Signal processing with stochastic memories

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A very large memory store ($2^{100-10000}$ addresses) can be simulated by building a very small part of it. Memory storage and retrieval then become probabilistic with some interesting behaviors. Resolution between different addresses in memory are a Gaussian function of how many bits differ between address locations and the dimensions of the memory. It is proposed that such a memory be used for local (e.g. Fourier) integral transforms.

Algorithm

Choose an address size greater than 100 bits. Randomly select a subset of this memory space (e.g. a million values) that can be stored on a current computer. We *store* in a target address by storing into all subset addresses which are 'near' the target address. We *retrieve* from the target address by retrieving from all subset addresses which 'near' the target address. The nearness measure is the number of bits that differ between two addresses.

The number of distinguishable memory locations depends upon the parameters of address size, subset size, and nearness threshold length. As the address size increases then more memory locations can be distinguished within the same size subset. Figure 1 illustrates this behavior. It is the @impulse response' of storing an unit value into one address location. Leakage into nearby addresses decreases with address size, thereby increasing address resolution. Note the distribution is normal.

This memory resolution behavior is like to packing smaller spheres within a larger bounding sphere in various dimension spaces. In this analogy, the address size is the spatial dimension and the nearness measure is the relative diameter of the sub-sphere to the bounding sphere. To pack a constant number of small spheres within the boundary sphere, the ratio of the diameter of the small sphere to the bounding sphere increases

