Prestack residual migration

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INTRODUCTION

Is it possible to define a prestack residual migration? That is, is the result of prestack migration with one velocity followed by a second similar migration with another velocity, equivalent to a single migration with some appropriate replacement velocity? Rothman et al. (1983) demonstrated that such a residual migration process is feasible for post-stack migration, and showed how to use it to decrease computational effort and to correct errors in migration velocity. As an extension of that work, Larner and Beasley (1985) cascaded residual migrations at small velocity increments to obtain steep-dip accuracy from inexpensive 15 degree finite-difference operators.

It is natural then to ask whether a similar residual migration can be done for unstacked data, e.g. for migrating individual shot profiles or constant offset sections. Clearly, when we ask this question, we exclude the final step of stacking over surface datum location (or over wavenumber) that collapses the various images of the subsurface obtained from different shots into a single image. Beyond that, there seems at first sight to be no insuperable objection, since both NMO and post-stack migration can be iterated. Unfortunately, this conclusion proves too hasty. The purpose of this paper is to examine the obstacles that make residual prestack migration infeasible.

IMAGING METHODS

Prestack migration can be done by a variety of methods, all of which have the same general goal. Here we examine the difficulty in doing residual prestack migration from four different viewpoints that illuminate different aspects of the problem. However, each approach will yield the same negative conclusion, as would any other method; the barriers to doing residual prestack migration are not just caused by the choice of a particular algorithm.

One class of prestack migration methods separately extrapolate the shot and geophone wave fields. As an example of such methods we will look at the hybrid-domain algorithm described by Al-Yahya and Muir (1984). In this approach, instead of downward continuing both the upcoming field and the downgoing fields, only the upcoming field is downward continued and then a time-shift equal to the one-way travel-time from the source to each geophone is applied. This form of imaging can be summarized as:

$$R(x,\tau) = \int \left[\int U(k_x,\omega) e^{-ik_\tau(k_x,\omega)\tau - ik_x x} dk_x \right] e^{i\omega\sqrt{\tau^2+(\frac{x}{v})^2}} d\omega \quad ,$$

where $R(x,z)$ is the reflectivity of the subsurface and $U(k_x,\omega)$ is the Fourier transform of the upcoming wave field recorded at the surface.

Another approach to prestack migration is given by the processing sequence of normal moveout (NMO), dip-moveout (DMO), and zero-offset migration. A good summary discussion of DMO methods is given by Deregowski (1986). DMO at first looks very different from shot-geophone type prestack migration, but Hale (1983) gives a formal proof of the effective equivalence for constant velocity media. Here the imaging sequence can be written as:

$$image = \mathbf{Mig\ DMO\ NMO}\ data \quad .$$

Stolt (1978) presented an F-K domain method for constant velocity migration of a prestack data set. It is based on the double square root dispersion equation

$$k_\tau = \sqrt{\omega^2 - v^2 k_s^2} + \sqrt{\omega^2 - v^2 k_g^2} \quad ,$$

where ω is the frequency in the unmigrated section, k_τ is the frequency in the migrated section, and k_s and k_g are the wavenumbers for the shot and geophone axes respectively. This approach operates on the entire data set at once, not just on single gathers or section.

Finally, we will look at diffraction curves in constant-offset sections and in shot gathers, and examine the kinematics of collapsing these curves to focused images by migration. This method lends itself to simple pictorial representation of the stationary phase zone where the energy is concentrated.

RESIDUAL MIGRATION

The hybrid method

First let's consider the hybrid method. Since time shifting is similar to NMO, we might consider doing residual prestack migration by doing residual NMO and residual migration. Residual migration is done by a residual velocity $v_r = \sqrt{v^2 - v_m^2}$ while residual NMO is done by $\frac{1}{v_r} = \sqrt{\frac{1}{v^2} - \frac{1}{v_s^2}}$, where v_m is the velocity used in migration, v_s is the velocity used in NMO shifting, and v is the correct velocity for the medium. This means that to do migration with a velocity v in two equal steps, we can migrate in two steps with $(v_m, v_s) = (v/\sqrt{2}, \sqrt{2}v)$. The result is shown in Figures 1 and 2. The figures clearly show that this concept is WRONG!!

