Interval velocity estimation from beam-stacked data

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ABSTRACT

Beam stacks, like local slant stacks from which they are derived, select data based on the direction of wave propagation. The goal is to estimate the low-wavenumber components of the velocity function from beam-stacked data using a tomographic approach. The velocity estimation does not require picking the data but it is solved by searching for the maximum of an objective function that measures how well the velocity model predicts the beam-stacked data.

The inversion was successful in estimating the velocity function in a horizontally layered medium both from synthetic data and from field data. The results of this simple inversion indicate directions for future research into method's application to two-dimensional velocity estimation.

INTRODUCTION

A good estimate of interval velocity is required for successful imaging of the earth subsurface by seismic migration. In the presence of lateral variations in velocity and structure in the geology, conventional velocity analysis using stacking velocities is inadequate to reconstruct the velocity model.

Various methods to improve the estimate of velocity have been proposed; most of these are based on a tomographic approach that reconstructs the velocity model from measures of traveltimes in the data. Tomographic methods can resolve only the lower-wavenumber components of the velocity model because these methods are based on measures of traveltimes, which are integral measures of velocity. The method that I propose in this paper is based on a tomographic reconstruction of the velocity model and has the same limitations. In practice this limitation is irrelevant since, in order to properly image the reflectors, migration needs a smoothed version of the true velocity function.

Conventional tomographic inversion (Bishop et al., 1985) is performed in the following two steps: first, picking traveltimes and reflecting horizons; and second, searching for the velocity model that minimizes the mismatch between the observed traveltimes and the traveltimes predicted by the model. Sword (1986) presented a tomographic inversion that used events picked from locally slant-stacked data. He substituted the picking of ray parameters in the local slant-stacks' domain for the picking of horizons, with the important advantage that the picking procedure can be then executed automatically.

The velocity estimation that I propose is based on beam-stack transformation of data. The method can be related to Sword's inversion, but it uses beam stacks instead of local slant stacks and, more importantly, does not require the picking of events. I prefer to avoid picking because when the data are contaminated by noise, picking becomes difficult, and the picking errors might not follow the Gaussian distribution, on which the least-squares inversion to the velocity model is based (Rothman, 1984). On the other hand, using all the transformed data for the inversion, without picking events, increases the CPU-time and storage requirements of the algorithm.

Instead of minimizing the mismatch between the picked events and the events predicted by the velocity model, I try to maximize the amplitude of beam-stacked data at the traveltimes predicted by the velocity model. In this respect my inversion method resembles more the methods presented by Toldi (1985) and Fowler (1985), who maximize the stacking power as a function of stacking velocity and migration velocity. Beam stacks depend on the data more locally than do stacking velocity and constant-velocity migration panels, and therefore beams stacks are more appropriate for the localization of velocity anomalies.

BEAM STACKS

The proposed velocity estimation is carried out in the beam-stacks' domain. Beam stacks are a variation of local slant stacks (Harlan and Burridge, 1983; Sword, 1984) and are presented in another paper in this report (Kostov and Biondi, 1987). Beam stacks differ from local slant stacks because the stack-trajectory is hyperbolic instead of being a straight line.

For velocity analysis, beam stacks have the useful feature of selecting those waves generated and recorded at assigned surface locations and propagating in given directions. A beam is determined by the shot location and the receiver location together with the shot ray parameter and the receiver ray parameter. For each beam the value of beam stacks as a function of traveltime depends on the velocity model along the beam path; this value also depends on the geometry of the reflectors. By considering several beams traveling within a region one can reconstruct the velocity model from the information provided by beam stacks.

To explain how the beam-stacked data are related to the velocity function I consider a shot profile recorded on a horizontally stratified Earth. In this case the shot ray parameter is equal to the receiver ray parameter and the beam stacks of a shot profile are defined as a function of traveltime, the ray parameter, and the recording location.

Figure 1 shows a shot profile from a marine survey in the Gulf of Mexico. In the area the geology is flat and lateral variations in velocity are mild; thus the assumption of horizontally layered medium is approximately satisfied.

Figure 2 shows a beam-stack decomposition of the profile shown in Figure 1, for a fixed ray parameter. Ideally, beam stacks should select only the reflected energy arriving at the

