Cascaded one-step 15-degree migration versus Stolt migration

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ABSTRACT

Claerbout's new time-velocity (t', w) wavefield extrapolation method was introduced in his paper "Velocity Extrapolation by Cascaded 15 Degree Migration" (Claerbout, 1986). This paper attempted first to understand and relate Claerbout's method to Larner and Beasley's cascaded migration method (Larner and Beasley, 1985) and then to compare results of applying the velocity-extrapolation method (or cascaded one-step migration method) and a Stolt method to migrations of a field data set from the Gulf of Mexico.

The results of the field data migrations show that taking the number of depth step being one in cascaded 15-degree migration is as or more computationally efficient than Stolt migration when dips of reflections are moderate.

INTRODUCTION

The method of 15-degree cascaded migrations was presented by Larner and Beasley at the 1985 SEG convention (Larner and Beasley, 1985), to improve accuracy in migrations of steep-dip reflections. It turned out that the time-space domain implementation of Claerbout's method is exactly one special form, with the number of depth steps being one, of Larner and Beasley's cascaded 15-degree migration. Larner and Beasley mentioned that the number of depth steps in their cascaded migrations should generally be proportional to migration velocity used. Claerbout took one step further and pointed out that the number of depth steps can be one when the velocity used is small enough. The resulting cascaded one-step migration method not only improves the accuracy in migrations of steep-dip reflections, but also reduces computational costs for seismic data migrations.

In variable-velocity media, the cascaded one-step migration method uses the root-mean-square (RMS) velocity obtained directly from conventional common-midpoint velocity analysis (instead of the interval velocity) as its migration velocity. Experimental processing of some field data sets shows the cascaded one-step method has surprisingly good results with extremely low computational costs.

Other than the approach of the cascaded migrations, using a conventional 15-degree migration and imaging beyond the diagonal line $t' = \tau'$ will also be able to improve accuracy in migration of steep-dip reflections.

CASCADED MIGRATIONS

Larner and Beasley's formulation

Most poststack migrations in (t, τ, x) space use the one-way (upcoming wave) scalar wave equation,

$$\begin{cases} \dfrac{\partial P}{\partial \tau} = \dfrac{\partial}{\partial t}\left[1 - \dfrac{v^2\, \partial^2/\partial x^2}{\partial^2/\partial t^2}\right]^{1/2} P \quad \text{and} \\ Image = P(t=0, \tau, x)\,, \end{cases} \tag{1}$$

where $P(t, \tau, x)$ is the wavefield. A poststack section $P(t, \tau=0, x)$ is the input of the migration.

The imaging process given by equation (1) can be implemented by following iterative steps (Larner and Beasley, 1985):

$$\begin{cases} \dfrac{\partial^2 P^{(n)}}{\partial \tau \partial t} = \dfrac{\partial^2 P^{(n)}}{\partial t^2} - \dfrac{v^2}{2N}\dfrac{\partial^2 P^{(n)}}{\partial x^2} \quad \text{and} \\ P^{(n)}(t, \tau=0, x) = P^{(n-1)}(t=0, \tau, x)\,, \end{cases} \tag{2}$$

where $n = 1,2,\ldots,N$ is the index of iteration and $P^{(0)}(t, \tau=0, x) = P(t, \tau=0, x)$.

Applying the retarded transform,

$$\begin{cases} t' = \tau + t\,, \\ \tau' = \tau\,, \text{ and} \\ x' = x\,, \end{cases} \tag{3}$$

we have

$$\begin{cases} \dfrac{\partial^2 P^{(n)}}{\partial \tau' \partial t'} = -\dfrac{v^2}{2N}\dfrac{\partial^2 P^{(n)}}{\partial x'^2} \quad \text{and} \\ P^{(n)}(t', \tau'=0, x') = P^{(n-1)}(t', \tau'=t', x')\,. \end{cases} \tag{4}$$

Equation (4) is the fundamental formulation of the 15-degree cascaded migration (Larner and Beasley, 1985).

Claerbout's velocity extrapolation

Claerbout extends equation (4) to time-velocity space and obtains the velocity-extrapolation equation

$$\begin{cases} \dfrac{\partial^2 P}{\partial w \, \partial t'} = -\dfrac{t'}{2} \dfrac{\partial^2 P}{\partial x'^2} \quad \text{and} \\ Image = P(t, w = v^2, x), \end{cases} \tag{Jon-8}$$

where $w = v^2$ and $\partial w = v^2/N$.

