Test your migration IQ

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INTRODUCTION

The ten questions posed below came up in various discussions about post-stack migration, modeling, and conjugate-gradients during the last year. I found them challenging and learned several interesting things trying to answer them.

STOLT

Migration and modeling are best understood for the isotropic, constant-velocity earth model. Stolt's migration algorithm is so simple and transparent that I always wondered why Bill Harlan needed to formalize its properties in his three page SEP-35 article "Linear properties of Stolt migration and diffraction." There he worked out formulas for the transpose (adjoint) and least-squares inverse of Stolt migration. In the Fourier domain his formulas may be described as

	Operator	**Transpose**	
Migration	*NMO* $\cos\theta$	*INMO*	(1)
Diffraction	*INMO* $\sec\theta$	*NMO*	

where *NMO* represents the Stolt frequency downshift $\omega \rightarrow k_\tau$ with $\omega^2 = k_\tau^2 + v^2 k_x^2$ and $\cos\theta$ is the ratio k_τ/ω. *INMO* is the (least-squares) inverse *NMO* which zero fills the evanescent region.

1) Back in the time-space domain, Stolt migration
   a) preserves the amplitude of dipping events, or
   b) decreases the amplitudes of dipping events, or
   c) increases the amplitudes of dipping events?

2) True or False? This definition of diffraction is unbounded in the L^2 (root-mean-square) sense.

Jon Claerbout, in "What is the transpose?" (SEP-42) gave his own versions of Stolt migration and its transpose. These are

$$\begin{aligned} Stolt &= FT \rightarrow NMO \rightarrow FT^{-1} \\ Stolt^{T} &= FT \rightarrow NMO^{T} \rightarrow FT^{-1} \end{aligned} \tag{2}$$

with the conclusion that "... *the transpose to Stolt modeling is Stolt migration, provided that you forget the Jacobian, as I usually do.*" Comparing, we see that Jon has chosen to define Stolt migration as the transpose of Harlan's definition of diffraction.

3) The proper conclusion is that
   a) $NMO^{T} = INMO \sec\theta$, or
   b) one of them has made a mathematical error, or
   c) we're trying to comparing apples and oranges.

PHASE-SHIFT

Stolt's $NMO \cos\theta$ migration is derived by the change of variables $\omega \rightarrow k_{\tau}$ in the (constant-velocity) phase shift migration

$$M(k_x, \tau) = \frac{1}{\sqrt{2\pi}} \int e^{i\tau k_{\tau}} P(k_x, \omega) d\omega \tag{3}$$

after removing evanescent energy.

In SEP-42, Claerbout also showed that the transpose of this (i.e. INMO) is modeling by "upward continue and add":

$$\hat{P}(k_x, \omega) = \frac{1}{\sqrt{2\pi}} \int e^{-i\tau k_{\tau}} M(k_x, \tau) d\tau \tag{4}$$

with the same restriction to the nonevanescent region.

An alternative to zeroing the evanescent region is to allow k_{τ} to go imaginary thereby applying exponential gain or decay in that region. Let's now choose exponential decay in both (3) and (4) for evanescent waves in the interests of stability. It is easy to verify that (3) and (4) are still transposes.

A more direct method of phase-shift modeling is by marching the "exploding reflectors" $t=0$ snapshot of the subsurface forward in time:

$$P(k_x, t) = \frac{1}{\sqrt{2\pi}} \int e^{-it\omega} M(k_x, k_\tau) dk_\tau \qquad (5)$$

4) Which of the modeling equations (4) and (5) produce evanescent waves? Under what restrictions?

5) Reverse-time migration would run equation (5) backwards, using the time section as a source at the surface $\tau=0$. Is this equivalent to

   a) the transpose of Stolt diffraction, or

   b) Stolt migration, or

   c) the transpose of equation (5), or

   d) none of the above?

