Slant stacks from velocity stacks

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Recently, data processing in hyperbolic stack coordinates (velocity stacks) has been an area of intense SEP research (see appendix). There are many analogies to processing in linear stack coordinates (slant stacks). Thorson (1983) and Beylkin (1982) established that both linear and hyperbolic stacks were invertible transforms and special cases of the generalized Radon integral transform. This paper closes the triangle with the direct transformation between linear and hyperbolic stacks.

Algorithms in linear stack coordinates are often make fewer approximations than those in hyperbolic stack coordinates. For example, linear stack migration (Ottolini 1983) theoretically handles all vertical velocity variations and slow lateral velocity variations while hyperbolic stack migration (Fowler 1983) is valid where the root-mean-squared velocity approximation holds. This may be partly due to that we have been studying linear stacks longer and partly due to that hyperbolic stacks are quadratic. On the other hand, because hyperbolic stacking trajectories more closely follow stepouts in seismic data than do linear stacking trajectories, hyperbolic stacks tend to be more resistant to numerical artifacts caused by undersampling of seismic data in the spatial dimensions. Therefore, some data processing algorithms may be improved by analyzing them in linear and hyperbolic stack coordinates together. This paper gives an example of improving the quality of slant stacks by first constructing velocity stacks.

Coordinate transforms

The following figure summarizes the adjoint transforms between data, linear stacks, and hyperbolic stacks coordinates. These transforms only account for the kinematics. The must be added to insure correct amplitude and phase, particularly when cascading transforms together.

Slant stacks from velocity stacks

There are two paths from data to linear stack coordinates shown in the above figure—direct transform, and transform via hyperbolic stacks. Figure 2 demonstrates that both paths are kinematically identical on a synthetic dataset. Figure 3 shows slant stacks of a 24-fold Western Geophysical Gulf of Mexico obtained by each path. The hyperbolic slant stack seems more resistant to artifacts. In addition, the signal to noise could be enhanced while in hyperbolic stack

Linear Stacks $u_p(\tau,p)$

$$u_p(\tau,p) = \int dv \; u_v\left(\frac{\tau}{\sqrt{1-p^2v^2}}, v\right)$$

$$u_v(t_0,p) = \int dp \; u_p\left(t_0\sqrt{1-p^2v^2}, p\right)$$

Hyperbolic Stacks $u_v(t_0,v)$

$$u_d(t_h,x) = \int dp \; u_p(t_h - px, p)$$

$$u_p(\tau,p) = \int dx \; u_d(\tau + px, x)$$

Data $u_d(t_h,x)$

$$u_v(t_0,v) = \int dx \; u_d\left(\sqrt{t_0 + x^2/v^2}, x\right)$$

$$u_d(t_h,x) = \int dv \; u_v\left(\sqrt{t_h - x^2/v^2}, v\right)$$

FIG. 1. Equations for transforming between data, linear stack, and hyperbolic stack coordinates.

coordinates (though it wasn't done in these examples). According to Thorson's and Harlan's SEP theses, signal detection is more robust when signal has been Gaussianity-decreasing transform has been applied. This is generally the case for the hyperbolic stacking of hyperbolically moved out events. The velocity filtering slant stack enhancement method of Schultz and the semblence enhancement method of Stoffa are antecedents of hyperbolic stack space enhancement of this paper. The drawback of the former methods is that they assume a single earth velocity model while hyperbolic stacks include *all* feasible earth velocity models.

Appendix: Data processing in linear and hyperbolic stack coordinates

Table 1 lists various seismic data processes in linear and hyperbolic stack coordinates with the first published reference (that I am aware of). The references listed are necessarily the most definitive study. There is some understanding of how to perform most seismic processing in both coordinate systems. Hyperbolic stack processing is less mature than linear stack processing. Also, there may not be enough improvement in results or efficiency to be worth the trouble of transforming into a new coordinate system for a given process.

