

## Inversion of seismic data in the Fourier domain and prestack Stolt migration

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### ABSTRACT

Born inversion in the Fourier domain produces results similar to prestack Stolt migration, at least for constant background (migration) velocity. The two methods are compared and some simple results are shown. Finally, some approaches to an inversion scheme that combines Born inversion, a high frequency method, with low frequency methods are described.

### INTRODUCTION

The past years inversion methods for seismic reflection data have been presented in two forms: Cohen and Bleistein (1979) found a analytical relation between data and model, allowing for a direct inversion and Tarantola (1984) defined an iterative method that gives the "best" model according to some predetermined norm. Both methods are based on linearization of the forward modeling, the so-called first order Born approximation, and are set up in the time-space domain. Clayton and Stolt (1981) proposed a Born inversion scheme in the Fourier domain and, just like Tarantola's method, their inversion procedure can be carried out in an iterative fashion. Frequency methods have the main advantage that they are computationally fast but their disadvantage is that they are not flexible with respect to velocity variations. In particular, the Born inversion in the Fourier domain that will be presented here assumes a constant background velocity and gives only small perturbations from the reference velocity. However, the method should be extendable to variable background velocity (Clayton and Stolt (1981)). Furthermore, Born inversion is a high frequency approximation.

Ideally, other methods that model the low frequency content of the seismic data should first give information about the background velocity and then the Born inversion should perturb this velocity model to fit the high frequencies. I will describe the Born inversion in the Fourier domain, present some preliminary results and briefly address low frequency methods and their relation to velocity analysis.

## BORN INVERSION

In this section I will derive the formulae for the Born inversion in the Fourier domain, based on Clayton and Stolt (1981) but mainly following Tarantola (1984) and Ikelle, Diet and Tarantola (1985). Unlike Clayton and Stolt (1981), only perturbations in velocity are inverted for, because density and bulk modulus perturbations are hard to resolve simultaneously (Tarantola et al. (1985)) and because density is often assumed to be constant anyway.

### Forward modeling

The acoustic wave equation for a homogeneous, two dimensional medium with a driving force  $s(x, z, \omega)$  is:

$$(\nabla^2 - \frac{\omega^2}{v_r^2})P_r(x, z, \omega) = s(x, z, \omega) \quad (1)$$

where  $P_r(x, z, \omega)$  is the pressure field and  $v_r$  the velocity. The subscript r stands for reference; its significance will become apparent in following equations. The solution to this equation can be expressed in terms of the Green's function  $G_r(x, z|x', z'; \omega)$  (see appendix):

$$P_r(x, z, \omega) = \int dx' \int dz' G_r(x, z|x', z'; \omega) s(x', z', \omega) \quad (2)$$

Assuming the driving force to be a point source at  $x_s, z_s$ :

$$s(x, z, \omega) = \delta(x - x_s)\delta(z - z_s)S(\omega) \quad (3)$$

where  $S(\omega)$  is the source spectrum, we get:

$$P_r(x, z, x_s, z_s, \omega) = G_r(x, z|x_s, z_s; \omega)S(\omega) \quad (4)$$

When the velocity is perturbed by  $\delta v(x, z)$ , the resulting pressure wave field is defined as the sum of the reference wave field  $P_r(x, z, \omega)$ , corresponding to the direct wave, and a scattered wave field  $\delta P(x, z, \omega)$ . We have

$$(\nabla^2 - \frac{\omega^2}{(v_r + \delta v)^2})(P_r(x, z, \omega) + \delta P(x, z, \omega)) = s(x, z, \omega) \quad (5)$$

Neglecting second and higher order perturbations (thus effectively performing a first order Born approximation) and using (1), we get for the scattered wave field:

$$(\nabla^2 - \frac{\omega^2}{v_r^2})\delta P(x, z, \omega) = \frac{2\omega^2 \delta v(x, z)}{v_r^3} P_r(x, z, \omega) \quad (6)$$

This means that in the Born approximation the scattered wave field is the solution of the reference wave equation with a source term consisting of the scattered direct wave. Therefore, like (2) the solution can be expressed in terms of the reference Green's function:

$$\delta P(x, z, \omega) = \int dx' \int dz' G_r(x, z|x', z'; \omega) \frac{2\omega^2 \delta v(x', z')}{v_r^3} P_r(x', z', \omega) \quad (7)$$

For data recorded at geophone position  $x_g, z_g = 0$  and a source at  $x_s, z_s = 0$  this becomes (using (4)):

$$\delta P(x_g, z_g = 0, x_s, z_s = 0, \omega) = \int dx' \int dz' G_r(x_g, 0 | x', z'; \omega) \frac{2\omega^2 \delta v(x', z')}{v_r^3} G_r(x', z' | x_s, 0; \omega) S(\omega) \quad (8)$$

Fourier transforming over  $x_s$  and  $x_g$  and substituting the expression for the Green's function (see appendix) gives:

$$\delta P(k_g, 0, k_s, 0, \omega) = \int dx' \int dz' - \frac{2\pi^2 \omega^2 \delta v(x', z') S(\omega)}{k_{z,g} k_{z,s} v_r^3} \exp[i(k_{z,g} + k_{z,s})z' + i(k_g + k_s)x'] \quad (9)$$

where:

$$\begin{aligned} k_{z,g} &= \sqrt{\frac{\omega^2}{v_r^2} - k_g^2} \\ k_{z,s} &= \sqrt{\frac{\omega^2}{v_r^2} - k_s^2} \end{aligned} \quad (10)$$