KIRCHHOFF

Rothman, Levin, and Rocca (1985) noted " ... *the equivalence of NMO stretch in Kirchhoff migration and the frequency downshift of migrated dips, reconciling ray- and wave-theoretic views of migration.*" Consider now "hand migrating" a dipping segment. Let T be its unmigrated time duration and suppose its subsurface dip is θ. Let the constant migration velocity be v.

6) Compute the following quantities:

   a) the spatial width of the unmigrated event,

   b) the time duration of the migrated event, and

   c) the spatial width of the migrated event.

7) Taking into account the frequency downshift of NMO stretch mentioned above, by what factor would the sum-of-squares of the amplitudes on the dipping event change as a result of migration?

FINITE DIFFERENCE

Consider now the 15 ° equation

$$P_{t\tau} = -\frac{v^2}{2} P_{xx} \tag{6}$$

again assuming constant velocity. Figure 1 outlines pictorially how this is used in discrete form to do finite difference migration. In SEP-38, this author noted that reordering the computations along lines parallel to the diagonal image is one flavor of reverse-time migration as these are lines of constant time. In particular the diagonal is the image condition $t = 0$ and the area above the diagonal corresponds to $t < 0$.

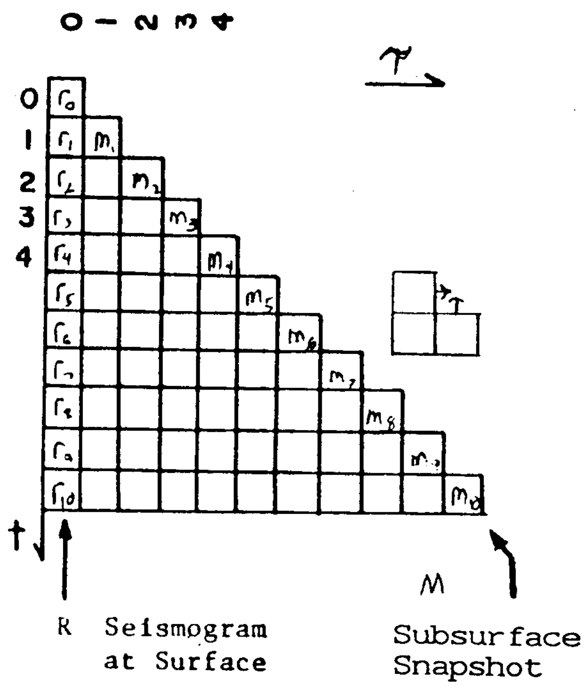

FIG. 1. 15 ° finite difference migration. The spatial coordinate x runs into and out of the page and is suppressed. The time section to be migrated is placed on the left-hand edge of the triangular grid, zeros placed along the bottom and a two by two differencing star is used to fill in the grid. The migrated image is then read off the diagonal.

Let's now try to run migration "backwards" to make a 15 ° modeling algorithm. After placing the subsurface model on the diagonal, we still can't get started. We need to specify values just above (or just below) the diagonal in order to begin marching.

8) Which of following determines suitable values for the desired initial off-diagonal information?

a) Place zeros above the diagonal since the exploding-reflectors model is zero before time zero.

b) Use the 15 ° modeling equation

$$P_{t\tau} = + \frac{v^2}{2} P_{xx} \tag{7}$$

to extrapolate the model one Δt forward in time to fill in just below diagonal.

c) Shift the diagonal sideways to extrapolate below the diagonal.

d) None of the above are satisfactory for 15 ° modeling.