Dan Rothman and I noticed that if $v^2 \Delta\tau'/N$ is replaced by $t'\Delta v^2$, then equation (Jon-8) is equivalent to equation (4) at the imaging line $\tau' = t'$. When velocity is constant and N is large enough, taking the step $\Delta\tau'$ at t' to be τ' (one-step imaging), then

$$v^2 \, \Delta\tau'/N = (v^2/N)\, \tau' = t' \, \Delta v^2 = t'\Delta w \, . \tag{5}$$

Therefore, Claerbout's equation (Jon-8) is actually a special form of Larner and Beasley's equation (4), when taking $\Delta\tau'$ to be τ'. This change is valid when velocity is constant and when N is sufficiently large.

Cascaded one-step migration in (t', τ', x')

Letting $t'\partial w = t' v^2/N$ in equation (Jon-8), or $\partial\tau' = \tau' = t'$ (one-step imaging) in equation (4), we have the following formulation:

$$\begin{cases} \dfrac{\partial P^{(n)}(t', \tau'=t', x')}{\partial t'} - \dfrac{\partial P^{(n)}(t', \tau'=0, x')}{\partial t'} = -\dfrac{t' v^2}{2N} \dfrac{\partial^2 P^{(n)}}{\partial x'^2} \quad \text{and} \\ P^{(n)}(t', \tau'=0, x') = P^{(n-1)}(t', \tau'=t', x') \, . \end{cases} \tag{6}$$

After N iterations, the final image is obtained as $P^{(N)}(t', \tau'=t', x')$. This is a special form of Larner and Beasley's cascaded migration, with the number of depth steps being one ($\Delta\tau' = \tau' = t'$). Finite-difference implementation of equation (6) is shown in Figure 1.

To achieve higher accuracy than that given by the algorithm shown in Figure 1, we can use linear interpolation between two adjacent points before moving the four-point finite-difference star to the next column of t'. Interpolation also suppresses dispersion effect of finite difference.

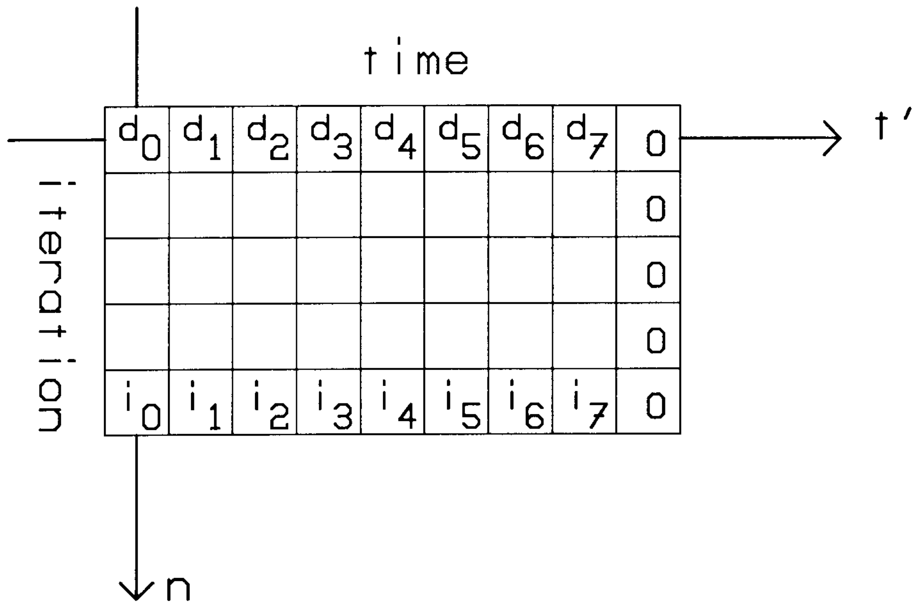

FIG. 1. Finite-difference grids for cascaded one-step 15-degree migration. d_i 's are surface data, 0's are zero values of boundary condition, and i_i 's are images.

The necessary number of iterations, N, depends on three factors: the number of samples per seismic trace, the earth's velocity, and the dips of the reflections presented in the data. The larger these three factors are, the larger the number N should be.

Figure 2 shows an impulse response of a cascaded one-step 15-degree migration with $N = 20$. Notice that the response is a semicircle.

CONSTANT-VELOCITY MIGRATION

The cascaded one-step 15-degree equation (6) gives us an accurate constant-velocity migration algorithm. Constant-velocity Stolt migration (Stolt, 1978) has been known for both its high accuracy and efficiency. Experiments show that cascaded one-step 15-degree migration may be even faster than Stolt migration, when dips of reflections are moderate.