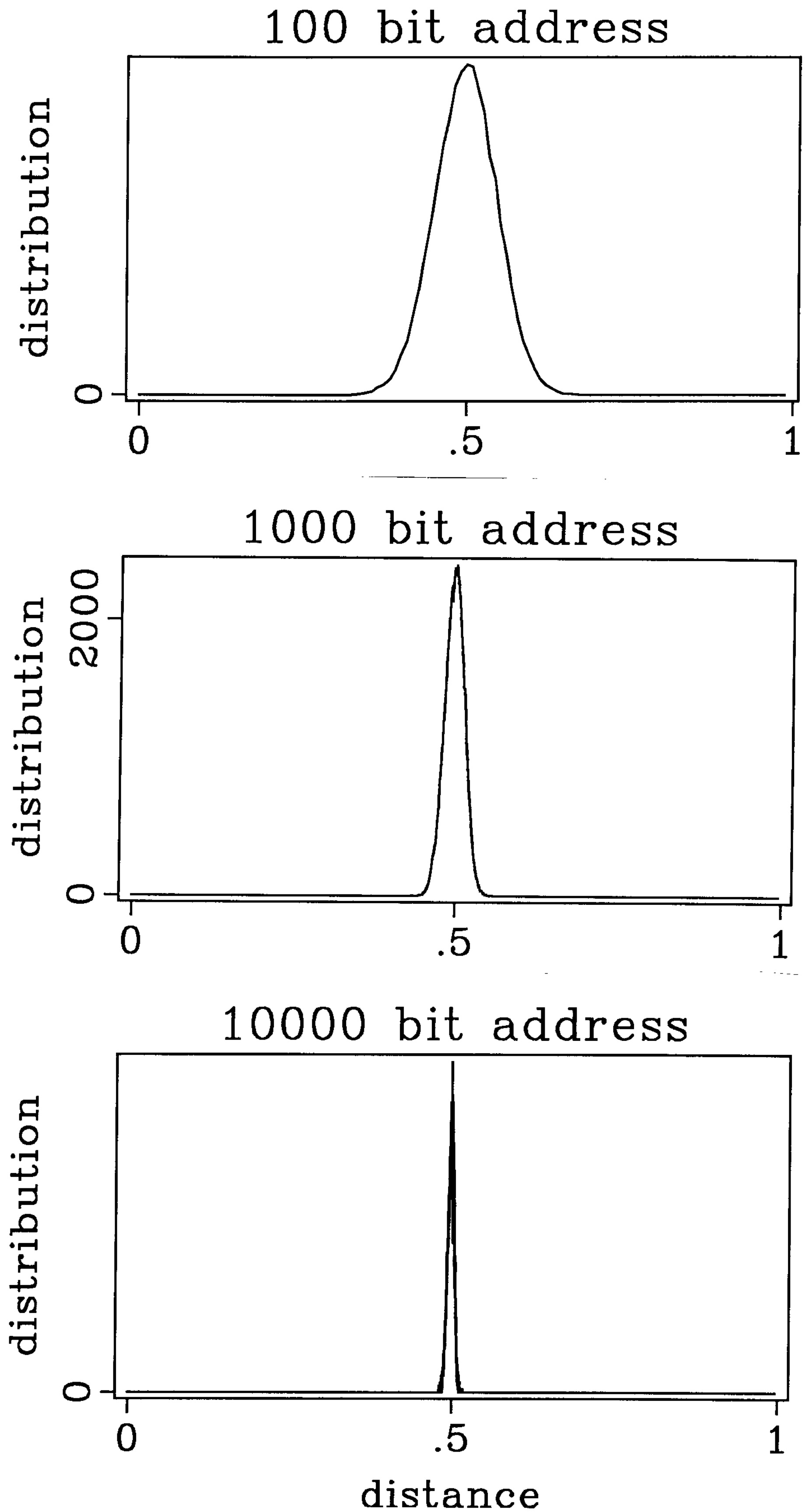

FIG. 1. Impulse response of a single store/retrieve operation for addresses of various sizes. The distance is normalized to the address size. The interpretation of this figure is (1) large addresses better resolve storage in a stochastic memory and (2) memory retrieval convolves a Gaussian upon memory contents. These figures were generated with the code in the appendix.

with the dimension. For example, the diameter of a million sub-circles is smaller than the diameter of a million sub-spheres. The monograph of Pentti Kanerva (1985), that inspired this article, tabulates the packing radii for a million sub-spheres at various dimensions:

address length in bits	100	1000	10000
radii of a million spheres	52%	84%	95.2%

The larger the relative diameter of a sub-sphere, the more likely a random probing of the boundary sphere would tell us in which of the sub-spheres one is at.

Properties of stochastic memories

The address is the data. Whether or not something has been stored at a given address becomes an important piece of information. Since the stochastic memories become more robust as the address size increases, more and more information can be stored in the address bits themselves. This is also called associative memory or content addressable memory.

Learning and forgetting behavior. Stochastic memory is non-local, but not entirely global. Repeated storage in the same or nearby address reinforces that information and tends to lower the signal content of infrequently used addresses.

Incomplete addresses. The exact address need not be known to retrieve memory contents. A nearby address often suffices. As the address length increases proportionally fewer of the exact bits need to be know. This can be explained with the sphere packing analogy. As the sub-sphere size increases for larger dimensions, they enclose a larger set of nearby addresses.

Applications

Simulating biological memories. This was Kanerva's interest. Biological memories learn and forget. From anatomical studies they seem not to be located at one place in the brain, but somewhat globally stored. Not the exact same stimulus is required to retrieve a memory as what created the memory in the first place.

Pattern recognition. Images or sounds are often interpreted by matching previously stored templates. However deterministic algorithms are somewhat brittle when pattern differs from a template. Stochastic memories offer a means to store large templates (map them into long addresses) and non-brittle matching (incomplete addresses).

Local integral transforms. Many integral transforms (e.g. Fourier) globally average the input data. Attempts to make them somewhat local usually comprise applying an some sort of window to the data or integral kernal. The stochastic memory itself is an smoothing integral with a window size as shown in figure 1. As the amount of computation increased (address size and subset size) the resolution of the window increased. A local integral transform could be implemented with stochastic memory. A randomly generated set of integrands could be stored in stochastic memory using the input transform coordinates as addresses. Then the output transform point could be retrieved using the output transform coordinates as addresses.

Problems

Coding data into addresses. If you are performing pattern recognition, similar patterns should have the nearby addresses while different patterns have far away addresses. How do you code data into addresses?

In the case of local integral transforms how do you get a large enough address size? Data or transform coordinates are not large enough, but short sequences of data may be large enough.

Speed. The computer simulation of stochastic memory requires a nearness calculation for every address subset for every memory access. This makes memory access prohibitively slow even on a conventional super-computer. However, these calculations can be done simultaneously in a parallel computer (probably how biological memories operate). An article in the June 1986 IEEE Spectrum describes an optical stochastic memory built for pattern recognition.

Appendix: Computer code

```
subroutine STORE (address,value)
integer address
real value
for each subset-address
    {
    distance = count-bits ( not-xor (address,subset-address) )
    if (distance < nearness-length) then
        {
        contents-of (subset-address) = contents-of (subset-address) + value
        }
    }
```

```
subroutine RETRIEVE (address,value)
integer address
real value
value = 0
for each subset-address
    {
    distance = count-bits ( not-xor (address,subset-address) )
    if (distance < nearness-length) then
        {
        value = value + contents-of (subset-address)
        }
    }
```

REFERENCES

Kanerva, 1984, P., Self-propagating search: a unified theory of memory: Center for the study of language and information Report 84-7, Stanford University.