Where did we go astray? The first pass of migration using the hybrid method can be represented by the equation

$$image = \mathbf{Sh}_1 \ \mathbf{Ex}_1 \ data \quad ,$$

where **Ex** represents extrapolation and **Sh** represents shifting (and imaging). One way to look at residual migration is to undo the first migration and then re-migrate with the new velocity. This means that the new operator looks like

$$\begin{aligned} res &= \mathbf{Sh}_2 \ \mathbf{Ex}_2 \ \mathbf{Ex}_1^{-1} \ \mathbf{Sh}_1^{-1} \\ &= \mathbf{Sh}_2 \ \mathbf{Ex}_{12} \ \mathbf{Sh}_1^{-1} \quad , \end{aligned} \tag{1}$$

where $\mathbf{Ex}_{12} = \mathbf{Ex}_2 \mathbf{Ex}_1^{-1}$ is the residual extrapolation (and imaging). However, this extrapolation is surrounded by the two shifting operators that don't commute with it. This suggests that we need to use a program different from the migration program to do the stepping in velocity. The *wrong* method effectively replaced equation (1) by

$$res = \mathbf{Sh}_{12} \ \mathbf{Ex}_{12} \quad ,$$

where $\mathbf{Sh}_{12} = \mathbf{Sh}_2 \mathbf{Sh}_1^{-1}$ is the residual shift.

DMO

Simple arguments that prestack residual migration should work are perhaps most seductive when one thinks in terms of DMO. Pre-stack migration by zero-offset migration after NMO and DMO can be written as

