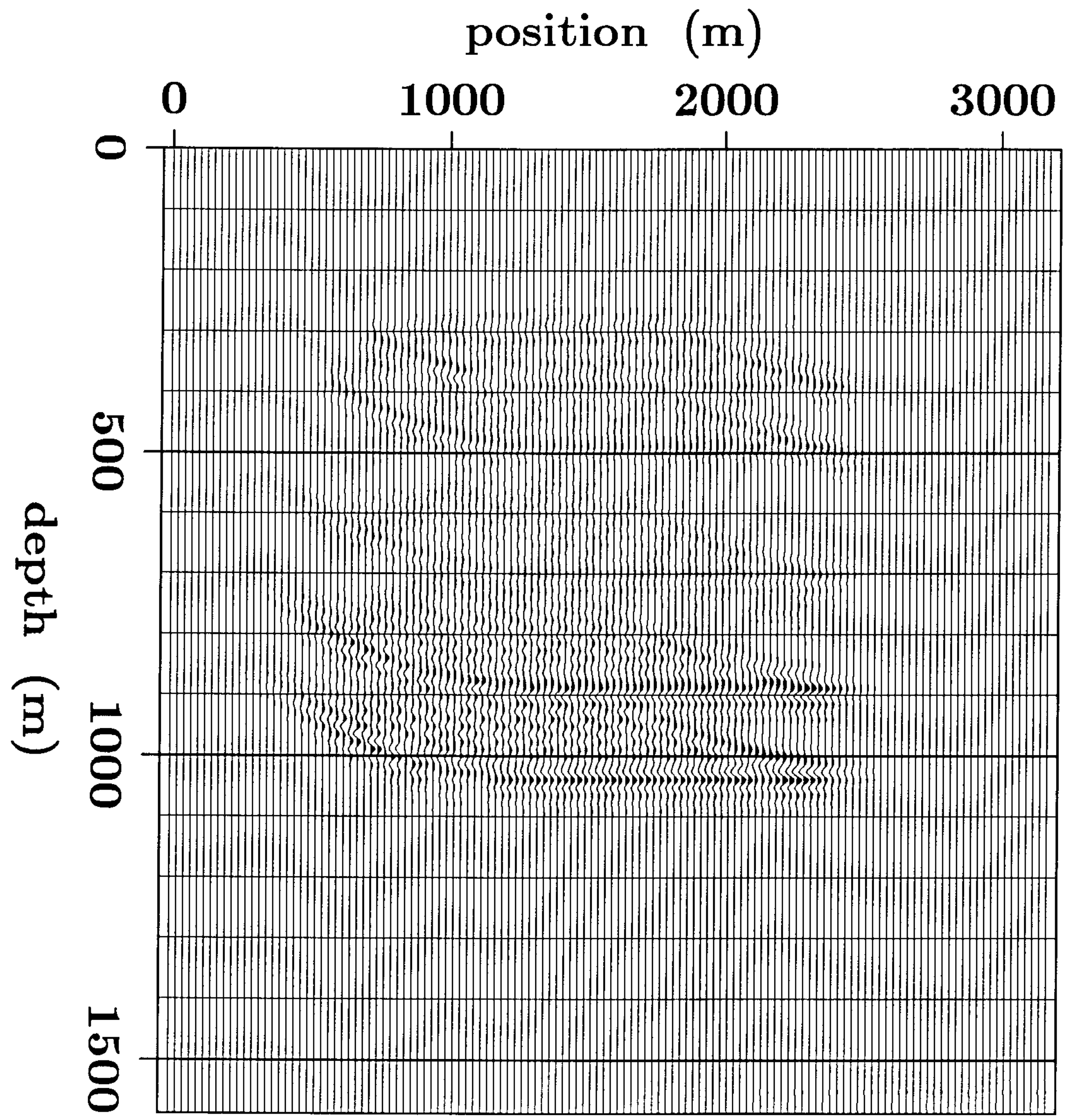

FIG. 7. Prestack depth migration of the ten profiles using the starting guess velocity.

migration, and velocity analysis using migration. I would also like to thank Jos van Trier for informative discussions on optimization techniques.

REFERENCES

Al-Yahya, K., 1984, Velocity analysis using prestack migration: SEP-38, 105-112.

Al-Yahya, K., 1984, Velocity analysis using prestack migration-applications: SEP-41, 121-148.

Bleistein, N., Cohen, J.K., and Hagin, F.G., 1987, Two and one-half dimensional Born inversion with an arbitrary reference: Geophysics, **52**, 26-36.

Claerbout, J.F., 1985, Imaging the Earth's interior: Blackwell Scientific Publications., London, 46-47.

Deregowski, S.M., 1985, Prestack depth migration by the boundary integral method: Presented at the 55th annual International SEG meeting October, 1985, in Washington D.C.

Etgen, J.T., 1987, Fast prestack depth migration using a Kirchhoff integral method: This report.

Etgen, J.T., 1986, Prestack reverse time migration of shot profiles: SEP-50, 151-169.