Note that the roots in the above equations have to have positive arguments to assure imaginary exponentials in (9). In one of the next sections I will discuss this matter in detail. The right hand side of expression (9) is recognizable as a Fourier transform:

$$\delta P(k_g, 0, k_s, 0, \omega) = - \frac{2\pi^2 \omega^2 S(\omega)}{k_{z,g} k_{z,s} v_r^3} \delta v(k_g + k_s, k_{z,g} + k_{z,s}) \quad (11)$$

We want to express the velocity variations as a function of the coordinates of our two dimensional earth model and therefore we first change to midpoint(m), offset(h) coordinates:

$$\begin{aligned} x_m &= \frac{x_g + x_s}{2}; & k_m &= k_g + k_s \\ x_h &= \frac{x_g - x_s}{2}; & k_h &= k_g - k_s \end{aligned} \quad (12)$$

and then define a new variable  $k_z$ :

$$k_z = k_{z,g} + k_{z,s} = \frac{\omega}{v_r} \left[ \sqrt{1 - \frac{v_r^2 k_g^2}{\omega^2}} + \sqrt{1 - \frac{v_r^2 k_s^2}{\omega^2}} \right] \quad (13)$$

In this expression  $\omega$  has been factored out to show that  $\omega$  and  $k_z$  have the same sign. Equation (13) defines the dispersion relation:

$$\omega = \omega(k_m, k_h, k_z) = \frac{v_r k_z}{2} \sqrt{\left(1 + \frac{k_m^2}{k_z^2}\right) \left(1 + \frac{k_h^2}{k_z^2}\right)} \quad (14)$$

It seems as if there is no restriction on  $\omega$  in this equation (the argument of the root is always positive). However, nonevanescence imposes an implicit constraint on  $k_z$  (see equation (21)) and

it is this limitation that restricts the range of frequencies. Relation (14) can be used to write the perturbed wave field  $\delta P$  as a function of  $k_m, k_h, k_z$  and after some algebra it can be shown that:<sup>1</sup>

$$\delta P(k_m, k_h, k_z) = A(k_m, k_h, k_z) \delta v(k_m, k_z) S(\omega) \quad (15)$$

where:

$$A(k_m, k_h, k_z) = -\frac{2\pi^2 (k_z^2 + k_h^2)(k_z^2 + k_m^2)}{v_r (k_z^4 - k_m^2 k_h^2)} \quad (16)$$

The coefficient  $A(k_m, k_h, k_z)$  is similar to  $A_1(k_m, k_h, k_z)$  of Clayton and Stolt (1981). The only difference between equation (15) and their equation (34) is a factor  $8\pi^2/\rho_r v_r$  because Clayton and Stolt (1981) use a slightly different definition of the Green's function and invert for bulk modulus instead of velocity. In first order approximation, bulk modulus and velocity perturbations are linearly related. The change of coordinates from  $\omega$  to  $k_z$  is in practise achieved by interpolating the discrete wave field  $\delta P(k_m, k_h, \omega)$  onto a new mesh  $(k_m, k_h, k_z)$ .

To invert equation (15), we have to remove the source effect and therefore define:

$$\delta P'(k_m, k_h, k_z) = \frac{\delta P(k_m, k_h, k_z)}{S(\omega)} = A(k_m, k_h, k_z) \delta v(k_m, k_z) \quad (17)$$

### Inversion scheme

The relation between data and model (equation (17)) allows us to set up an inversion scheme. When we define the following optimization function:

$$F(\delta v) = \|\delta P' - A\delta v\|_2 + \epsilon \|\delta v\|_2 \quad (18)$$

where  $\|\cdot\|_2$  defines the Euclidian (L2) norm and  $\epsilon$  is a constant, the model that minimizes  $F$  is given by (see e.g. Ikelle et al. (1985)):

$$\delta v(k_m, k_z) = \frac{\int dk_h A(k_m, k_h, k_z) \delta P'(k_m, k_h, k_z) C_{\delta P'}^{-1}(k_m, k_h, k_z) \left| \frac{d\omega}{dk_z} \right|}{\epsilon C_{\delta v}^{-1}(k_m, k_z) + \int dk_h A^2(k_m, k_h, k_z) C_{\delta P'}^{-1}(k_m, k_h, k_z) \left| \frac{d\omega}{dk_z} \right|} \quad (19)$$

with:

$$\left| \frac{d\omega}{dk_z} \right| = -\pi^2 \sqrt{\left(1 + \frac{k_m^2}{k_z^2}\right) \left(1 + \frac{k_h^2}{k_z^2}\right)} \frac{1}{A(k_m, k_h, k_z)} \quad (20)$$

where  $\frac{d\omega}{dk_z}$ , the Jacobian, enters because coordinates change from  $\omega$  to  $k_z$ .  $C_{\delta P'}$  and  $C_{\delta v}$  are the diagonal elements of the covariance matrices of  $\delta P'$  and  $\delta v$ . The off-diagonal elements are assumed to be zero, corresponding to uncorrelated noise in the data and independent model parameters. In practise,  $C_{\delta P'}$  and  $C_{\delta v}$  are chosen to be constant and under this assumption the result is identical to a classical damped least squares solution where  $\epsilon$  is the damping constant.