Letting nx be the number of traces in the input poststack section and nt be the number of samples per trace, we can figure out the number of operations for both cascaded one-step 15-degree and Stolt migrations. When an implicit finite-difference scheme (along the x axis) is used to implement cascaded one-step 15-degree migration, the basic computation involves solving a tridiagonal matrix system $AX = D$, which requires $2nx-2$ operations (multiplications) for LU (lower and upper bidiagonal matrices)

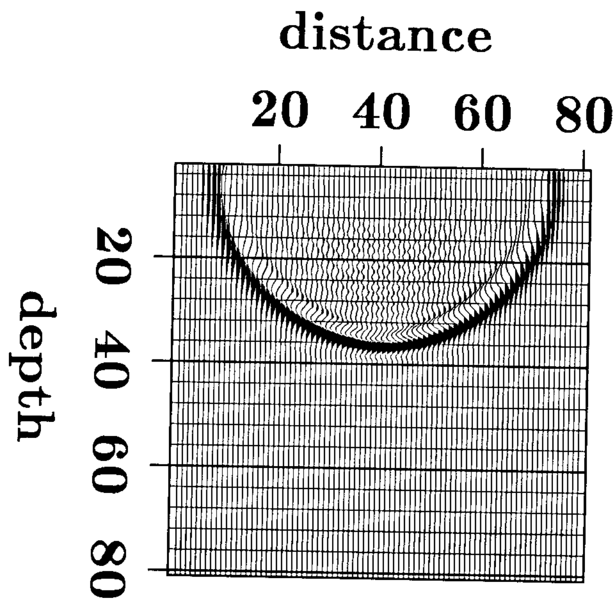

FIG. 2. Impulse response of a 20-iteration, cascaded one-step 15-degree migration.

decomposition ($A = LU$), $4nx$ operations for computing vector D, and $3nx-2$ operations for solving $LUX = D$. If we do the computation first along iteration axis n and then along the time axis t' (Figure 1), the LU decomposition needs to be done once every column of t' for all N iterations. Therefore, the total number of operations for cascaded one-step 15-degree migration is $nt \times (7nx \times N + 2nx - 2)$, or on the order of $nt \times nx$, because N (about 20 when dips of reflections are moderate) is much less than both nx and nt.

For Stolt migration, assuming no zero padding (so that nt and nx are powers of 2 to use FFT), we need $nx \times nt \times \log_2(nt \times nx)$ complex operations (multiplications) for FFT and inverse FFT (one complex-number operation corresponds to four real-number operations). The computation for FFT and inverse FFT already costs about the same as that for cascaded one-step 15-degree migration. In addition, Stolt migration needs $nt \times nx \times N_s$ complex operations for transforming $P(\omega, k_x, z=0)$ to $P(t=0, k_x, k_z)$, where N_s is the number of operations involving one square-root operation, six multiplications, and operations for interpolations (two when linear interpolation is used, or more when sinc interpolation is used), plus efforts to suppress wrap-around artifacts.

Therefore, cascaded one-step 15-degree migration takes about the same or fewer number of operations as Stolt migration, when N is much less than both nx and nt. It will be shown later, by migrations of a field data set (nt = 500 and nx = 512) containing fault-plane reflections of steep-dips (up to about 45 degrees), that no visible differences can be observed between cascaded one-step migrated and Stolt migrated sections, even when N is taken to be 10 in cascaded one-step migration. When dips of reflections are moderate (hence N can be small in cascaded one-step migration) and padding is needed in Stolt migration, the cascaded one-step 15-degree approach is less expensive than the Stolt method. The following table compares the speeds of these two constant-velocity migrations, when they are applied to different sizes of data sets.

Convex C-1 CPU Time (seconds)					
data	**nt×nx**	**Stolt**	**cascaded one-step**		
			N=10	N=20	N=30
synthetic	80×80	7.9		2.1	
synthetic	128×128	7.9		4.4	
field	500×512	121.7	31.8	59.2	
field	1024×512	243.8		114.8	170.5

The Stolt migration in the table uses a 7-point-sinc frequency interpolator to suppress wrap-around artifacts (Harlan, 1982). For the same sizes of computation ($nt \times nx$), the number of iterations needed for cascaded one-step migration of synthetic data is often larger than that needed for cascaded one-step migration of field data, because synthetic data usually contain much more steep-dip events than field data.