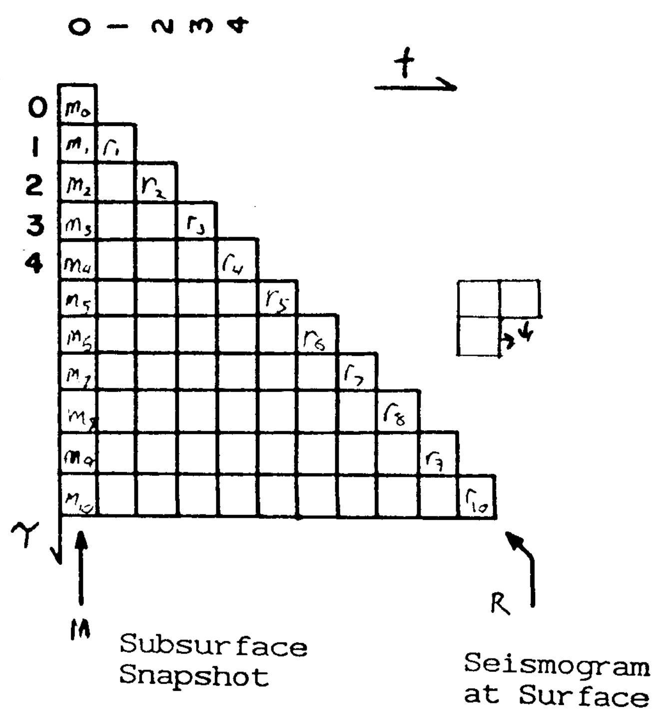

FIG. 2. 15 ° finite difference modeling. Again the x-axis is suppressed. The input subsurface t =0 snapshot is placed on the left-hand edge of the triangular grid and a two by two differencing star is used to fill in the grid. The output time section is then read off the diagonal.

Of course all this fuss could be avoided by simply using equation (7) to do 15 ° modeling directly. This setup is pictured in Figure 2. The subsurface model is placed on the left edge of the grid and the modeled time section extracted from the diagonal. The direction of recurrence is now, unlike in migration, from top to bottom. So again we need to determine values above the diagonal image in order to get started.

9) Which of following determines suitable values for the desired initial off-diagonal information?

   a) Place zeros above the diagonal to force zero pressure above the earth's surface.

   b) Reflect information across the diagonal to specify a zero-slope free surface condition.

   c) Extend the triangular grid above the diagonal to form a square and place zeros at the top of the square.

   d) None of the above are satisfactory for 15 ° modeling.

10) Which, if any, of the methods outlined in questions 8) and 9) above is the transpose of 15 ° migration? the inverse?

ANSWERS

10 – none are the inverse or the transpose of 15 ° migration

9 – c) extend to a square grid with zeros at the top

8 – c) copying the diagonal sideways is satisfactory for 15 ° modeling

7 – $\cos\theta$

6 – a) $vT/\sin\theta$; b) $T\cos\theta$; c) $vT\cos^2\theta/\sin\theta$

5 – d) none of the above

4 – equation (4) whenever M is not identically zero

3 – a) $NMO^T = INMO \sec\theta$

2 – True

1 – a) amplitudes are preserved

DISCUSSION

1) Surprisingly, Stolt migration generally preserves amplitudes of dipping events unless they contain evanescent energy in which case amplitudes will decrease. The easiest way to see this is the phase-shift integral (3) from which Stolt migration is derived. In terms of θ we can rewrite (3) as

$$M(k_x, \tau) = \frac{1}{\sqrt{2\pi}} \int e^{i\omega\tau\cos\theta} P(k_x, \omega) d\omega \tag{8}$$

so for a fixed dip, $M(k_x, \tau) = P(k_x, \tau\cos\theta)$ and amplitudes are redistributed but preserved. So Stolt migration steepens and stretches out the wavelet on a dipping event without changing amplitudes.

2) The presence in the formula of $\sec\theta$, which goes to infinity for 90° dips, does not automatically mean the operation is unbounded because the *INMO* operator may (and indeed does) cancel out some or all of the apparent singularity. A short proof of unboundedness is the following:

Let D be the diffraction operator. Then $D^T D$ is scalar multiplication by $\sec\theta$ and so is unbounded. If D were bounded, then (Reed and Simon, 1972, 185-187) D and D^T would have the same norm and the norm of $D^T D$ would be its square and hence also bounded. So we conclude this definition of diffraction is unbounded in the L^2 sense.

3) It's interesting that a similar conclusion also applies to ordinary normal-moveout correction of CMP gathers. So the primary effect of pseudo-unitary NMO (Biondo and Claerbout, SEP-44) is to apply dip-dependent scale corrections.