<sup>1</sup>this expression normally means: "after major computations using several, for the reader not so obvious, algebraic tricks." However, here the algebra is pretty straightforward.

### Evanescent zone

The Born inversion as derived above is only valid in the nonevanescence zone, although the reconstructed earth's image spans the complete  $k_m, k_z$ -space (as commented by Stolt (1984)). The condition of nonevanescence requires the arguments of the two square roots in (10) to be positive. This can be translated into one implicit constraint on  $k_z$ :

$$|k_z| > |k_m k_h| \quad (21)$$

The above condition introduces two computational artifacts in the inversion scheme.

First, the coefficient  $A$  blows up near the evanescent zone (the denominator in (16) gets infinitesimally small). To avoid this, I have added a small constant to the denominator.

Furthermore, by imposing the above condition on  $k_m, k_h, k_z$  the summations in expression (19) get truncated in the Fourier domain, introducing artifacts in the space domain. My suggestion is to let the solution exponentially decay near the evanescent zone, instead of truncating the sum.

## PRESTACK STOLT MIGRATION COMPARED TO BORN INVERSION

Prestack Stolt migration (Stolt (1978)) can shortly be described as follows:

first, the recorded data (at  $z=0$ ) are Fourier transformed over time, offset and midpoint resulting in  $\delta P = \delta P(k_m, k_h, \omega, z = 0)^2$ . Then, the shots and geophones are downward continued:

$$\delta P(k_m, k_h, \omega, z) = \delta P(k_m, k_h, \omega, z = 0) \exp[-ik_z z] \quad (22)$$

where  $k_z$  is defined as in equation (13). The next step is imaging at  $t = 0, h = 0$  and we get for the migrated image  $M_{\delta P}$ :

$$M_{\delta P}(k_m, z) = M_{\delta P}(k_m, h = 0, t = 0, z) = \int dk_h \int d\omega \delta P(k_m, k_h, \omega, z = 0) \exp[-ik_z z] \quad (23)$$

Converting the integral in  $\omega$  to one over  $k_z$ , equation (23) can be rewritten as:

$$M_{\delta P}(k_m, z) = \int dk_h \int dk_z \left| \frac{d\omega}{dk_z} \right| \delta P(k_m, k_h, k_z) \exp[-ik_z z] \quad (24)$$

Substituting the expression for the Jacobian (equation (20)), we get:

$$M_{\delta P}(k_m, z) = \int dk_h \int dk_z \cdot -\pi^2 \sqrt{\left(1 + \frac{k_m^2}{k_z^2}\right) \left(1 + \frac{k_h^2}{k_z^2}\right)} \frac{\delta P(k_m, k_h, k_z)}{A(k_m, k_h, k_z)} \exp[-ik_z z] \quad (25)$$

Now we use equations (17) and (14) to write:

$$M_{\delta P}(k_m, z) = \int dk_h \int dk_z \cdot -\frac{2\pi^2 \omega}{v_r k_z} \delta v(k_m, k_z) S(\omega) \exp[-ik_z z] \quad (26)$$

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<sup>2</sup>I will write the recorded (reflected) field as  $\delta P$ . This may be confusing to some who are used to classical papers that use  $P$  as notation for the recorded field but it must be remembered that here  $P$  stands for the direct wave.

When we now change the data to:

$$\delta D(k_m, k_h, \omega) = -\frac{v_r k_z}{2\pi^2 \omega S(\omega)} \delta P(k_m, k_h, \omega) \quad (27)$$

the result of prestack Stolt migration on  $\delta D$  is:

$$M_{\delta D}(k_m, z) = \int dk_h \int dk_z \delta v(k_m, k_z) \exp[-ik_z z] = \text{constant} \cdot \delta v(k_m, z) \quad (28)$$

The migrated image represents the velocity perturbation model and this shows that, at least for the constant background velocity case, Born inversion can be performed as a prestack Stolt migration with modified data. Cheng and Coen (1984) find a similar result but their transformation of the data is different from the one presented here. However, their result applies to poststack data.

## RESULTS

In this section some simple results of the Fourier domain Born inversion are presented.

First, the impulse response of Born inversion is compared with the impulse response of prestack Stolt migration. Figure 1 shows the result of a prestack Stolt migration.

The input time-offset gathers are zero for all midpoints, except for midpoints 320 and 480 m, where spikes are located at zero-offset. The positions of the spikes in time correspond to depths of 100, 200 m respectively, when the velocity is chosen to be 1250 m/s. The midpoint gathers consist of 64 traces, just as there are 64 midpoints. The shot and geophone spacing is 10 m, the shot geometry split-spread. The time sampling interval is 4 msec and a trace contains 256 time samples. The depth sample interval was chosen to be 2.5 m. All these parameters will be the same throughout the rest of this section.