Figure 3 shows the results of constant-velocity migrations of a dip-moveout-corrected (DMO) and normal-moveout-corrected (NMO) stack section (Figure 3.a). The field data set contains steep-dip (up to 45 degrees) reflections from some growth-fault planes in the Gulf of Mexico. Both cascaded one-step 15-degree and Stolt migrations are applied to the data. The cascaded one-step 15-degree method produces the same good-quality results in migrated sections as Stolt migration, yet it saves about three-fourth of computation time of Stolt migration in this field-data example (nt = 500 and nx = 512). However, when input data contain dips of reflections greater than 45 degrees, Stolt method is certainly better than cascaded one-step method in constant-velocity migrations.

The number of iterations in the above example is 10. Using more iterations does not give visible improvement in the migrated sections (Figure 4), because dips of reflections are moderate in this field-data set. For other data with larger dips of reflections, larger iteration number N should be expected.

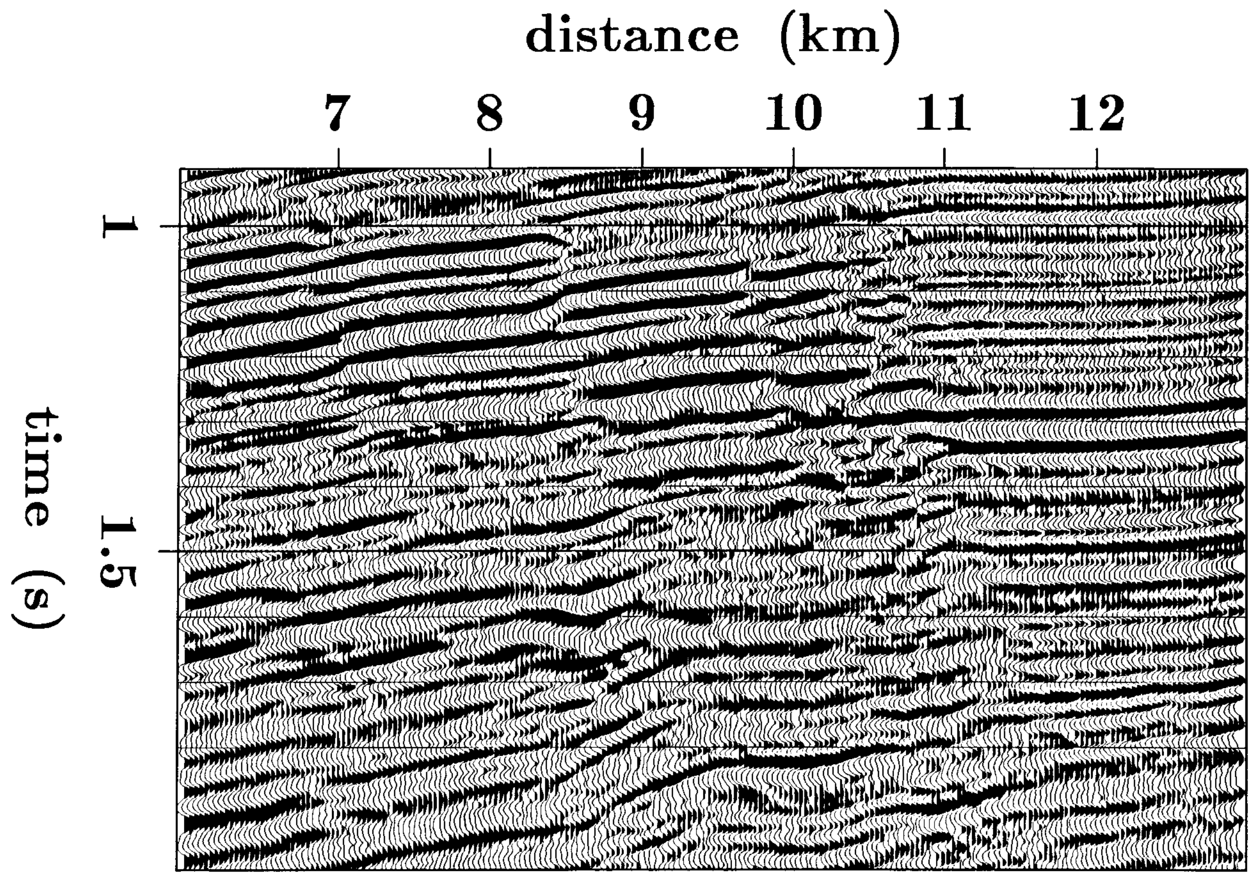

FIG. 3a. A window of a DMO- and NMO- corrected stack section from the Gulf of Mexico. The data contains wide-angle reflections from growth-fault planes and diffractions generated by the truncated edges of fault planes.