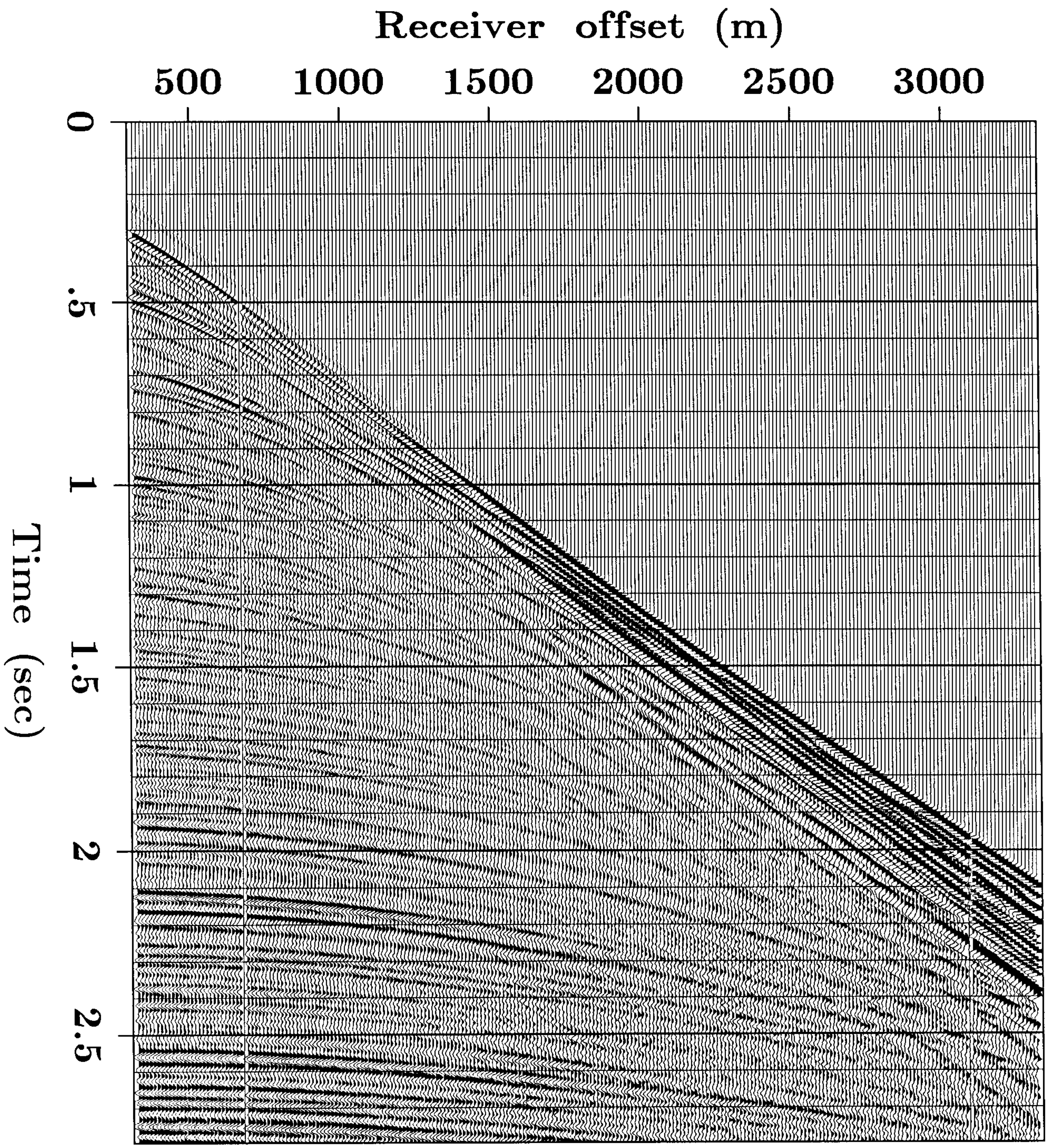

FIG. 1. Shot profile recorded in the Gulf of Mexico.

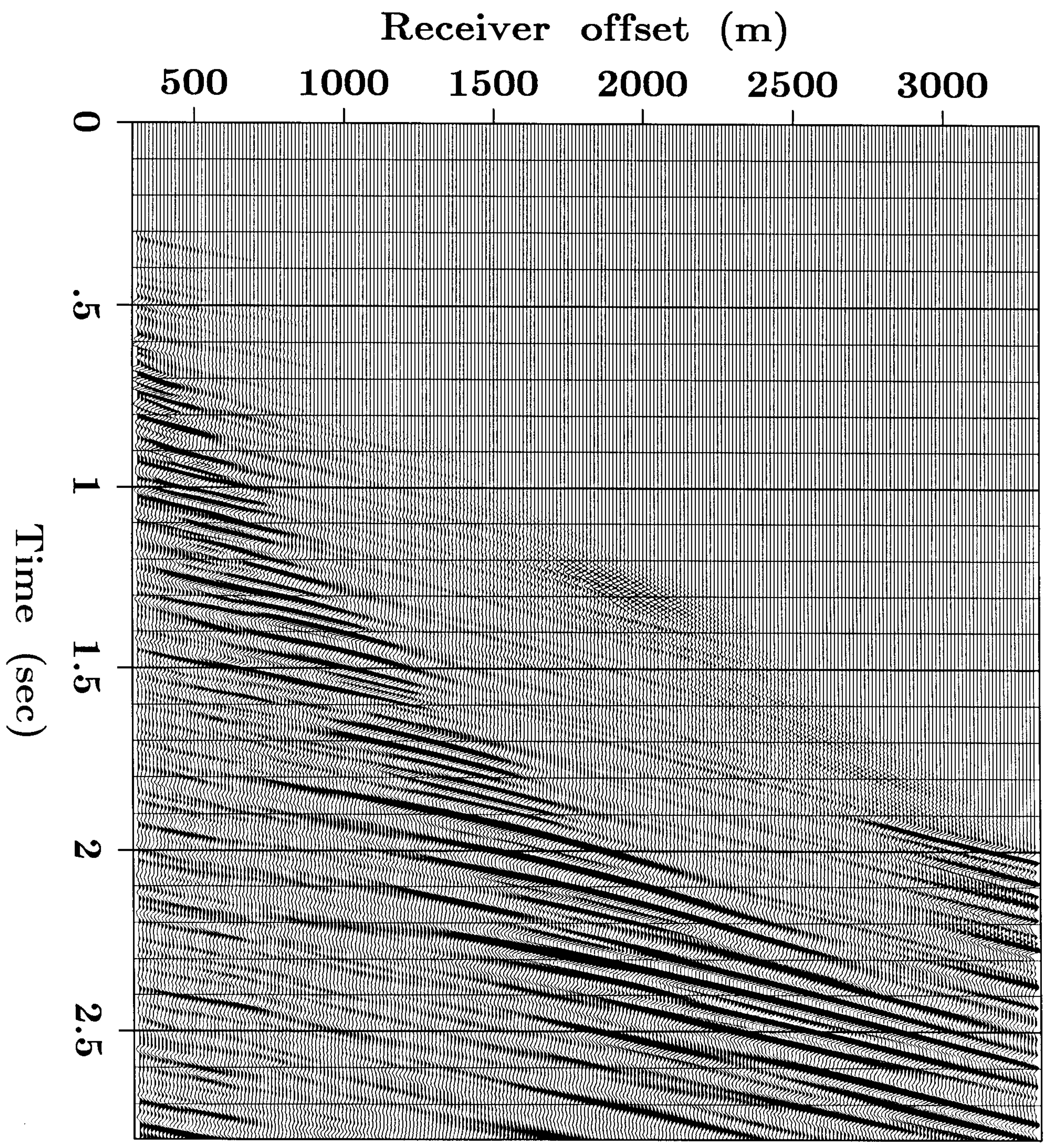

FIG. 2. Beam stacks of the profile shown in Figure 1, for a fixed ray parameter. The energy is localized in a wedge-shaped area around the peaks; this area corresponds to the reflections traveling with the specified ray parameter. The traveltime and the receiver-offset of the peaks is dependent on the velocity model.

surface with the specified ray parameter and the result of the stacks should be a sequence of peaks in correspondence to each reflection. In practice, the resolution of beam stacks is finite and the peaks are smeared along the Fresnel zone of the reflections; the smear increases with traveltime because the Fresnel zone becomes wider. In the beam stacks shown in Figure 2 the energy is thus localized in a wedge-shaped region around the peaks. In the transformed profile there are also some aliasing artifacts caused by the strong arrivals of head waves at large offsets.

The paths of the beams arriving at the surface with the same ray parameter are sketched in Figure 3. The traveltimes along the beams, and the receiver locations at which the the beams arrive, are the traveltimes and the receiver offsets of the peaks in beam stacks. Since the beam paths and the traveltimes along the beams depend also on the velocity function, the positions of the peaks are functions of the velocity model. The true velocity model predicts exactly the traveltimes of the peaks, for every receiver location and ray parameter. The inversion method described in the next section searches for the velocity model that best predicts the peaks in the beam-stacked domain.