I have not included the evanescent zone in computing the migrated image. Furthermore, the Jacobian has been left out as well because it didn't appear to have much influence on the result, neither in the migration algorithm nor in the inversion scheme (Fowler (pers. comm.) made the same observation for prestack Stolt migration). This seems to be in contrast with poststack migration where the Jacobian changes the result a great deal, particularly in the "tails" of the semicircle. Also, the data have not been zero padded. A sinc-interpolator has been used to regrid the data in the Fourier domain.

The same impulses were used as input for Born inversion and the result is shown in figure 2.

The most striking difference between the result of Stolt migration and that of Born inversion is that the low frequency "tails" have disappeared. As Born inversion is a high frequency approximation, this can be expected. For this inversion result I have used a relatively small value for  $\epsilon$  (as defined in (18)). Choosing a higher value, thus damping the inversion more heavily, the tails start to show up again and the result looks more like a Stolt migration (figure 3). Inspecting equation (19), we see that the influence of the coefficient  $A$  on the inversion result gets less

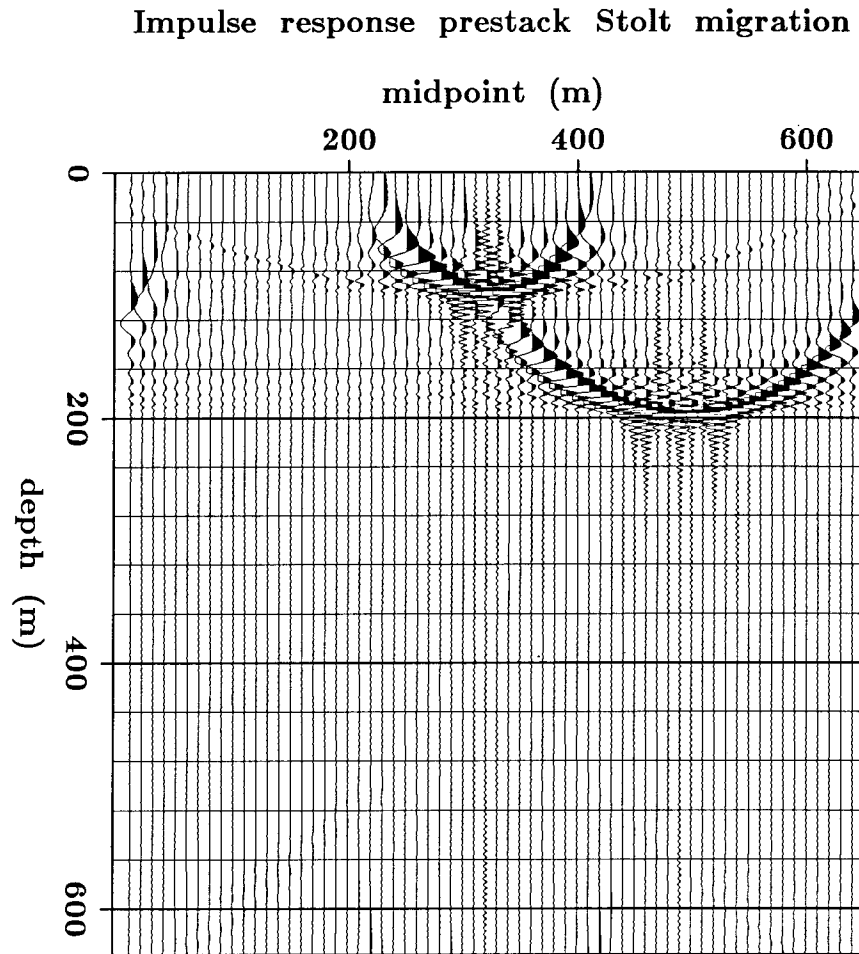


Figure 1: Impulse response of prestack Stolt migration. Input spikes are located at zero-offset at midpoints 320 and 480 m. Velocity is 1250 m/s. Note that the figure is plotted with a low clip-value to illustrate the artifacts.