Table 1: Processing Algorithms

Algorithm	*Linear Stacks*	*Hyperbolic Stacks*
forward transform	Rieber 36	Taner 69
inverse transform	Thorson 78	Beylkin 82
filtering	Phinney 81	Thorson 83
missing data	Ottolini 81	Thorson 83
multiple decon	Morley 78	Claerbout 86
migration	Rybinkin 62	Fowler 83
velocity analysis	Schultz 76	Taner 69
statics	Schultz 74	Toldi 82
interpretive sections	Schultz 76	unknown

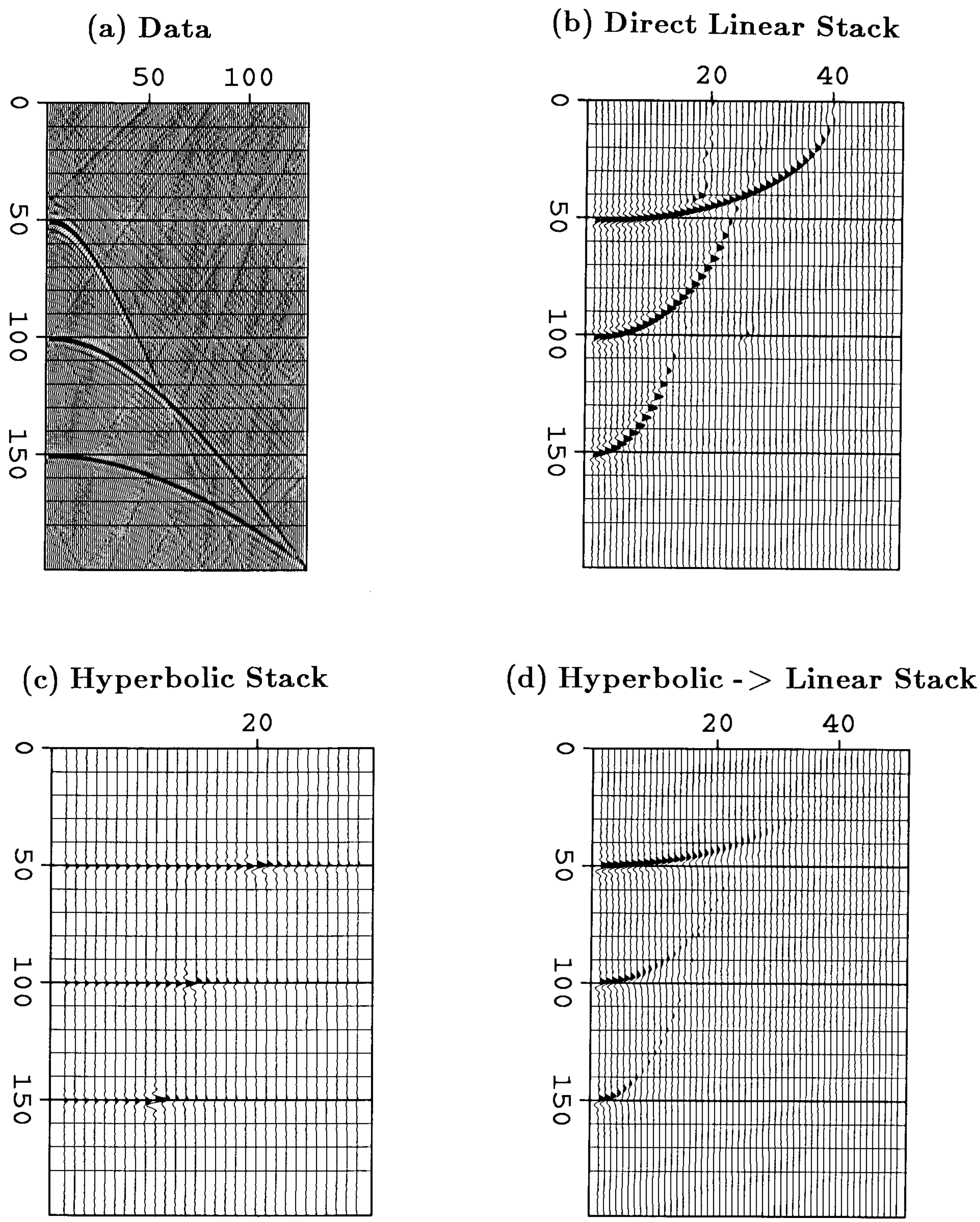

FIG. 2. Two slant stack results using the input data (a). (b) is the regular slant stack, while the slant stack of (d) had first been transformed into velocity stacks (c).