$$image = \mathbf{Mig}_1 \ \mathbf{DMO}_1 \ \mathbf{NMO}_1 \ data \quad .$$

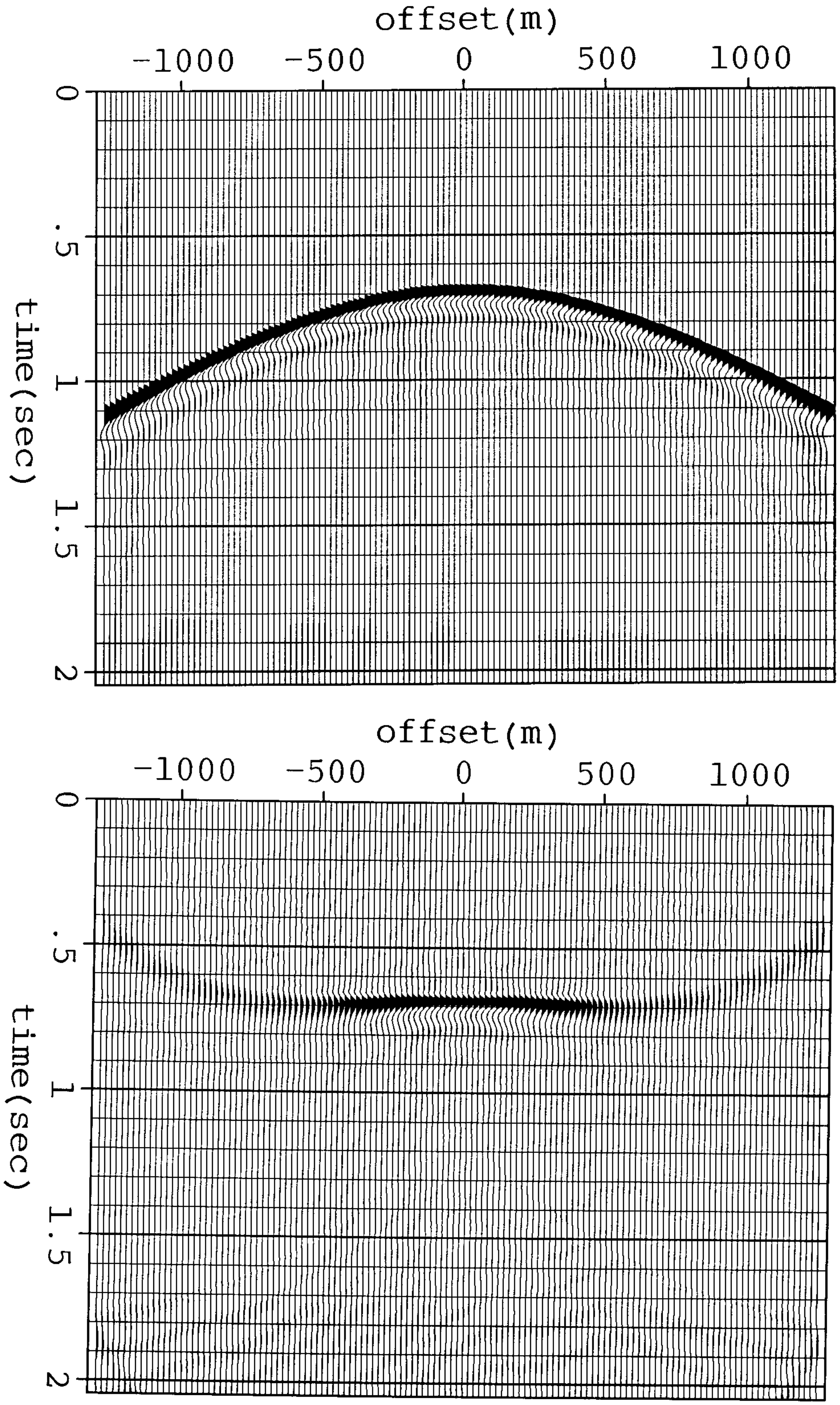


FIG. 1. Top: A synthetic profile for a single horizontal reflector in a constant velocity. Bottom: The result of migrating the profile with the correct velocity.