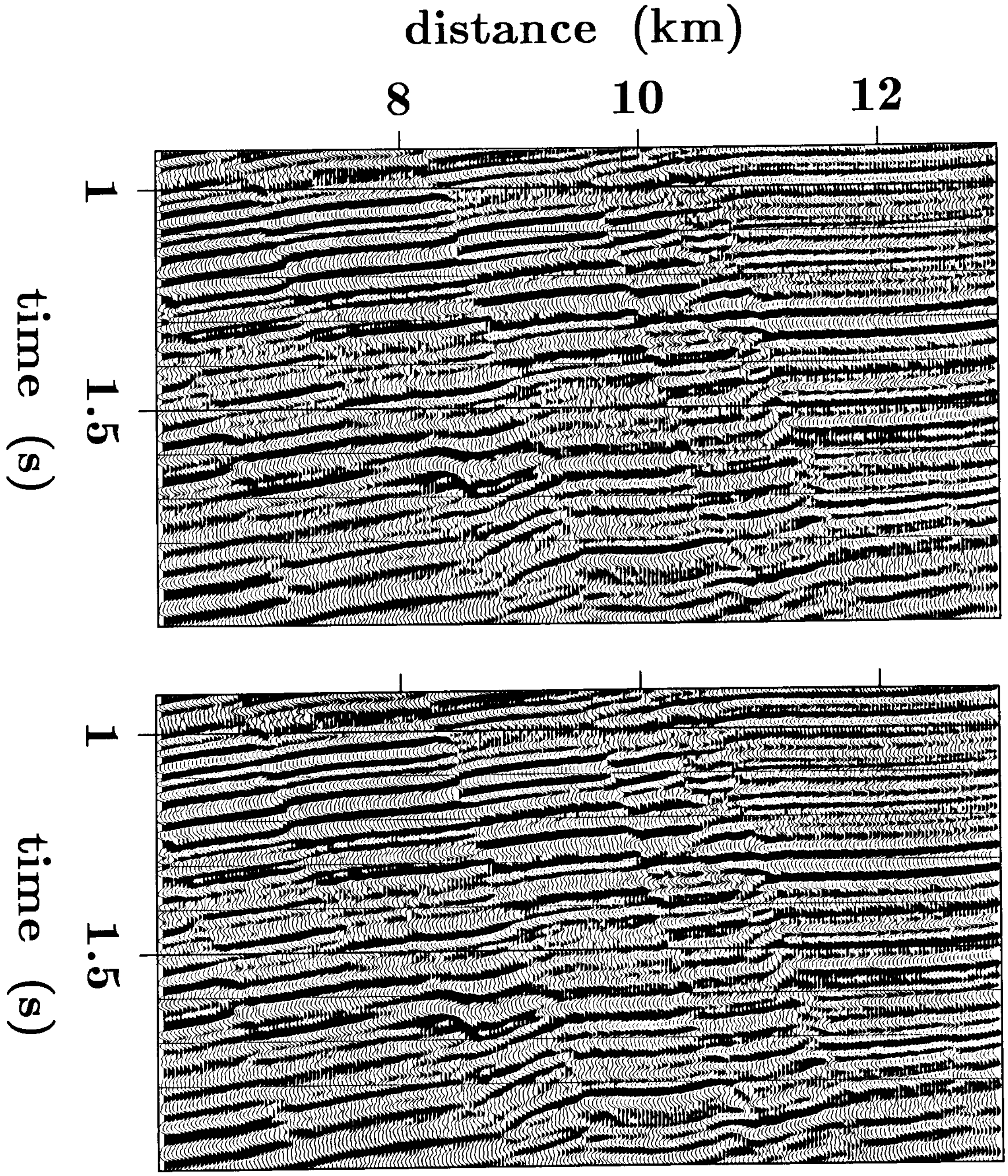

FIG. 3b. Constant-velocity ($v = 1.676$ km/s) migrated sections using 10 iterations of cascaded one-step migration (in the upper section) and Stolt migration (in the lower section). The upper parts in both sections have been fully imaged, while the lower parts are undermigrated because the lower parts of the section should be migrated with higher migration velocity than the upper parts. No visible differences between these two sections can be observed. The 10 iterations of cascaded one-step migration took 31.8 s of Convex C-1 CPU time, while Stolt migration took 121.7 s.