Fowler, P.J., 1985, Velocity analysis by optimization - linear theory: SEP-44, 1-20.

Fowler, P.J., 1986, Extending Toldi's velocity analysis algorithm to include geologic structure: SEP-48, 65-78.

Gray, S.H., 1986, Efficient traveltime calculations for Kirchhoff migration: Geophysics, **51**, 1685-1688.

Schneider, W.A., 1978, Integral formulation for migration in two and three dimensions: Geophysics, **43**, 49-76.

Stolt, R.H., and Benson, A.K., 1986, Seismic migration, theory and practice: Geophysical press, London, 16-17.

Toldi, J.L., 1985, Velocity analysis without picking: Ph.D. Thesis, SEP-43, 1-103.

Van Trier, J., 1987, Velocity analysis by nonlinear optimization of phase-contoured shot profiles: This report.

ii. Minimum spread length (about 10 to 40 meters)

A certain spread length is necessary for the identification of the *S*-waves because their vertical amplitudes are in theory very small at small angles of incidence (near the borehole)

iii. Wide band recording and the use of *low frequency geophones*

Both points have some importance, since the frequencies of the *S*-waves can be as low as 15 or 20 cps.

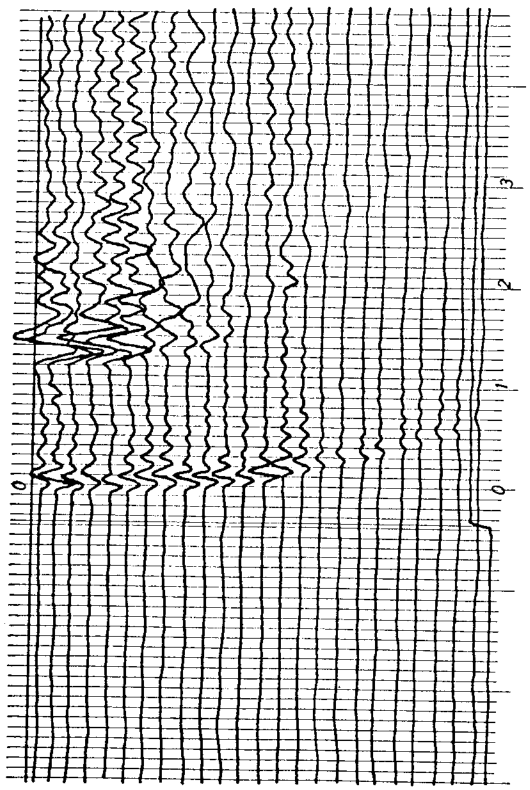

Fig. 3 Uphole-time seismogram from a depth of 45m with *P*- and *S*-waves.

Fig. 3 shows a seismogram with recorded *S*-waves. A normal blasting cap was used together with 24 refraction geophones spaced at two to ten meters intervals covering a range of about 170 meters. The seismogram was recorded without AVC and with constant gain for frequencies over 10 cps. Similar seismograms have been obtained at other shooting depths at that shotpoint. A second wave group can be determined from a depth of about 10 meters onward, exact measurement of travel times and amplitudes is possible from about 20 meters onward.

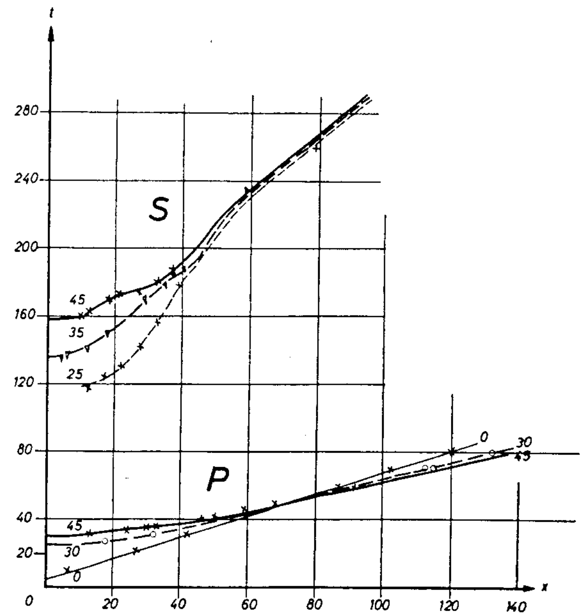

Fig. 4 Time-distance curves of *P* and *S* waves, *x-t* diagram.

The evidence for this second wave group being *S*-waves followed from travel time observations. (See Fig. 4, 5, 6.) The velocity v of the *S*-waves is about 450 m/sec in comparison to a *P*-velocity of $V=2030$ m/sec. This high velocity relation $V/v = 4.5$ leads to a Poisson's ratio of $\sigma=0.475$ and was found quite often. In this case, similar values have been obtained at distances of up to 12 km. The layers involved consisted of diluvial sand and clay.

R. Meissner, "*P- and SV-Waves from Uphole Shooting,*" *Geophysical Prospecting,* XIII, 3(1965), 433-459.