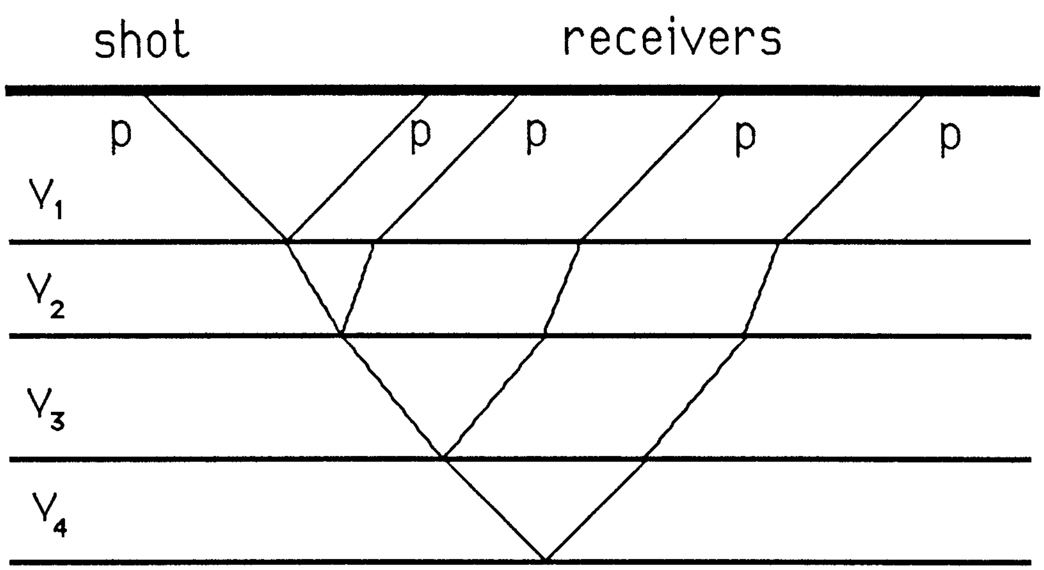

FIG. 3. The paths of the beams arriving at the surface with the same ray parameter. The beam paths and the traveltimes along the beams depend on the velocity function.

THE INVERSION METHOD

The proposed interval-velocity inversion searches for a velocity model that best explains the beam-stack transformed data. The basic principle of the inversion is a tomographic fitting to the beam-stacked data of the traveltimes along the beams.

Figure 4 shows the path of a beam that starts from the shot at location s_0 with ray parameter p_s, bounces on a segment of reflector, and travels to the surface to arrive at the receiver location r_0 with ray parameter p_r. The beam path and the traveltime along the beam are functions of the velocity model. The beam path can be determined by ray tracing of the down-going ray and the up-going ray until they meet, and the traveltime t_V is the integral of slowness along the trajectory; thus $t_V = t_V(s_0, r_0, p_s, p_r)$. If the velocity along the beam path is correct, and there is a reflector at the position at which the down-going ray meets the up-going ray, the value of $Beam(s_0, r_0, t_V(s_0, r_0, p_s, p_r), p_s, p_r)$ is different from zero. This value of the beam-stacked data is dependent on the strength of the reflector on which the beam bounced and on the angle of reflection. On the contrary, if the velocity along the beam path is wrong, the value of the beam stack at the predicted traveltime t_V is approximately zero. The value of the beam-stacked data at the predicted travel time t_V is thus a measure of the correctness of the velocity model along the beam path.

When all the beams in the dataset are considered, the inversion can be formulated as the solution of the non-linear optimization problem of finding the maximum, with respect

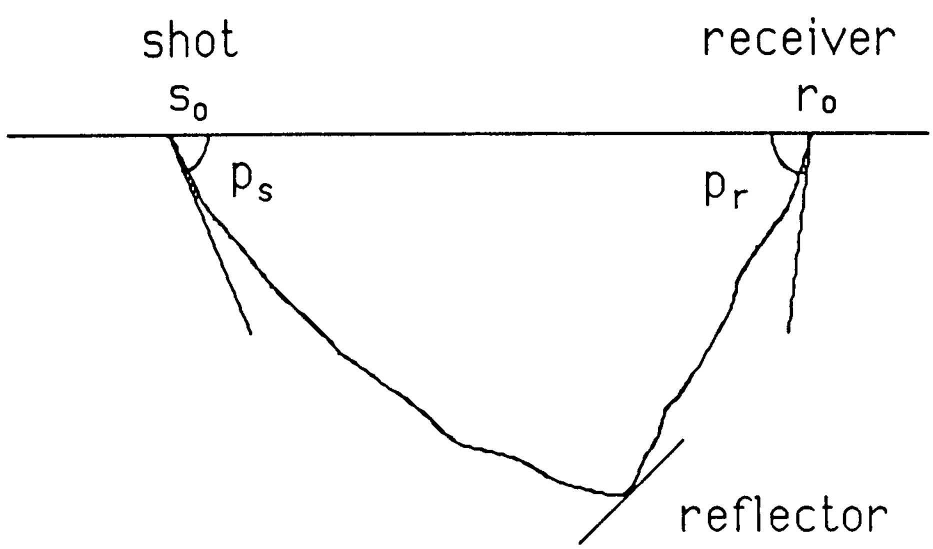

FIG. 4. The path of a beam that starts with ray parameter p_s from the shot location s_0, bounces on a reflector, and arrives with ray parameter p_r at the receiver location r_0. The beam path and the traveltime along the beam are functions of the velocity model.

to the velocity model $V(x,z)$, of the total energy

$$E(V) = \sum_{s_0} \sum_{r_0} \sum_{p_s} \sum_{p_r} [Beam(s_0, r_0, t_V(s_0, r_0, p_s, p_r), p_s, p_r)]^2 . \tag{1}$$

If we have a good initial guess of the velocity model, as we hope might be obtained by use of conventional velocity analysis, the optimization problem can be efficiently solved with a gradient algorithm. The implementation of a gradient algorithm requires that the gradient of the objective function be computed with respect to the model. The predicted traveltimes t_V along the beams are a function of velocity, and the value of the objective function depends on the predicted travel times t_V. It is therefore natural to express the gradient of the objective function as

$$\frac{dE(V)}{dV} = \frac{dE(V)}{dt_V} \frac{dt_V}{dV}, \tag{2}$$

where the derivatives are computed at fixed s_0, r_0, p_s, and p_r.