4) This came up while reading a comment by Stolt on 3-D migration (Stolt, 1984). There he pointed out that from $\hat{P}$ of equation (4) one can reconstruct the same M by means of either the propagating or the evanescent part of $\hat{P}$. Doing the "Stolt" change of variable $k_\tau \rightarrow \omega$ in equation (5) shows it is the same as Harlan's definition of diffraction and so has no evanescent component.

5) This is a tough one. At first view reverse-time migration would seem to be "backwards-continue and add" and so would be the transpose of equation (5) just as equation (4) was the transpose of (3). But reverse-time migration would be more appropriately termed "backwards-continue and replace" with the time section specifying the wavefield at the earth's surface rather than adding to an existing wavefield as modeling by "upward-continue and add" would. So it's not answer (c). And since it includes evanescent energy as well as propagating energy it isn't (a) or (b). In fact the correct integral expression is equation (3) with the above-mentioned choice of exponential decay in the evanescent region.

6) These formulas assume the velocity v has been previously halved as is customary in discussing post-stack migration.

7) The wavelet is stretched out over a length sec θ longer after migration. The width of the event is shrunk by $\cos^2\theta$. So the sum-of-squares is changed by $\sec\theta \cos^2\theta$ or $\cos\theta$. So we conclude that migration reduces the energy in a section even though it preserves amplitudes. Correspondingly, diffraction increases energy by the same amount sec θ, verifying the conclusion for question 2).

8) Until recently I hadn't given any real thought to 15 ° modeling as a multitude of high-dip accurate modeling programs have been readily available. I got into 15 ° modeling when studying the transpose of 15 ° migration. So I was startled to find that method a), which seems so logical and straightforward, does not leave horizontal reflectors alone but instead turns them into step functions as illustrated in Figure 3.

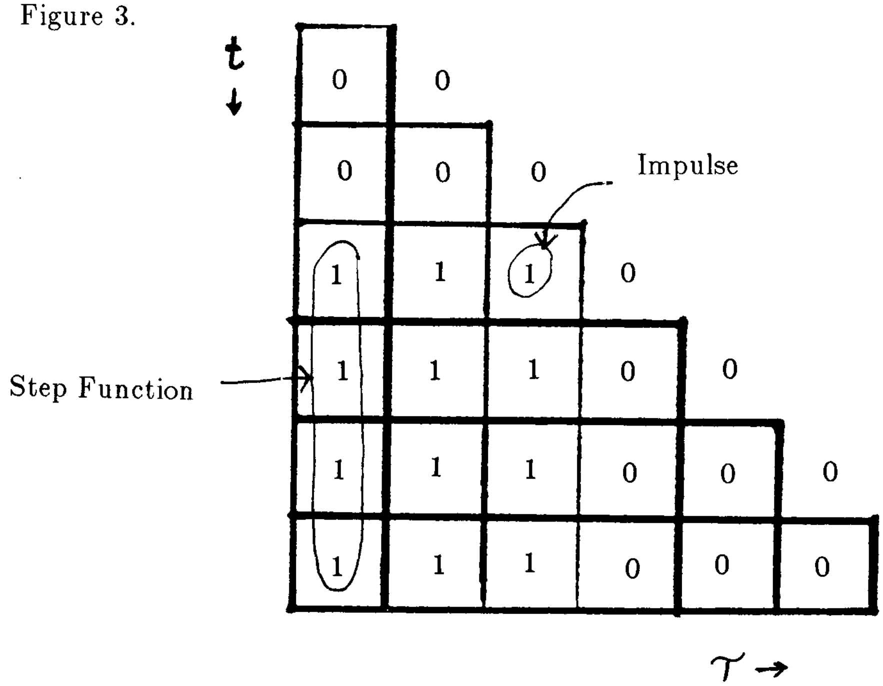

FIG. 3. The computational grid produced when method a) is applied to a single, flat event. The impulse on the diagonal turns into a step function on the time section.