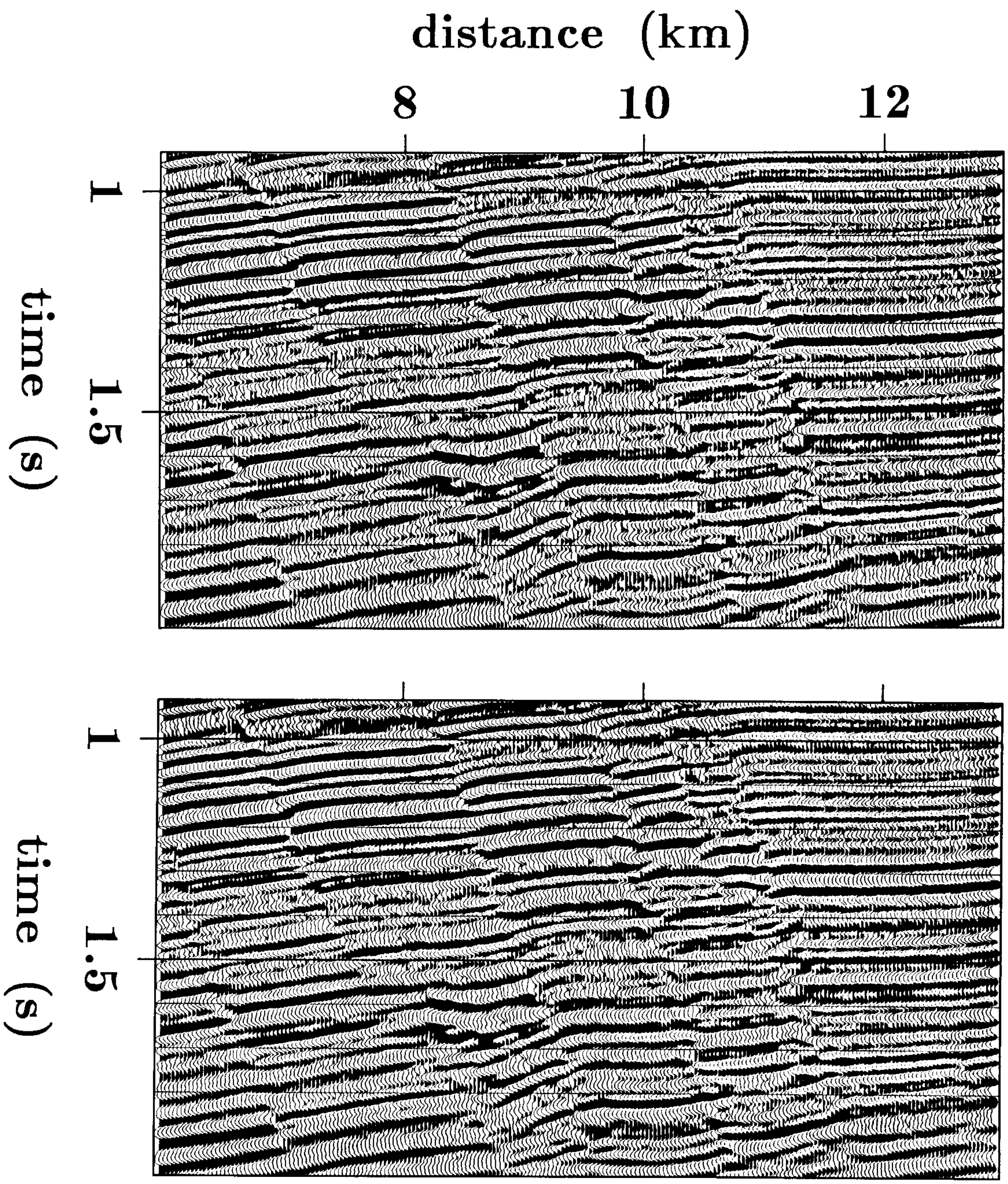

FIG. 3c. Constant-velocity ($v = 2.286$ km/s) migrated sections using 10 iterations of cascaded one-step migration (in the upper section) and Stolt migration (in the lower section).