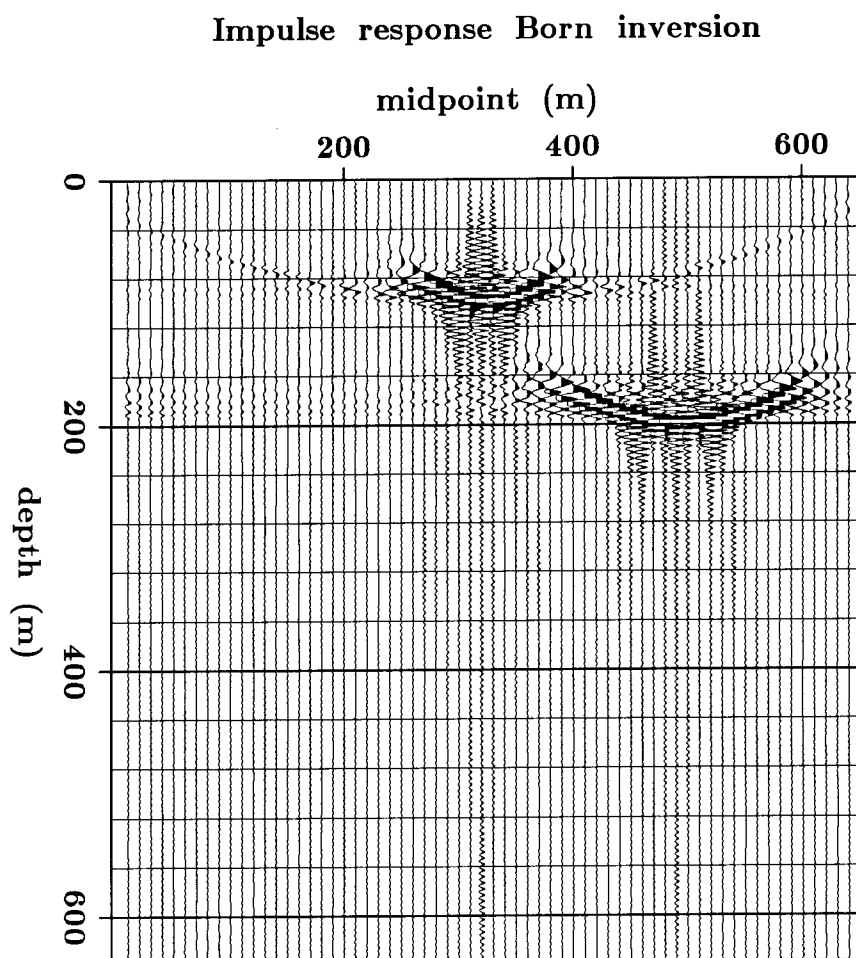


Figure 2: Impulse response of Fourier domain Born inversion. Input data and plotting parameters are the same as in figure 1.



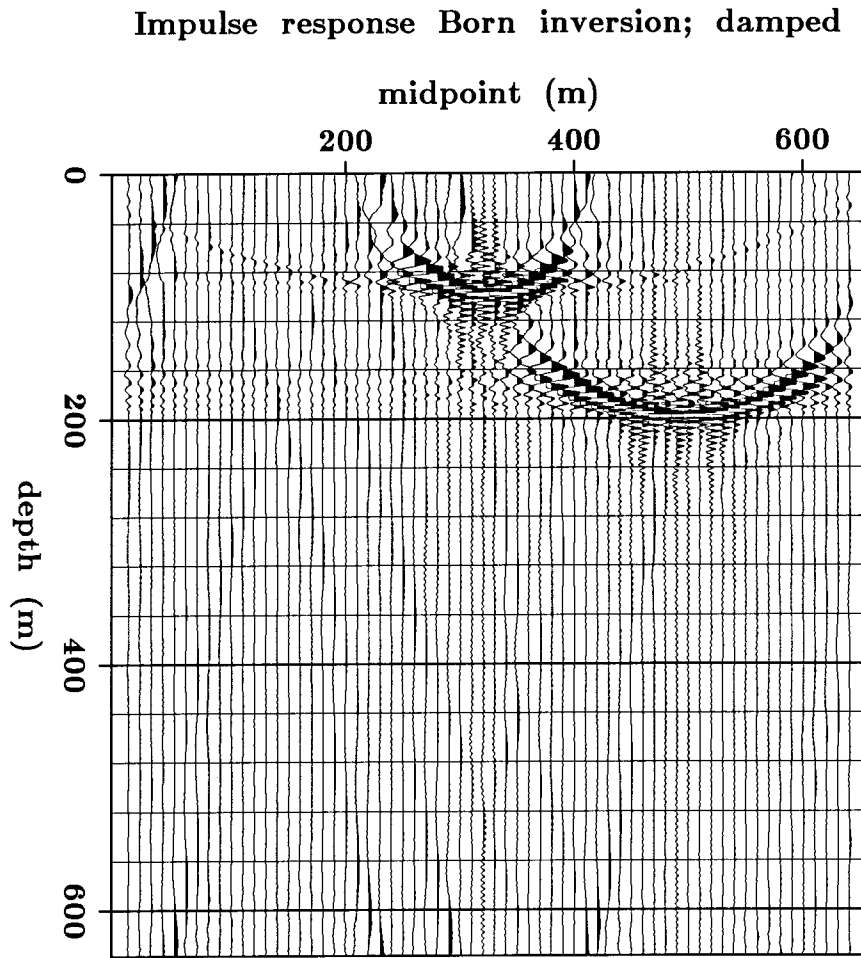


Figure 3: Impulse response of a strongly damped Fourier domain Born inversion. Input data and plotting parameters are the same as in figure 1.

pronounced for larger  $\epsilon$  and so the result approaches a prestack Stolt migration.

We now study the point scatterer response: if we have a spike in the midpoint-depth section, the response looks like Cheops' pyramid in the  $(m, h, t)$ -space (Claerbout (1985)). Figure 4 shows the zero- and far-offset slices through pyramids corresponding to spikes at the same location as in previous figures.

The pyramids were calculated using a simple modeling program that puts a wavelet at the appropriate arrival time and were then tapered to avoid truncation artifacts. The results of prestack Stolt migration and Born inversion are shown in figures 5 and 6.

Again we see that the Born inversion gives a high frequency response while the prestack Stolt migration result is dominated by low frequencies. In both the results of Born inversion and Stolt migration, the spike at midpoint 480 m is not symmetric because the corresponding input pyramid is not symmetric.