The first term on the right side of equation (2) is easily computed from the beam-stacked data with a finite-difference approximation of the derivative operator, and represents the interaction of the inversion algorithm with the actual data. The second term of equation (2) is more difficult to compute and will require a sophisticated ray tracing. The difficulties arise because traveltime is a function of the velocity model in two ways: directly, as the integral of the slowness along the beam path, and indirectly through the beam path itself.

In the examples presented in this paper the gradient of traveltime with respect to the velocity model was computed by use of finite differences. I perturbed the velocity model and computed the induced perturbations in the traveltimes. This procedure is accurate but it is too expensive because it requires a new ray tracing for each perturbed model parameter. I would rather find a more efficient way for computation of the gradient.

Among the several gradient algorithms for the solution of a non-linear optimization problem, a classic version of the conjugate gradient algorithm, derived by Polak-Ribiere (Luenberger, 1984), was chosen. This is an efficient method for the solution of nonquadratic problems.

A priori assumption of the velocity-model's smoothness

A tomographic inversion can resolve only the lower wavenumbers of the velocity model because it is based on traveltimes, which are integral measures of velocity. A common approach for dealing with the poorly determined components of the velocity model is to impose a smoothness condition on the velocity function during the inversion procedure. The smoothness condition is usually incorporated in the algorithm by the addition of a penalty term to the objective function. I chose to use as a penalty term the sum of the squares of the first derivatives of the velocity function. When this smoothness condition is added to the objective function in equation (1), the objective function of the problem becomes

$$Q(V) = E(V) \ - \ \lambda_x \sum_i \left(\frac{\partial V}{\partial x}\right)_i^2 \ - \ \lambda_z \sum_i \left(\frac{\partial V}{\partial z}\right)_i^2 , \tag{3}$$

where i is the index over model velocities, and λ_x and λ_z are constants that determine the strength of the smoothness assumption. This approach has some drawbacks that will be discussed in the last section.

RESULTS OF ONE-DIMENSIONAL INVERSION

A layered-medium synthetic example

I have tested the proposed inversion method on simple synthetic data. Figure 5 shows the synthetic shot profile that I used as data for the inversion. The profile was generated by use of a finite-difference modeling program and with the assumption of a horizontally stratified medium composed of five layers. When the data was inverted, the stratified medium was assumed to be composed of 140 layers, each 5 meters thick.

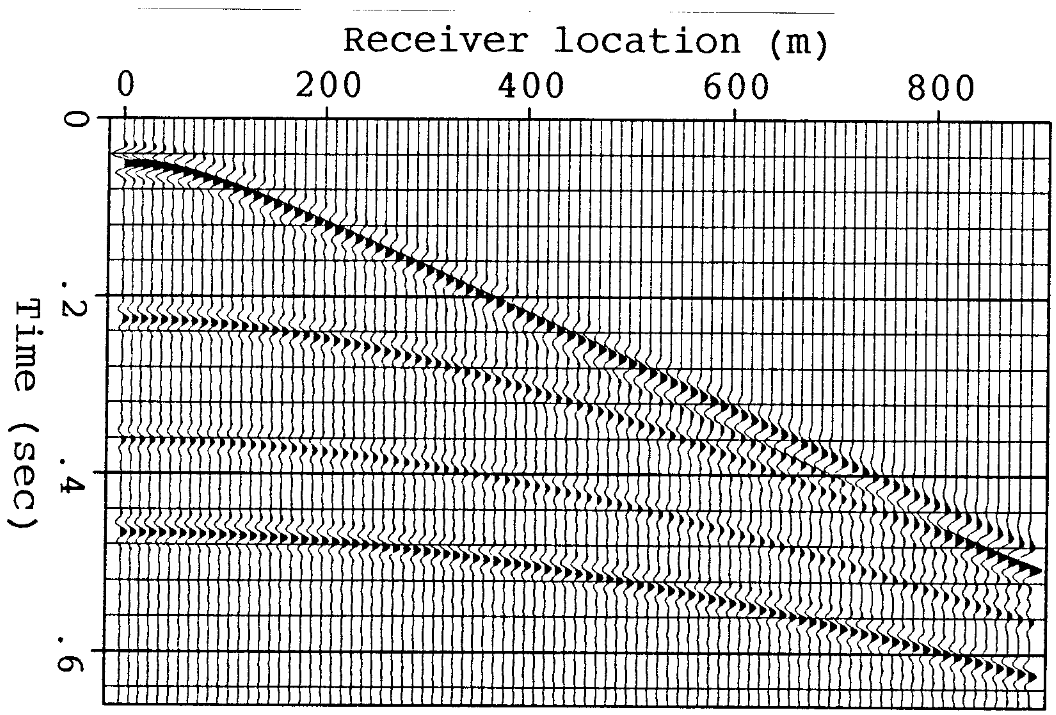

FIG. 5. The synthetic shot profile that I used as data for the testing of my inversion algorithm. The earth model is a stratified medium composed of five layers. The profile was generated with a finite-difference modeling program.

Because the model is laterally homogeneous the take-off angle and the arrival angle of the beams are equal. Therefore I decomposed the shot profile only into receiver ray parameter components; the resulting beam stacks form a data cube. The three axes of the cube are the time t_0, the receiver location r_0, and the receiver ray parameter p_r.

In the practical implementation of the inversion algorithm I computed the beam stacks for 18 receiver locations, 10 ray parameter values, and at all the discrete-time values. I also precomputed the square of the beam-stacks' value and slightly smoothed the stacks along the time axis.

Figure 6 shows the contour plot of a slice, taken at constant ray parameter, of the beam-stack data cube that I used for the inversion. The traveltime curves as a function of the