I pondered this and realized that migration would produce nonzero values above the diagonal, so I needed to figure out what these ought to be. I asked around and found out that Dave Hale had simply copied the diagonal sideways when generating synthetics for classroom exercises. This is the correct thing to do for flat dip events, but migration would do something other than this to dipping events. So I decided what I needed to do was b), extrapolate values for the off-diagonal using 15 ° modeling equation (7). This worked fine for flat dip synthetics but was unsatisfactory when applied to a 15 ° migrated CMP gather of field data as seen in Figure 4.

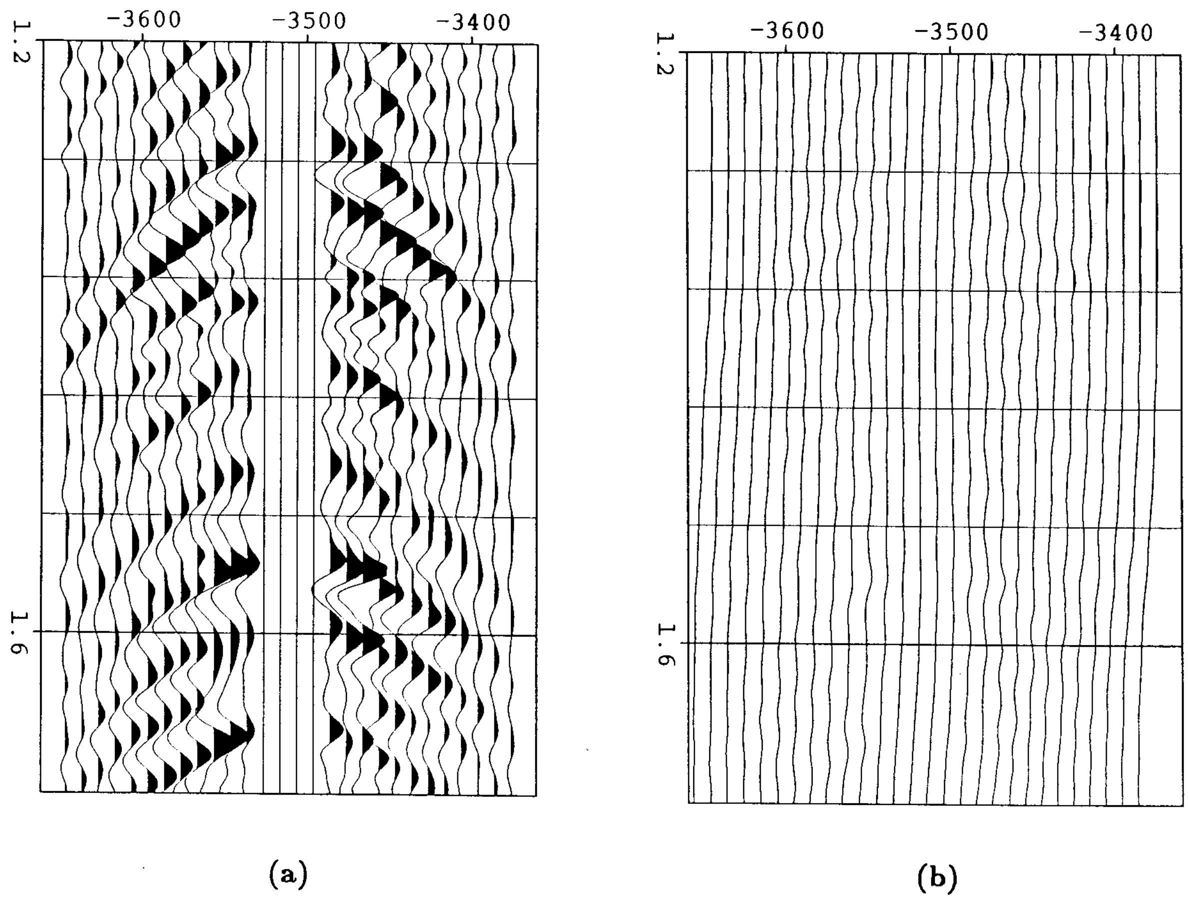

FIG. 4. A CMP gather (a) and (b) the result of 15 ° migration followed by 15 ° modeling using extrapolated off-diagonal values.