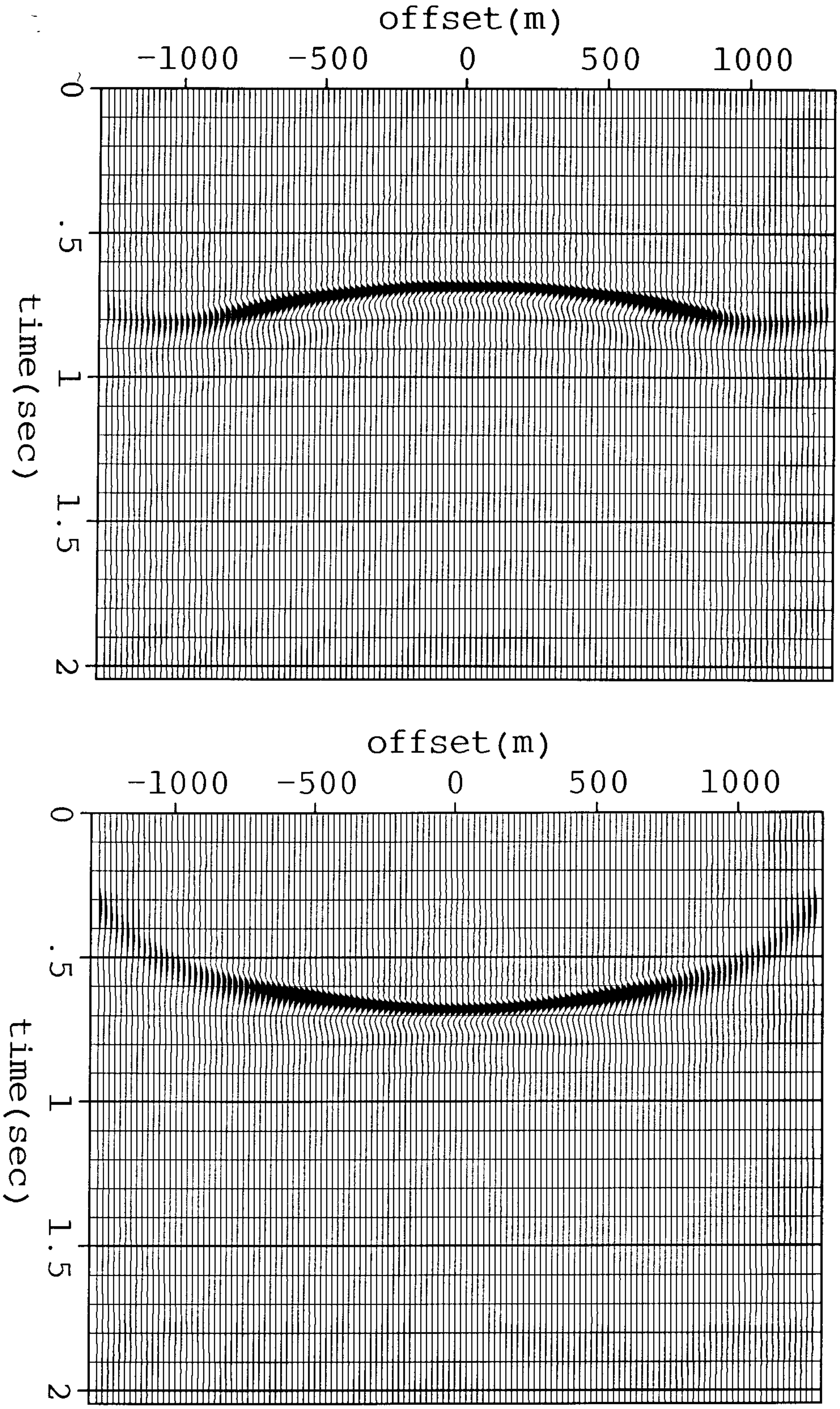


FIG. 2. Top: the first pass; data in Figure 1 migrated with $v_m = v/\sqrt{2}$ and $v_s = v\sqrt{2}$. Bottom: the second pass; result of migrating the output of the first pass with $v_m = v/\sqrt{2}$ and $v_s = v\sqrt{2}$.

Since NMO and zero-offset migration both can be done residually, and DMO is essentially velocity independent, there seems to be no problem. But if we now undo the first migration and then migrate with the new velocity, we are applying the operator

$$\begin{aligned} res &= \mathbf{Mig_2}\ \mathbf{DMO_2}\ \mathbf{NMO_2}\ \mathbf{NMO_1^{-1}}\mathbf{DMO_1^{-1}}\ \mathbf{Mig_1^{-1}}\ \ data \\ &= \mathbf{Mig_2}\ \mathbf{DMO_2}\ \mathbf{NMO_{12}}\ \mathbf{DMO_1^{-1}}\ \mathbf{Mig_1^{-1}}\ \ data \end{aligned} \tag{2}$$

where $\mathbf{NMO_{12}}$=$\mathbf{NMO_2}\ \mathbf{NMO_1^{-1}}$. But NMO, DMO, and migration do not commute, so the same type of problem with non-commutative operators that we saw with the hybrid shot-geophone method above has arisen again in equation (2).

Pre-stack Stolt migration

The Stolt algorithm is based on a mapping from unmigrated frequency ω to migrated frequency k_τ:

$$k_\tau = \sqrt{\omega^2 - v^2 k_s^2} + \sqrt{\omega^2 - v^2 k_g^2} \quad . \tag{3}$$

Equation (3) can be solved for ω to get an equation for modeling:

$$\omega(k_\tau) = k_\tau \left[1 + \frac{v^2(k_g - k_s)^2}{4k_\tau^2}\right]^{1/2} \left[1 + \frac{v^2(k_g + k_s)^2}{4k_\tau^2}\right]^{1/2} \tag{4}$$

(Stolt, 1979).

We can then model at one velocity v_1, followed by migrating at a second velocity v_2 to derive a residual operator. Substituting equation (4) into equation (3) we get

$$\begin{aligned} k_{\tau_2} = & \left[k_\tau^2 \left(1 + \frac{v_1^2(k_g - k_s)^2}{4k_\tau^2}\right)\left(1 + \frac{v_1^2(k_g + k_s)^2}{4k_\tau^2}\right) - v_2^2 k_s^2\right]^{1/2} \\ & + \left[k_\tau^2 \left(1 + \frac{v_1^2(k_g - k_s)^2}{4k_\tau^2}\right)\left(1 + \frac{v_1^2(k_g + k_s)^2}{4k_\tau^2}\right) - v_2^2 k_g^2\right]^{1/2} \quad . \end{aligned} \tag{5}$$

Equation (5) tells us how to map from one migration to another. But there is a problem: it is not a migration equation!

To be a true prestack residual migration operator equation (5) would have to look like:

$$k_{\tau_2} = \sqrt{k_\tau^2 - v_r^2(v, v_2) k_s^2} + \sqrt{k_\tau^2 - v_r^2(v, v_2) k_s^2} \quad ,$$

where the residual velocity v_r is given by some general function $v_r(v, v_2)$. But equation (5) cannot be forced into such procrustean form. So there *does* indeed exist an operator that converts one prestack migration into another, but unlike the post-stack case, the requisite operator is *not* itself simply a migration operator using some residual velocity function.