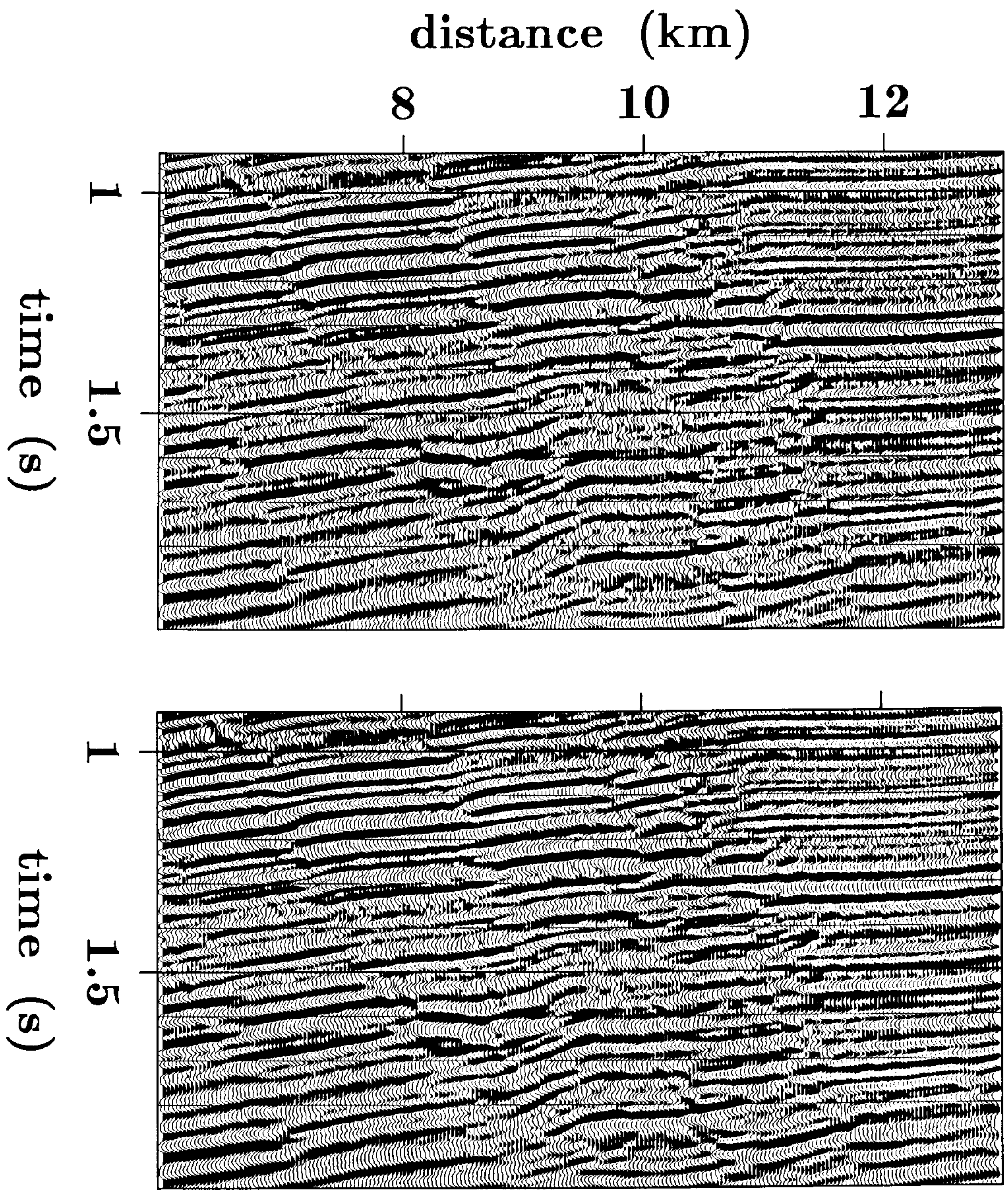

FIG. 3d. Constant-velocity ($v = 2.743$ km/s) migrated sections using 10 iterations of cascaded one-step migration (in the upper section) and Stolt migration (in the lower section). The upper parts of both sections are overmigrated (because migration velocity used is too large for the upper parts), while the lower parts are imaged with the correct velocity.