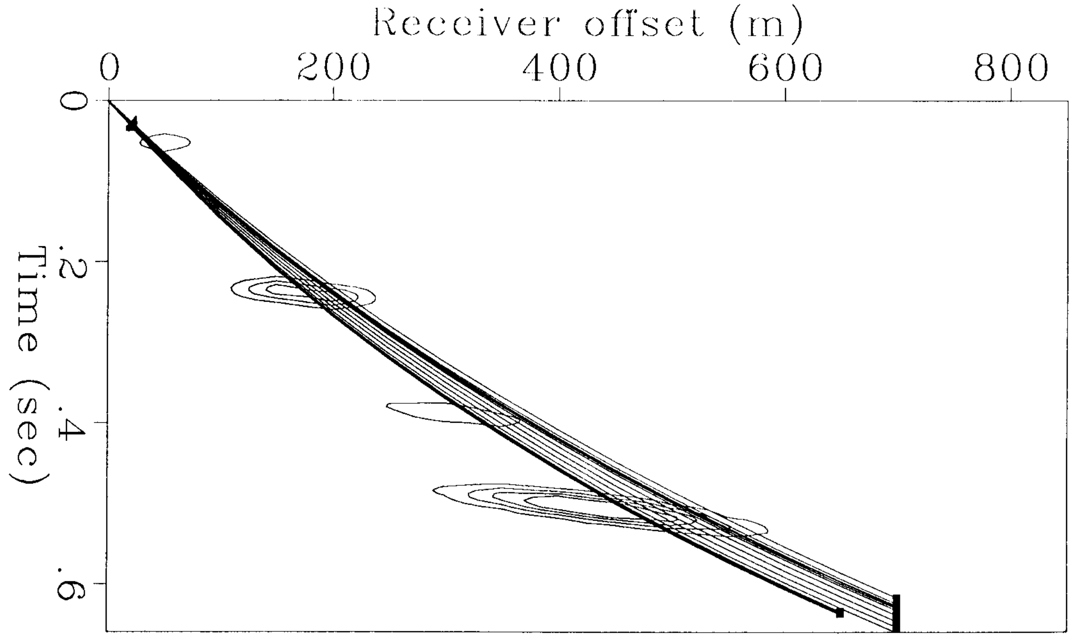

FIG. 6. Contour plot of a slice of the beam-stacks data cube, taken at constant ray parameter p_r. The traveltime curves are overlaid on the contour plot for 20 iterations of the inversion algorithm.

receiver location, for 20 iterations of the inversion algorithm, are superposed on the contour plot. Figure 7 shows the results of 20 iterations of the velocity-inversion algorithm, starting from an initial guess of velocity linearly increasing with depth. The true velocity model is drawn with a dashed line and the result of the final iteration is drawn with a thicker line.

The result of the final iteration is a smooth approximation of the true velocity model. The result is smooth because the high-wavenumber components of the velocity model are unresolved by the inversion, and because I imposed a smoothness condition on the velocity function; a penalty term was added to the objective function, as shown in equation (3). Figure 6 shows that the traveltime of the peaks in the beam-stacked profile are fairly well predicted by the final velocity model.

A field-data example

I tested the proposed velocity inversion also on a field shot profile, using the data shown in Figure 1. The inversion was carried out with assumption of a horizontally layered medium; this assumption is approximately satisfied by the geology of the region where the profile was recorded.

I computed the beam stacks for 48 receiver locations, 20 ray parameter values, and at all the discrete-time values. When a stratified Earth composed of 500 layers, each 10 meters thick, was assumed, the inversion algorithm converged to a final result after 50 iterations. Figure 8 shows the result of one iteration out of ten iterations; the result of the last iteration is drawn with a thicker line. The velocity of the initial model is equal to the velocity of water

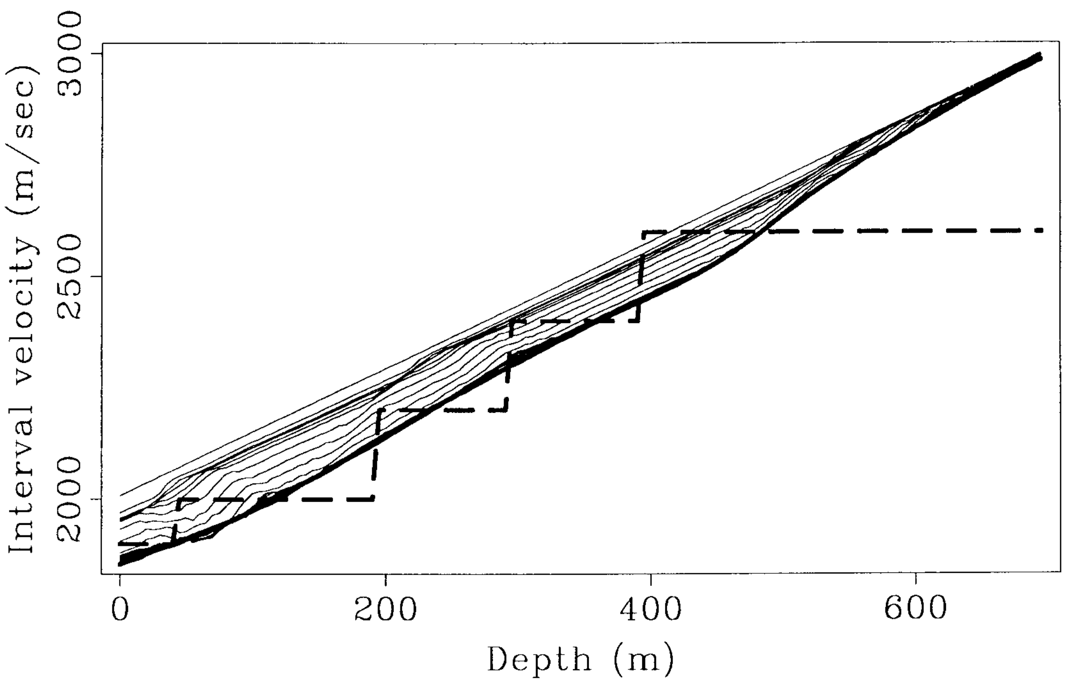

FIG. 7. The result of 20 iterations of the iterative-inversion algorithm. The starting guess is a velocity function linearly increasing with depth. The true velocity function is drawn with a dashed line and the result of the last iteration is drawn with a thicker line. The velocity model is composed of 140 layers, each 5 meters thick.

through the depth of the sea bottom, and increases linearly with depth in the subsurface.

Figure 9 shows the contour plot of beam stacks at constant ray parameter. A traveltime curve is superposed on the contour plot every 10 iterations of the inversion algorithm. All the peaks of the beam stacks except one are well fitted by the traveltime curves. The exception corresponds to a reflection that I think is a water-bottom peg-leg.

I did also a conventional stacking-velocity analysis on the same field profile, to verify the results of the inversion algorithm. Figure 10 compares the root mean square (rms) velocity profile obtained by use of stacking-velocity analysis to the rms velocity profiles computed from the results of the iterative inversion. The velocity function obtained by use of stacking velocities is drawn with a dashed line, and the result of the last iteration is plotted with a thicker line.

Figure 11 compares the interval-velocity profiles as a function of the zero-offset time. The interval-velocity function was computed from the rms velocity function, obtained from stacking velocities, by use of Dix's formula. Also in this Figure the dashed line is the result of the conventional method, and the result of the last iteration of the velocity inversion is plotted with a thicker line.