Why then is the post-stack residual migration operator a migration, when it is not in the prestack case? The dispersion relation for the post-stack case is the single square root equation

$$k_\tau = \sqrt{\omega^2 - v^2 k_x^2} \quad .$$

If we write this in terms of squared variables it becomes a linear equation, and by simple substitution the result of modeling followed by migration is found to be

$$k_{\tau_2} = \sqrt{k_\tau^2 - (v_1^2 + v_2^2) k_x^2} \quad .$$

The more complicated and less satisfactory form of equation (5) arises because the double square root equation, unlike the single square root, allows no such simple linearization.

Kinematics

As a last approach, let us consider the kinematics of migrating a point diffractor for a zero-offset section, for a constant-offset section, and for a shot profile. Consider a single diffracting point at (x, z). If we forward model, we get a curve in (x, t). Now treat each point on this curve as a spike in the data that corresponds to an elliptical reflector in the earth and superpose these impulse responses. For comparison with the original data, the depth axis z for these superposed images can be converted back into time t. Where the curves constructively add (i.e., the stationary phase path) a residual operator is defined; this tangent curve would be the summation path used for a space-time integral implementation of such an operator.

Figure 3 shows the zero-offset case. The diffraction curve is the hyperbola

$$t_1 = \sqrt{t_0^2 + \frac{4(x_1 - x_0)^2}{v_1^2}} \quad ,$$

if the diffractor is at (x_0, t_0). Each point along this hyperbola now corresponds to the semi-circular mirror in the earth:

$$t_2 = \sqrt{t_1^2 - \frac{4(x_2 - x_1)^2}{v_2^2}} \quad .$$

Figure 3(a) shows diffraction and migration at the same velocity; the energy is focused at the apex of the hyperbola. Figure 3(b) shows migration with a velocity that is about 7% too low. The superposition of the semi-circles for a sequence of points along the diffraction hyperbola now outlines the residual migration operator, which is itself hyperbolic as expected. For comparison, the computed residual migration hyperbola is also shown.