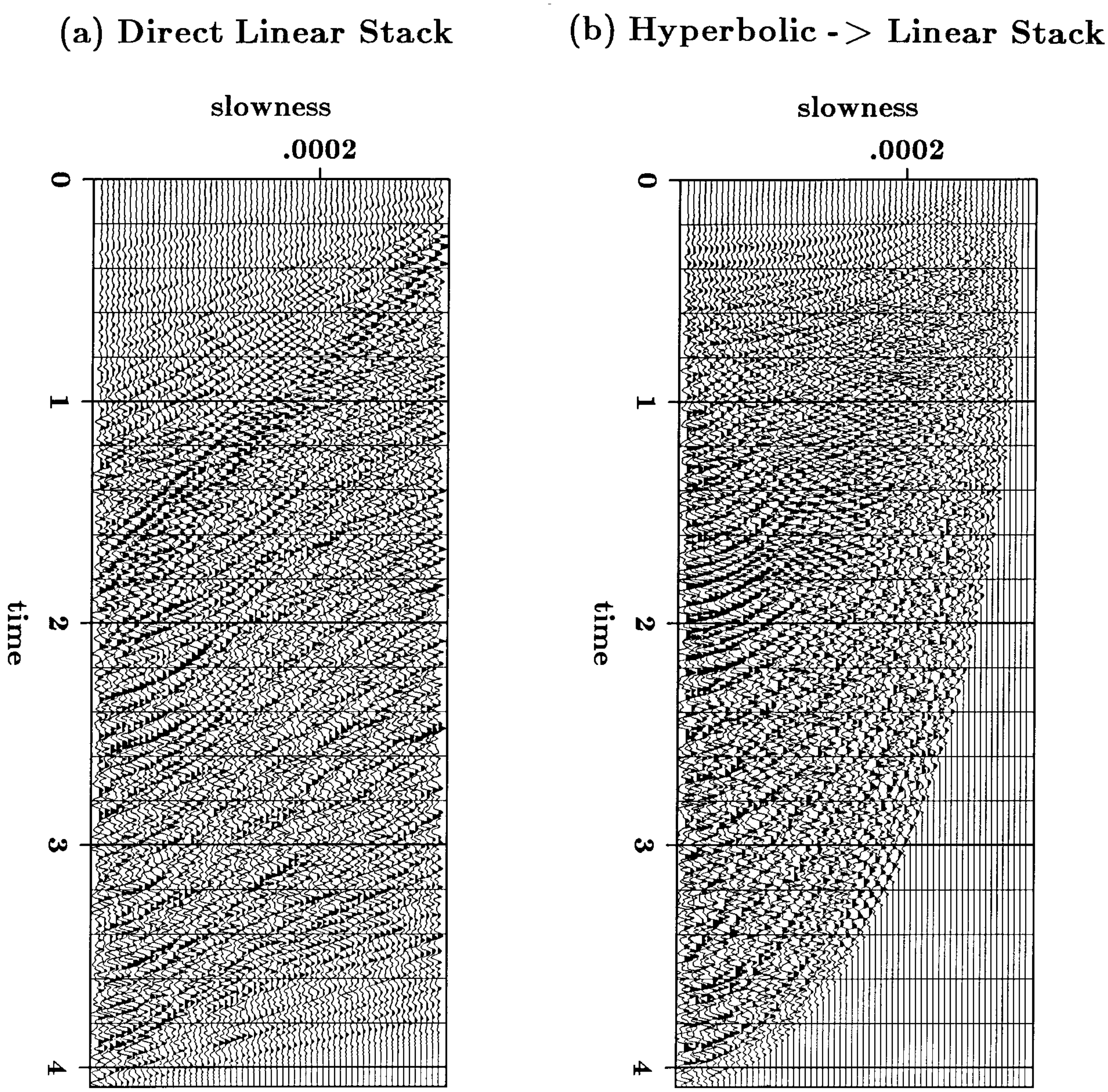

FIG. 3. Slant stacks of a 24-fold Western Geophysical gather from the Gulf of Mexico. (a) is the regular slant stack, while (b) is the hyperbolic slant stack.

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