The resemblance between the velocity functions obtained by use of conventional velocity analysis and the velocity functions obtained by use of the proposed velocity inversion confirms the soundness of the new method. I think that these results are promising although the inversion problem was simple.

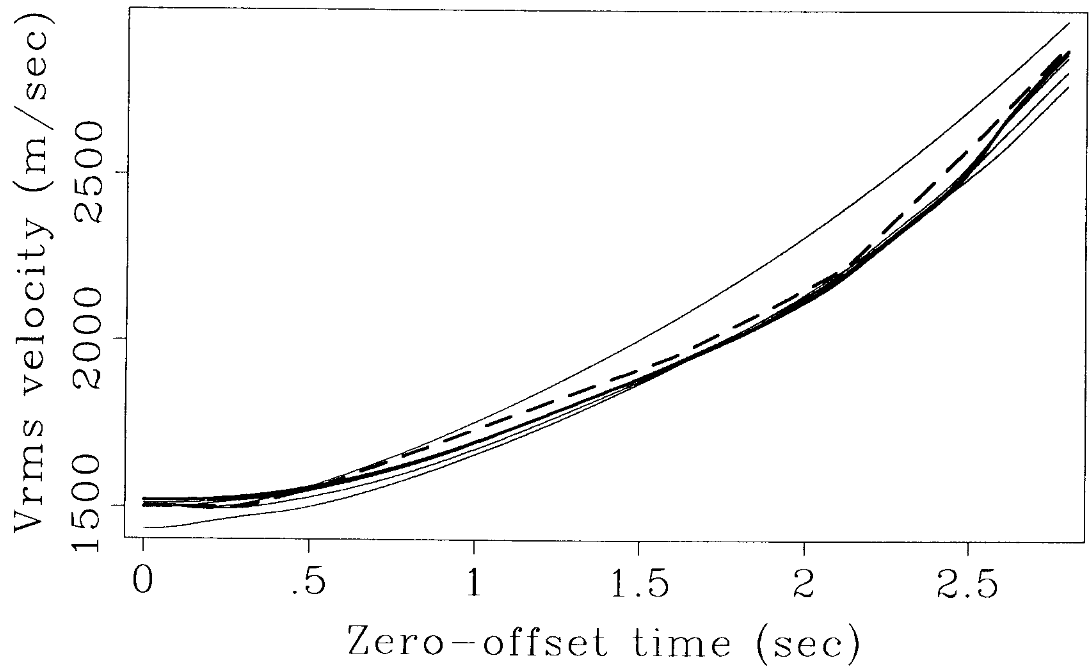

FIG. 10. Comparison between the rms velocity profile obtained by stacking-velocity analysis and the rms velocity profile computed from the results of the velocity inversion. The result of conventional velocity analysis is drawn with a dashed line, and the result of the last iteration is plotted with a thicker line.

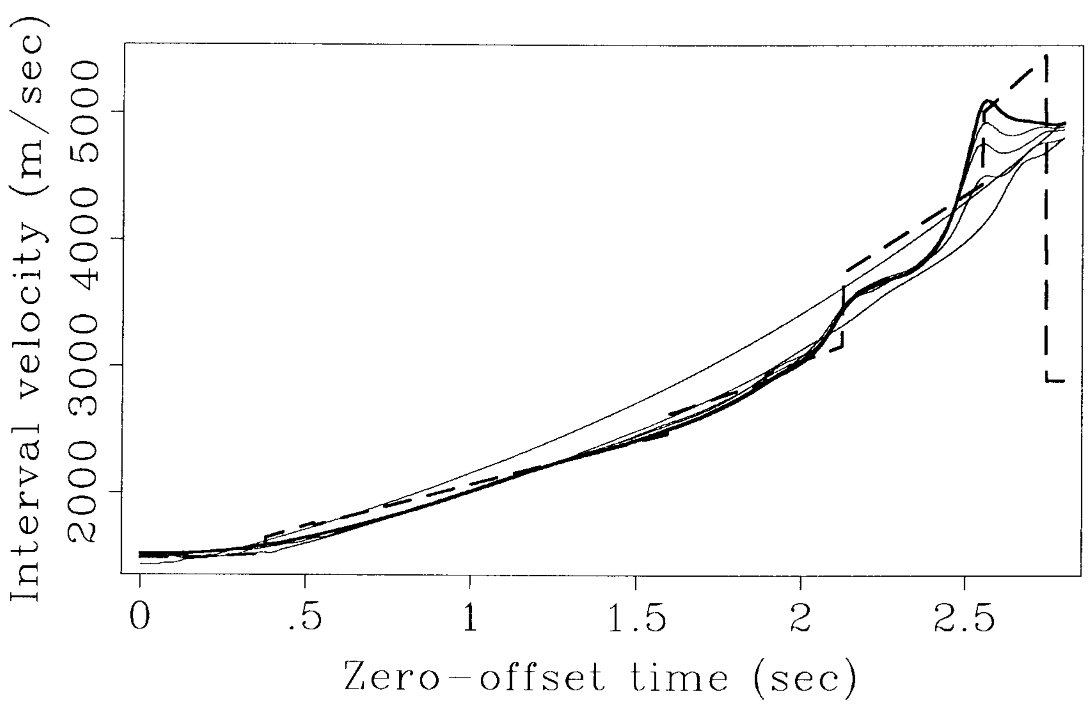

FIG. 11. Comparison between the interval-velocity function computed from the rms velocity profile (obtained by use of stacking velocity analysis) and the results of the velocity inversion. The profiles are plotted as a function of the zero-offset traveltime. The dashed line is the result of conventional velocity analysis, and the result of the last iteration is plotted with a thicker line.