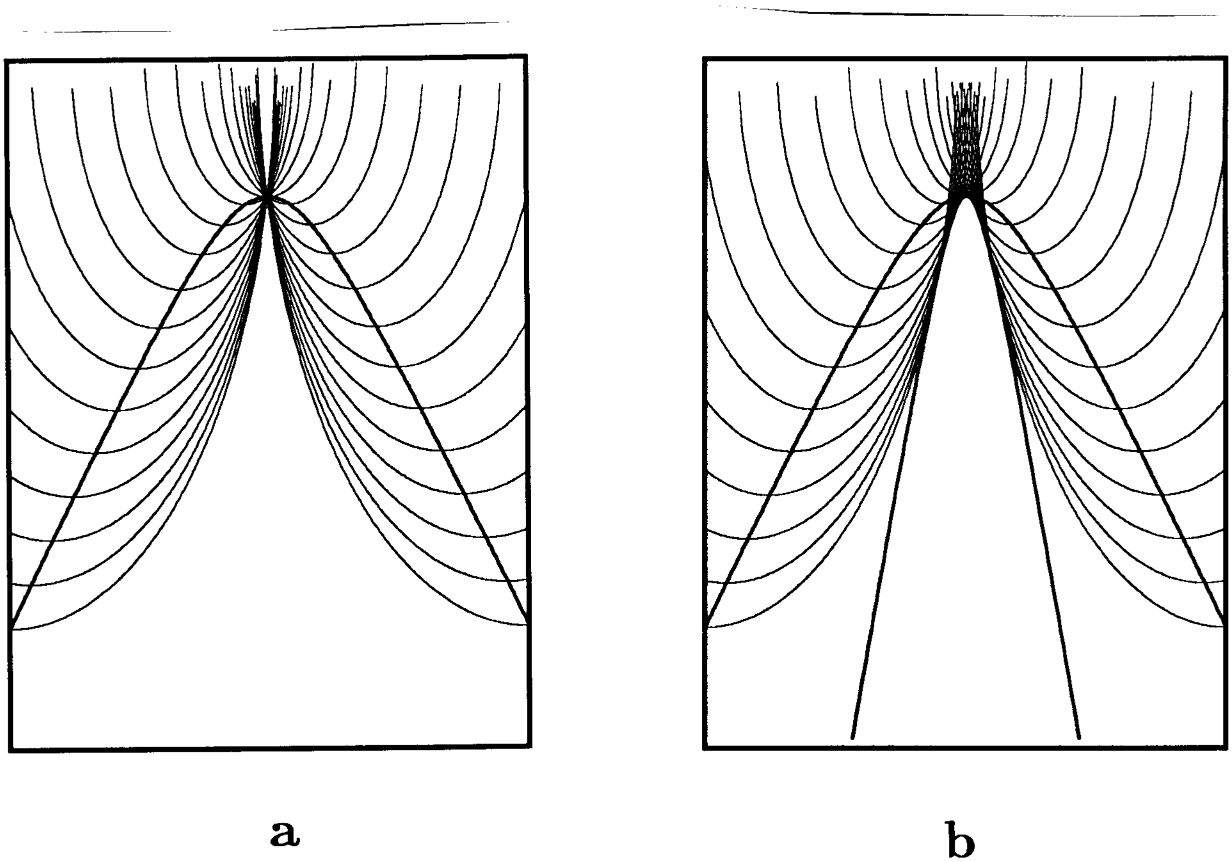


FIG. 3. (a) The travel-time curve for a diffractor in a zero-offset section is a hyperbola. Superposing semi-circles that represent the impulse responses of points along the hyperbola focuses the energy to the hyperbola apex. (b) The same diffraction hyperbola as in (a), but the migration semi-circles are now at a velocity 7% too low. The inner hyperbola is the residual migration operator that is tangent to all the semi-circles.

Figure 4 illustrates diffraction and migration for a constant (non-zero) offset section. The diffraction curve is now flattened on top:

$$t_1 = \frac{1}{2}\left[\sqrt{t_0^2 + \frac{4(y_1 - y_0 + h)^2}{v_1^2}} + \sqrt{t_0^2 + \frac{4(y_1 - y_0 - h)^2}{v_1^2}}\,\right] \quad ,$$

where y is midpoint, h is offset, and the diffractor is at (y_0, t_0). The migration ellipses are given by

$$t_2 = t_1\sqrt{\left[1 - \frac{4h^2}{v_2^2 t_1^2}\right]\left[1 - \frac{4(y_2 - y_1)^2}{v_2^2 t_1^2}\right]} \quad .$$

Figure 4(a) shows diffraction and migration at the same velocity. The energy focuses to a point that correctly represents the zero-offset location of the diffractor. In Figure 4(b) the migration velocity is 7% lower than the diffraction velocity. The stationary phase curve