I finally figured out that I wasn't using the transpose of 15 ° migration. The transpose algorithm extrapolates each element on the diagonal sideways using zero entries in the differencing star everywhere else. This is the finite-difference analogue of the phase-shift "upward-continue and add" algorithm discussed earlier. As Figure 5 shows, this yields a satisfactory 15 ° modeling scheme. And indeed this is what I will use in the future when I need to do 15 ° finite difference modeling.

As an afterthought I went back and tried the "flat-dip" method of copying the diagonal sideways. Surprisingly, as you may judge from Figure 6, this worked quite well, even though the more sophisticated extrapolation method b) did not. It

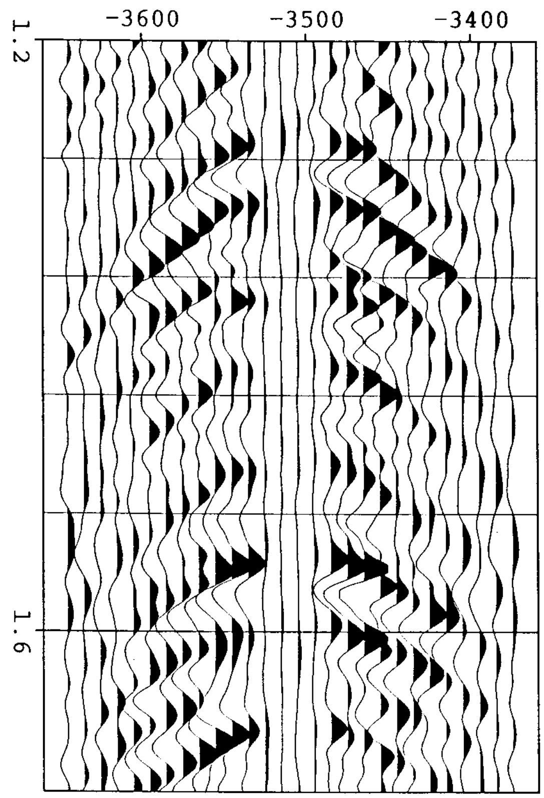

FIG. 5. The result of applying 15 ° migration followed by its transpose to the CMP gather of Figure 4a. This has done a fine job of reconstructing the original gather.

would appear then that to get a good result it is advisable not to use model information from shallower traveltimes to extrapolate below the diagonal.

9) As illustrated in Figure 7, method a) is unsatisfactory because it differentiates flat reflectors, method b) is significantly better, but it doubles the amplitudes of flat events, and method c), by extending the grid above the surface $\tau = 0$, removes any generated "downgoing" waves at the surface and so does not modify flat reflectors.

Of course results a) and b) are what would be produced by more sophisticated modeling programs as well when faced with zero value or zero slope boundary conditions at the surface of the earth. But migration is usually asked to leave flat reflectors alone. There are two factors that justify this. First, if a reflector were truly a source, wave theory predicts that the recorded wavefield would be an integrated version of the source. Fortunately, the exploding reflectors model deals with an effective source which is the original downgoing waveform differentiated by the process of reflection. So the integration and the differentiation cancel each other out and flat reflectors are unchanged. In this argument the surface of the earth is assumed to be transparent. However the second factor is that preprocessing, such as marine de-ghosting and deconvolution, largely correct for phase distortions produced by the nonzero reflectivity of the earth's surface. So it is indeed