## LOW FREQUENCIES AND VELOCITY ANALYSIS

The Born inversion is a high frequency approximation and needs a priori velocity information. Velocity analysis is often referred to as zero frequency modeling because it gives a step-like velocity function. To make full use of the frequency content of our data we have to combine the Born inversion with low frequency methods and in this section I will speculate on some of these methods.

First, we can combine velocity analysis and Born inversion in one iterative inversion scheme: in each iteration, a velocity analysis is carried out and the result is used as background model for the Born inversion. The updated velocity model is used for the velocity analysis in the next iteration. In this scheme the velocity analysis has to be defined as some optimization procedure that improves a given starting model. An example of such a procedure is tomography (Sword (1985)). The combination of tomography and wave equation modeling is exactly what has been proposed by Stork and Clayton (1985), who reconstruct the earth's image by iterative tomography and migration. Another approach would be to apply traditional NMO and maximize the power of the stack.

The problem is that these methods are set up in the time-space domain. If they are to be combined with Fourier domain inversion, it means that every iteration the data have to be transformed back and forth to the Fourier domain. For prestack data, this is a costly operation. However, the Stolt migration operator performs a combination of hyperbolic moveout and stack and wave equation moveout can be defined in the frequency domain (see Thorson and Yedlin (1980)). Maximizing the stack in the time-space domain corresponds to maximizing the energy in the zero

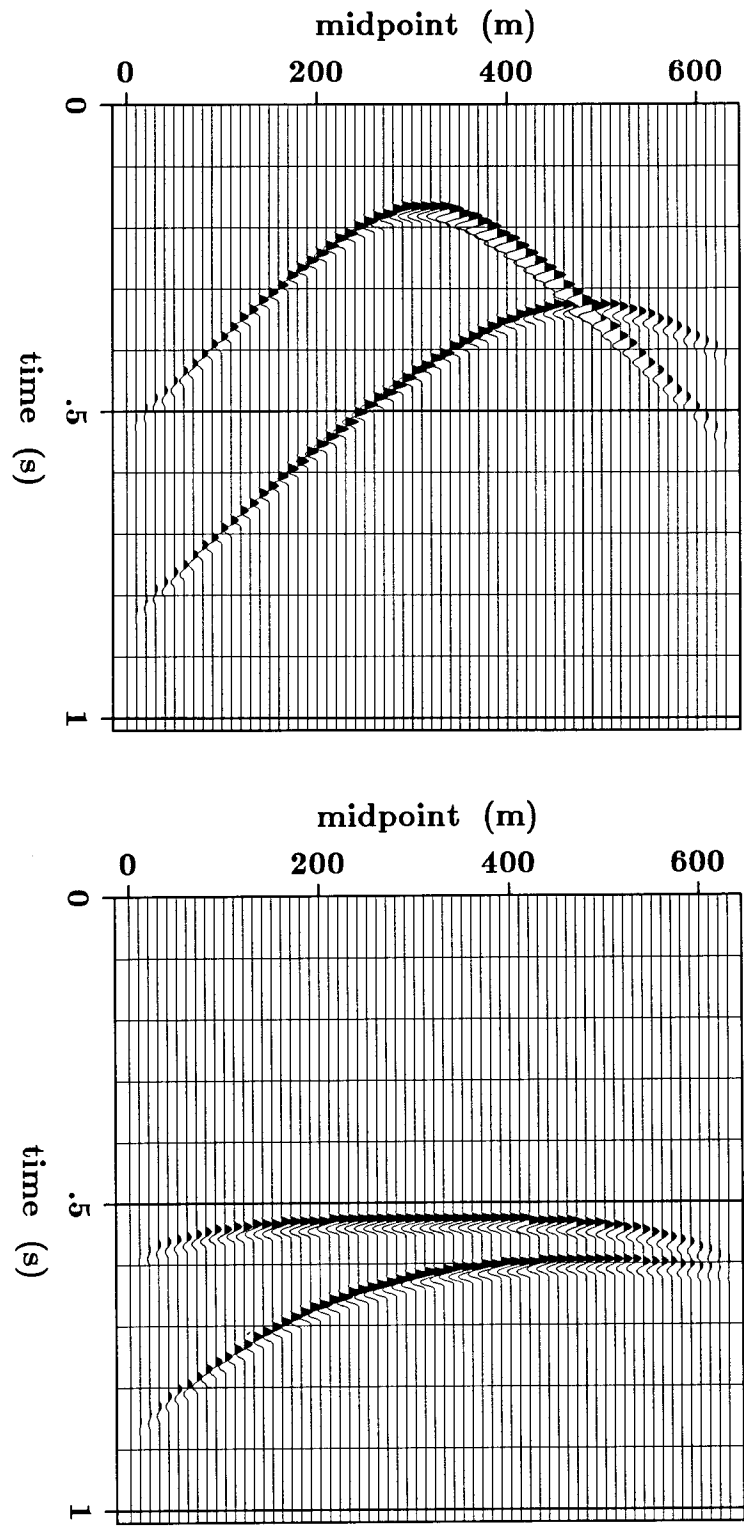


Figure 4: Point scatterer response; zero-offset section (*top*) and far-offset (310 m) section (*bottom*).