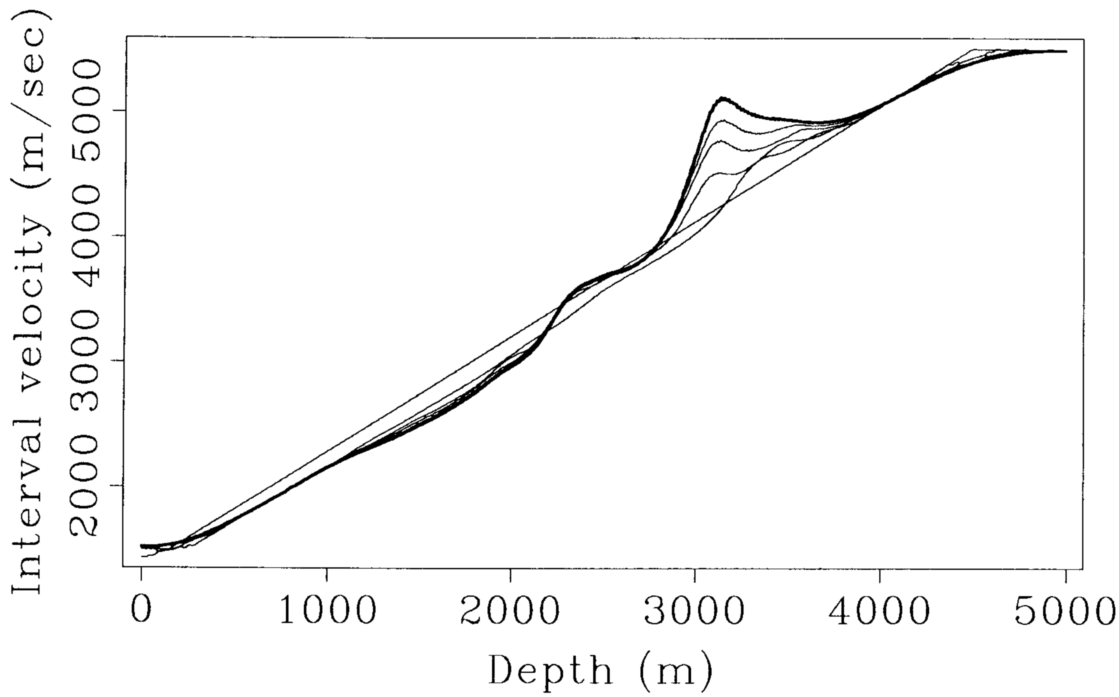

FIG. 8. The results of 50 iterations of the iterative-inversion algorithm. The velocity function is plotted every 10 iterations. The starting guess is a velocity function linearly increasing with depth. The result of the last iteration is drawn with a thicker line. The velocity model is composed of 500 layers, each 10 meters thick.

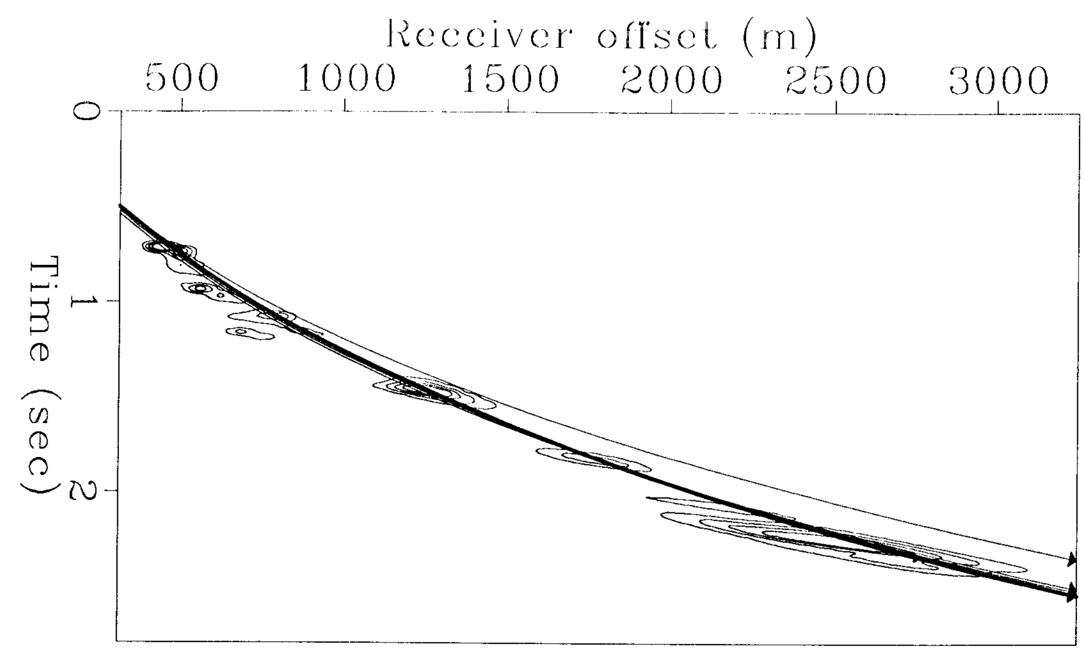

FIG. 9. Contour plot of a slice of the beam-stacks data cube, taken at constant ray parameter p_r. A traveltime curve is overlaid on the contour plot every 10 iterations of the inversion algorithm.

FUTURE INVESTIGATIONS

The results of one-dimensional velocity estimation using the proposed method are satisfactory but the purpose of the method is a two-dimensional velocity estimation. Two-dimensional inversion is more difficult than one-dimensional inversion and I think that my algorithm needs some improvements before it can successfully solve the problem. The aspects of the problem that I will study more in depth are discussed in the following sections.

Interactive migration of beam stacks

I would like to implement an algorithm for the migration of beam stacked data. In principle, the migration of beam stacks should be fast, although not extremely accurate. Beam stacks migration can be performed by use of ray tracing to find the beam paths and summation to image the contribution of each beam (Milkereit et al., 1986). A fast prestack-migration algorithm could be very useful for the interactive guidance of the inversion algorithm.

Objective function

The objective function that I proposed in equation (3) is simple, but has some important drawbacks. The major one is that the value of the objective function is too sensitive to the artifacts of local beam stacks. Local beam stacks have a finite spatial and directional resolution, so the stack energy is not zero around the peaks. Using the objective function in equation (3) we sum also the energy of these artifacts and we force the travel times to fit the artifacts as well as the correct peaks in the stacks. Because of these effects, the inversion needs a strong dumping of the velocity model to prevent the results from having an oscillatory behavior. I think that the objective function should take into account the finite resolution of beam stacks and exploit the knowledge of the expected shapes of the peaks in beam stacks.

Parameterization of the velocity model

In the section in which I described the inversion method I also presented a method for constraining the velocity model with a smoothing condition. A penalty term that decreases as the velocity function becomes smoother can be added to the objective function.

This approach has the advantage of being straightforward and easyly implemented but also has some disadvantages. The optimization problem is solved for many more unknown parameters than are neeeded to describe a smooth velocity model, and therefore the cost of each iteration is higher than is necessary. Furthermore the penalty term makes the optimization problem ill-conditioned and thus it often prevents the iterative inversion from converging to a model that predicts all the events in the data. Some components of the velocity model that are well determined in the data can be unresolved by the inversion.