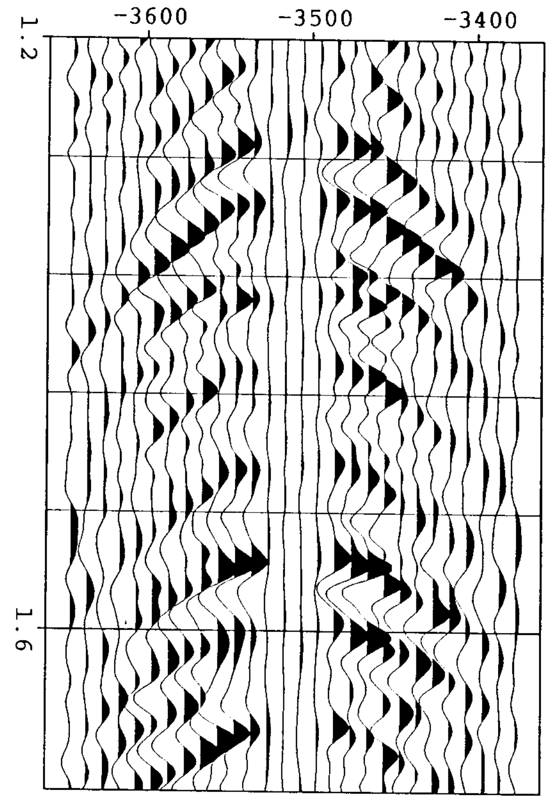

FIG. 6. The result of applying 15 ° migration followed by 15 ° modeling using "flat-dip" extrapolation to get started. This is quite as good a result as the transpose of 15 ° migration produced in Figure 5.

legitimate to assume the surface is transparent when we are ready to migrate seismic data.

10) The discussion of question 8) showed that none of the algorithms suggested in questions 8) or 9) are the transpose of 15 ° migration. Nor are any the inverse because the bottom zero boundary condition of migration is the same as assuming the time section to be zero for all sufficiently large t, a condition that does not hold for the various impulse responses which are nonzero for all computed times greater than or equal to the traveltime of the impulse. One could use an iterative, conjugate-gradient method to come up with suitable off-diagonal values so that a marching scheme would become close an inverse to migration. This approximation would start with, say, zeros for the off-diagonal and use the transpose followed by migration to get a residual mismatch with which to adjust the off-diagonal estimates. As the iteration proceeded, the residuals would in general decrease towards zero and we'd converge to an inverse to migration. While I see no point in concocting an inverse to 15 ° migration, especially with such nonphysical behavior at late times, it would be possible to modify this procedure to predict more appropriate bottom boundary conditions so as to reduce migration "smiles" in the deeper section.