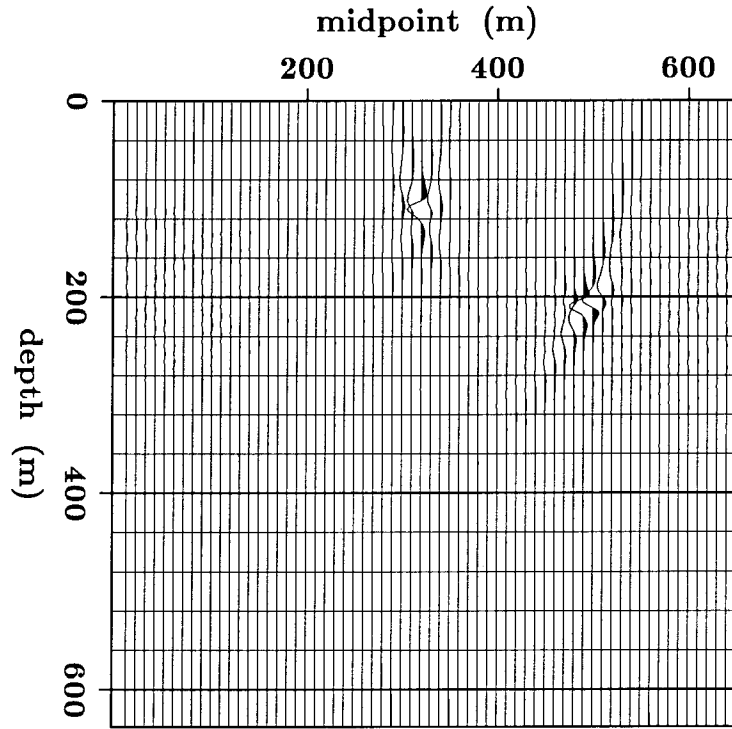


Figure 5: Result of prestack Stolt migration on Cheops' pyramids.

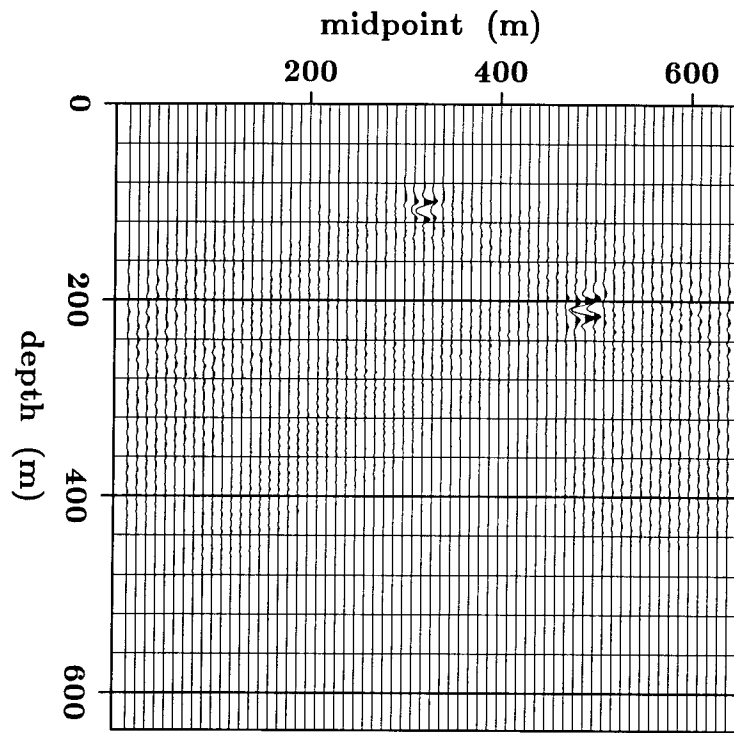


Figure 6: Result of Fourier domain Born inversion on Cheops' pyramids.

offset-wavenumber trace in the Fourier domain. This is a cheaper objective function than stacking over all the offsets and then maximizing the stack and it should be possible to incorporate the described procedure into an optimization scheme.

This brings us to the next approach to the problem. As the inversion in the Fourier domain is a relatively cheap method and moveout and stack is effectively performed by the inversion, it is possible to use the inversion itself to get (background) velocity information, in a way similar to Fowler (1985) and Al-Yahya (1986). They optimize migration to find the velocity model that migrates the data in the “best” way but use different objective functions to determine which model is the best.

I propose another objective function that can be better understood when we consider the following: assuming flat geometry and supposing the background velocity to be wrong, the data modeled by Born forward modeling look like hyperbolas with the wrong curvature and small velocity perturbations cannot change this for the better. This means that the norm as defined in (18) will have its minimum when the perturbations are zero (any non-zero perturbation will only increase  $\|\delta v\|_2$ , the modelnorm, and hardly change  $\|\delta P' - A\delta v\|_2$ , the datanorm).

We can now define an optimization scheme consisting of an outer and an inner loop. The objective in the outer loop is to maximize the velocity perturbations as function of the background velocity; the inner loop is a Born inversion as previously described. This may seem confusing as the Born inversion itself tries to *minimize* the sum of data- and modelnorm but we have to keep in mind that the outer loop deals with background velocities and low frequencies while the inner loop handles small velocity perturbations and high frequency variations.