June 1986 IEEE Spectrum (full reference not available at report publication time.)

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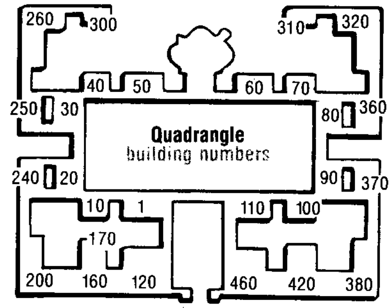

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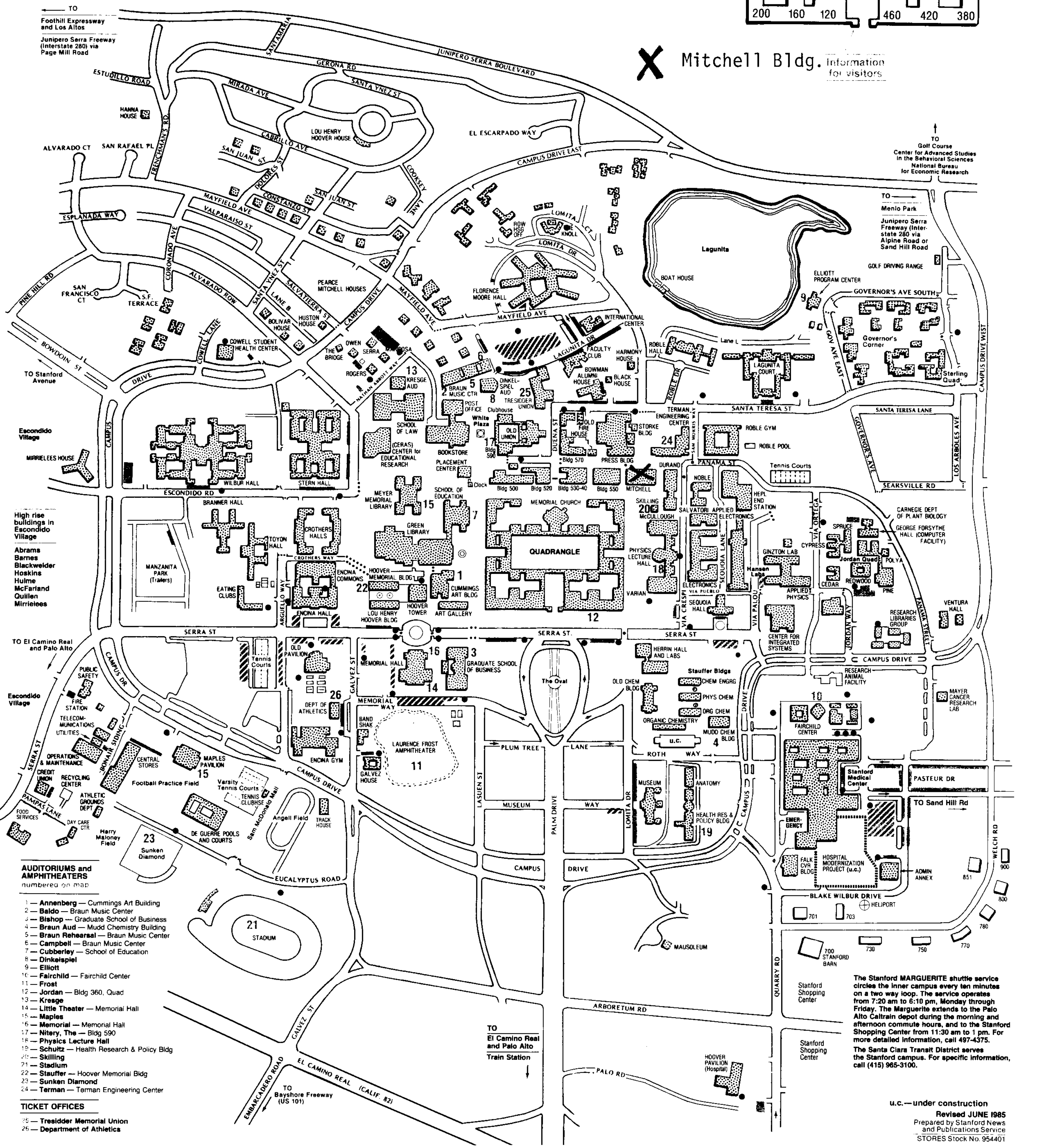

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Revised JUNE 1985

Prepared by Stanford News and Publications Service
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