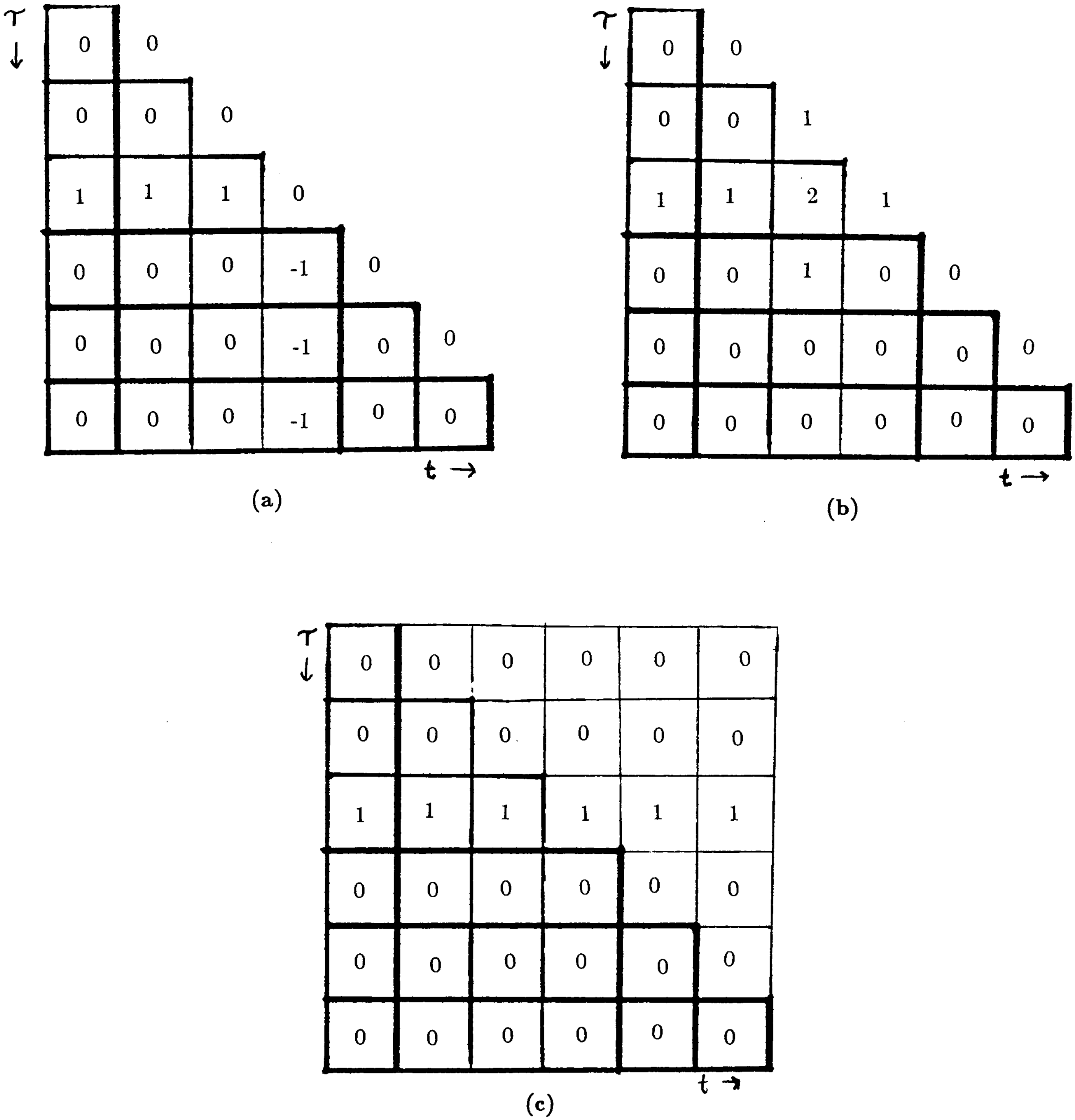

FIG. 7. Computational grids for forward 15 ° modeling under the three different assumptions of question 9). Only method c) leaves the flat event alone.

ACKNOWLEDGMENTS

I am grateful to my fellow SEP members, present and recently graduated, for passionate discussions of the various issues that led to this quiz. Special thanks go to John Sherwood who devoted a substantial portion of his visit to Stanford discussing theories and paradoxes of migration and modeling with me.

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Weird code, part 2

Here is another weird program which came over the net recently. Again, it is in the C programming language, and supposedly compiles and runs.

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2. The worst abuse of the C preprocessor:
(submitted by Col. G. L. Sicherman <decvax!sunybcs!colonel>)

```
#define C_C_(_)~' '&_
#define _C_C(_)('\b'b'\b'>=C_C>'\t'b'\n')
#define C_C _|_
#define b *
#define C /b/
#define V _C_C(
main(C,V)
char **V;
/*      C program. (If you don't
 *      understand it look it
 */     up.) (In the C Manual)
{
        char _,__;
        while (read(0,&__,1) & write((_=(_=C_C_(__),C)),
        _C_,1)) _=C-V+subr(&V);
}
subr(C)
char *C;
{
        C="Lint says "argument Manual isn't used."  What's that
        mean?"; while (write((read(C_C('"'-'/*"'/*"*/))?__:__-_+
        '\b'b'\b'|((_-52)%('\b'b'\b'+C_C_('\t'b'\n'))+1),1),&_,1));
}
```

[This program confused the C preprocessor so badly that it left some comments in the preprocessed version. Also, lint DID complain that "argument Manual isn't used".]