If the unknown velocity model could be parameterized in such a way that the smoothness constraint were taken explicitly into account, I could use a smaller number of parameters and avoid the necessity of a penalty term in the objective function. A possible alternative to the conventional approach might be the representation of the velocity function with use of finite elements (Zhu and Brown, 1987) or overlapping smooth basis functions (Van Trier, 1987).

A priori assumptions

A priori assumptions can be helpful for driving the inversion algorithm to reconstruct some of the poorly determined components of the velocity model. Continuity of some specific reflectors is often a possible assumption. I would like to incorporate into my inversion scheme the capability of constraining some reflectors to be continuous. The desired reflectors can be interactively picked from the result of a preliminary migration of beam stacks. I think that comparing the results of some trial inversions, each of them based on different constraints on the geometry of the reflectors, might be helpful in the discrimination between reflections that are deformed because of geologic structure and reflections that are deformed because of velocity anomalies.

The continuity assumption can be incorporated into the objective function by an additional term that rewards the continuity of reflectors. Events corresponding to a continuous reflector lie on a continuous curve in the beam-stacks' domain; rewarding the continuity of this curve reflects the assumption of continuity of the reflector.

CONCLUSIONS

The one-dimensional results of the proposed algorithm show that the inversion of beam-stacked data is a promising method for estimating the velocity model. The next step is to apply the method to a two-dimensional velocity estimation.

ACKNOWLEDGMENTS

Chuck Sword introduced me to the capabilities of local slant stacks as velocity analysis tools and advised me during several discussions. Fabio Rocca helped me to clarify the formulation of the method at the beginning of the project. Clement Kostov and I cooperated in the study of beam stacks and the discussions with him are always fruitful. I am grateful to all of them and I would like to thank them.

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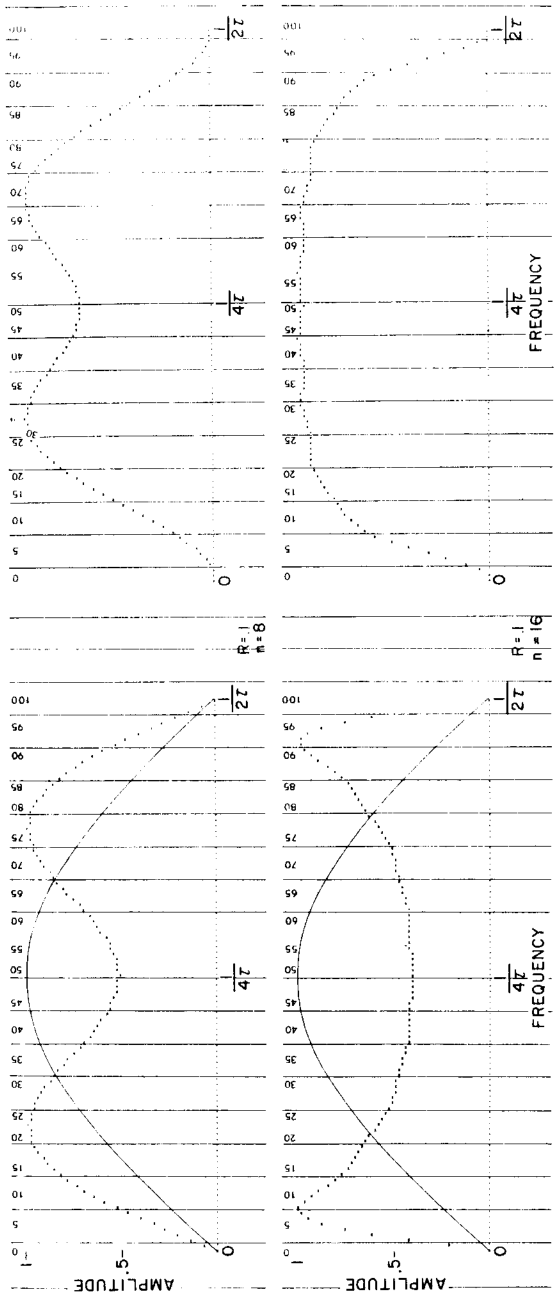

Fig. 5. Amplitude spectra of distortion filter, inverse filter, and their convolution.

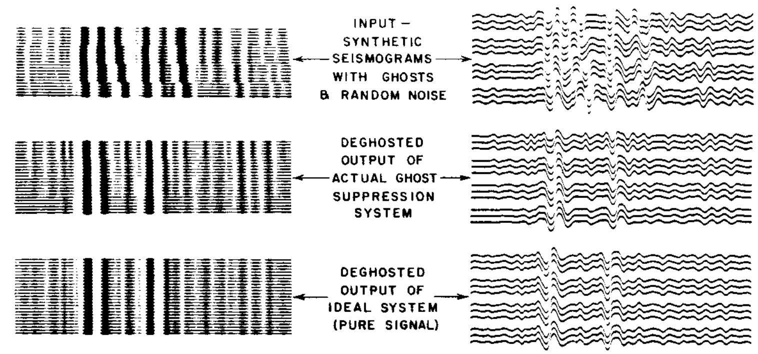

Fig. 6. Equivalent variable area and wiggle displays of input and output of sub-optimum ghost suppression system, together with the ideal output for comparison.

B. Velocity Bandpass Filtering

Velocity bandpass filtering is based on the assumption that the plane wave components of signal and coherent noise have apparent velocities largely confined to non-overlapping bands. The idea is to design a system which passes components in the signal band without distortion, but strongly rejects components in the noise band. Imagine a seismic record section made up of a large sequence of seismic traces from which we extract $2L$ adjacent traces. We label the extracted traces as in Figure 7, $-L \ldots -1, 1 \ldots L$. Then a signal component on the lth trace has a delay relative to a fictitious trace half-way between the -1 and $+1$ trace given by

$$\tau_l = a_l \frac{\Delta x}{v} \qquad (19)$$

Here Δx is the trace spacing, v the apparent velocity and

$$a_l = \begin{cases} l - 1/2, & l \geqslant 1 \\ l + 1/2, & l \leqslant 1 \end{cases}$$

It will be convenient to introduce the "moveout per trace", $\tau = \Delta x/v$. With Δx fixed the apparent velocity is uniquely specified by the moveout τ. In particular, a band of velocities corresponds to a band of moveouts. We therefore assume that the random variable τ lies in a band between the limits $\tau_T - \tau_C$ and $\tau_T + \tau_C$, where τ_T is the midpoint of the band and $2\tau_C$ is the bandwidth. If τ is uniformly distributed in this band, the appropriate probability density function is

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