How workable an objective function this is with respect to stability and sensitivity to local minima, remains to be seen and further research is needed to determine this.

Finally, it is possible to model the low frequency content of the data directly using normal mode theory. In this theory, so far only used in fundamental seismology, the spectrum of a seismic signal can be considered as a weighted sum of eigenfrequencies (normal modes). The values of the eigenfrequencies and the weights can be calculated for a given layered earth model. The advantage of this method is that it gives an independently determined velocity model and uses surface wave data that are now considered as noise in exploration geophysics. Note however that reflection data can be modeled as well by normal mode theory as long as enough eigenfrequencies are included in the spectrum (R. Geller (pers. comm.)). Indeed this is a necessity because surface waves only penetrate the top part of the subsurface.

In global seismology it is possible to invert the low frequency part of the data (using normal mode theory) and directly get a velocity model of the earth (Nolet, van Trier and Huisman (1986)).

The question is if the method is applicable in exploration geophysics and the main problem to be dealt with is the influence of near field terms on the solution.

### CONCLUSIONS

It has been shown that Fourier domain Born inversion produces results similar to prestack Stolt migration. Therefore, Born inversion should be applicable to more realistic examples than presented here and has indeed been applied to real data by Ikelle et al. (1985). However, the main advantage of Born inversion over Stolt migration is that it directly gives a physical parameter whereas the result of Stolt migration is hard to interpret in terms of earth quantities. The inversion scheme has to be tested further with respect to damping and the influence of the Jacobian and evanescent energy on the result. Also, the Born inversion should be extended to handle variable background velocity. Once variable background velocity can be incorporated for, an inversion scheme that models the complete frequency content of the data has to be set up.

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## APPENDIX: GREEN'S FUNCTION

The green's function is defined as the solution of the wave equation when the driving force is a point source at  $t = 0$  located at  $x_0, z_0$ :

$$(\nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2})G(x, z|x_0, z_0; t) = -\delta(x - x_0)\delta(z - z_0) \quad (\text{A1})$$

Using the following Fourier convention:

$$F(k_x, k_z, \omega) = \int dx \int dz \int dt f(x, z, t) \exp[ik_x x + ik_z z - i\omega t] \quad (\text{A2})$$

$$f(x, z, t) = \int dk_x \int dk_z \int d\omega F(k_x, k_z, \omega) \exp - [ik_x x + ik_z z - i\omega t] \quad (\text{A3})$$

we Fourier transform (A1) and find an expression for the Green's function in the Fourier domain:

$$G(k_x, k_z|x_0, z_0; \omega) = \frac{\exp[ik_z z_0 + ik_x x_0]}{(k_x^2 + k_z^2 - \frac{\omega^2}{v^2})} \quad (\text{A4})$$

In the  $k_x - z$  domain we obtain by inverse Fourier transforming:

$$G(k_x, z|x_0, z_0; \omega) = \int dk_z \frac{\exp[ik_z(z_0 - z) + ik_x x_0]}{(k_x^2 + k_z^2 - \frac{\omega^2}{v^2})} \quad (\text{A5})$$

The integral can be solved using the residues' theorem, thereby carefully choosing the contour to guarantee causality. We get two independent solutions that can be interpreted (for positive  $z$ ) as an imploding (reverse time) and exploding Green's function (Clayton and Stolt (1981)). We will only be concerned with the exploding one:

$$G(k_x, z|x_0, z_0; \omega) = \pi i \frac{\exp[ik_{z,x}(z_0 - z) + ik_x x_0]}{k_{z,x}} \quad (\text{A6})$$

with:

$$k_{z,x} = \sqrt{\frac{\omega^2}{v^2} - k_x^2} \quad (\text{A7})$$

An important property of the Green's function is based on reciprocity of source and geophone:

$$G(x, z|x_0, z_0; \omega) = G(x_0, z_0|x, z; \omega).$$



Jon Claerbout directs the Stanford Exploration Project

## Exploration Geophysics Society honors work of Jon F. Claerbout

The Society of Exploration Geophysics has conferred honorary membership upon Stanford Prof. Jon F. Claerbout to recognize his distinguished contributions to the field of geophysics. It is the third time the society has honored Claerbout.

The award was presented at the society's 55th annual meeting in Washington, D.C., Oct. 9. The citation praised Claerbout "for his considerable achievement in teaching us how to develop simple, universal, and automatic schemes for migrating seismic data."

Claerbout directs the Stanford Exploration Project, which began in 1973 and has since become a model for industrial affiliate programs at Stanford and at other universities.

The citation also mentioned Claerbout's active membership in John Kuo's International Affairs Committee, "which has a special concern in developing a closer working relationship with the Soviet Union" through the exchange of people and ideas between East and West.

The society conferred its Virgil Kauffman Gold Medal Award upon consulting Prof. William J. Ostrander, a Chevron geophysicist who has taught reflection seismology at Stanford for the last six years. Ostrander developed the angle dependent reflectivity method that aids geophysicists in the search for energy.