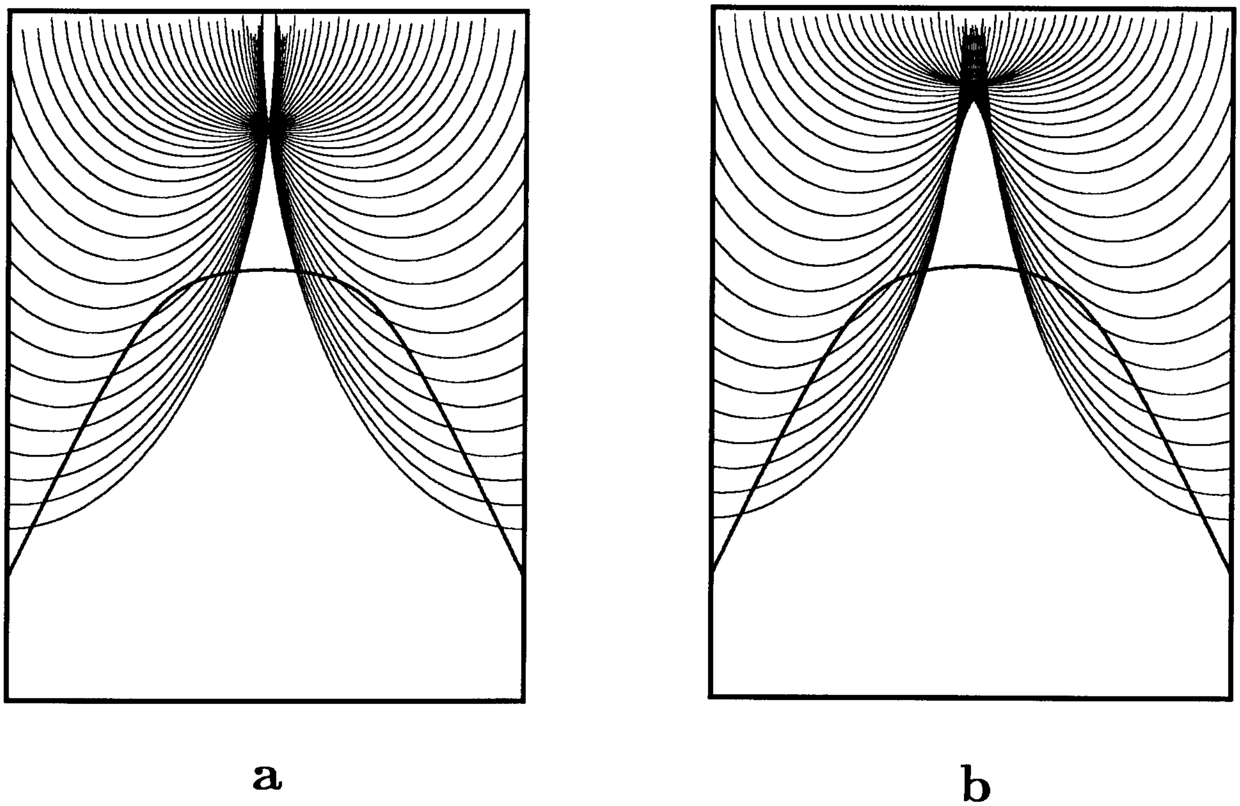


FIG. 4. (a) The travel-time curve for a diffractor in a constant-offset section is flat-topped. Superposing ellipses that represent the impulse responses of points along the diffraction curve focuses the energy to the (zero-offset) diffractor location. (b) The same diffraction curve as in (a), but the migration ellipses are now at a velocity 7% too low. The resulting stationary phase path is not a flat-topped diffraction curve.

is now clearly not a flat-topped diffraction hyperbola (indeed, it appears to triplicate), so the residual migration operator is not a migration.

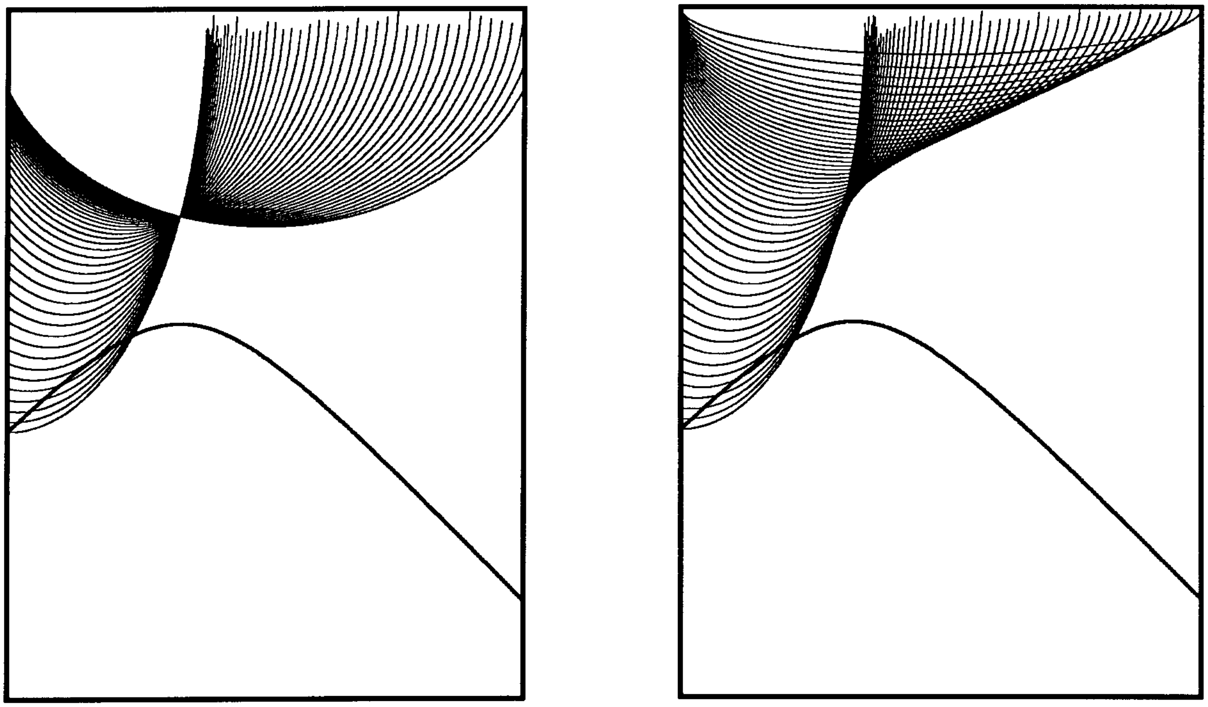
Figure 5 shows diffraction and migration for a shot profile. If the shot is at the origin and the diffractor is at (x_0, t_0), the diffraction curve is an offset hyperbola:

$$t_1 = \frac{1}{2}\left[\sqrt{t_0^2 + \frac{4(g_1 - x_0)^2}{v_1^2}} + \sqrt{t_0^2 + \frac{4x_0^2}{v_1^2}}\right] ,$$

where g is geophone offset. The migration ellipses are given by

$$t_2 = t_1\sqrt{\left[1 - \frac{g_1^2}{v_2^2 t_1^2}\right]\left[1 - \frac{(2g_2 - g_1)^2}{v_2^2 t_1^2}\right]} .$$

Figure 5(a) shows diffraction and migration at the same velocity. The energy focuses to a point that correctly represents location of the diffractor above the apex of the diffraction



a **b**

FIG. 5. (a) The travel-time curve for a diffractor in a shot-profile is a hyperbola. Superposing ellipses that represent the impulse responses of points along the hyperbola focuses the energy to the diffractor location. (b) The same diffraction hyperbola as in (a), but the migration ellipses are now at a velocity 7% too low. The resulting stationary phase path is not a hyperbola.

hyperbola. In Figure 5(b) the migration velocity is again 7% too low. The stationary phase curve is obviously not a hyperbola; again, we conclude that the residual migration operator is not itself a migration.

CONCLUSIONS

We have seen that two successive prestack migrations cannot be equivalent to a single migration. In other words, prestack migration cannot be partitioned into an iterative or residual migration process. For the methods we considered, this infeasibility appeared to arise from non-commutativity of various operators or from the nonlinearity of the double square root equation, but the conclusion is independent of the particular algorithm used.

A residual operator that takes one prestack migration into another can be defined by diffracting at the first velocity followed by migrating at the second. We have derived a frequency-wavenumber expression for such an operator, and illustrated the stationary phase curves for constant-offset sections and for shot profiles. The residual operator, however, is not itself a migration, and we have not explored possible finite-difference or space-time domain integral implementations.

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From dan%mit-erl.uucp@mit-eddie Wed Sep 10 17:28:38 1986
Received: from EDDIE (mit-eddie.arpa) by gregorio.stanford.edu with Sendmail; Wed,
Received: by EDDIE (5.31/4.7) id AA04250; Wed, 10 Sep 86 17:34:22 EDT
Message-Id: <8609102134.AA04250@EDDIE>
Received: from mit-ice.ARPA by mit-erl AA14839; Wed, 10 Sep 86 17:35:49 edt
Date: Wed, 10 Sep 86 17:37:26 edt
From: Daniel Rothman <dan%mit-erl.uucp@mit-eddie>
To: joe%hanauma@gregorio.stanford.edu, stew%hanauma@gregorio.stanford.edu
Status: RO

I found the problem with the self doc.
It is I think a compiler bug on the Alliant.
The problem is that the if((int) sntfEkd) statement always registered false
(I guess the alliant doesnt like integer casting of static doubles or
something like that) so the call to doc never got made, despite the fact
that noheader() = redout() = 0.
If I may, I might add that the if((int) snftEkd)) statement doesnt
strike me as the neatest way to shut off the pi complaint,
but then who am I to complain!
My fix was to just take the statement out -- at this point I don't care
if lint complains about pi, which I guess was why you did that.
```



```
  setbuf(stdin,(char *) NULL);  /* force unbuffered so data can be read
                                 * from in back of headers
                                 */
  if((int) snftEkd ) {  /* used to shut off complaint that pi was unused! */
/* Check to see if the program is one that has noheader=y,
 * So we shouldn't expect standard input to be redirected.
 * Then, if standard out is redirected, this must mean that
 * we shouldn't self-document.
 */
```