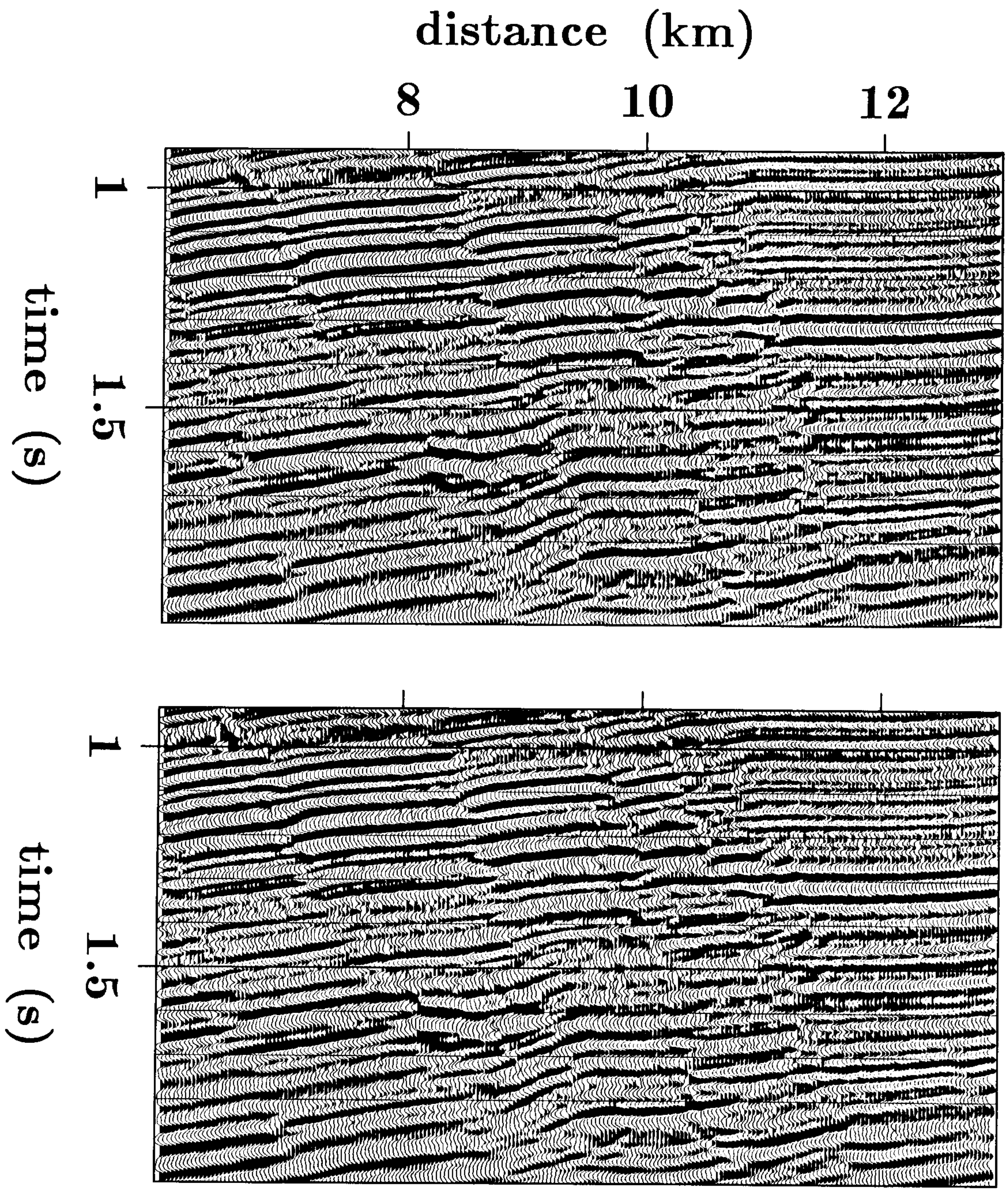

FIG. 4. Constant-velocity ($v = 2.286$ km/s) migrated sections using 10 iterations (in the upper section) and 20 iterations (in the lower section) of cascaded one-step migrations. No visible difference between these two sections can be found.

MIGRATION IN VELOCITY-VARYING MEDIA

The formulation of cascaded one-step 15-degree migration is valid in constant-velocity media. Approximations must be made when velocity changes vertically (Larner and Beasley, 1985).

Integrating equation (4) along the τ' axis from $\tau' = 0$ (data plane) to $\tau' = t'$ (image plane), we have

$$\frac{\partial P^{(n)}(t', \tau'{=}t', x')}{\partial t'} - \frac{\partial P^{(n)}(t', \tau'{=}0, x')}{\partial t'} =$$

$$- \frac{1}{2N} \frac{\partial^2 P^{(n)}(t', \tau' = \xi t', x')}{\partial x'^2} \int_0^{t'} v^2(\tau') d\tau' , \tag{7}$$

where $0 \leq \xi \leq 1$.

Comparison of equations (7) and (6) tells us that the migration velocity in equation (6) should be the RMS velocity, defined by

$$v_{rms}(t') = (1/t') \int_0^{t'} v^2(\tau) d\tau , \tag{8}$$

when the velocity of the medium, $v(\tau)$, changes vertically. The migration thus uses $v = v_{rms}(i\Delta t)$ when extrapolating samples at the i-th column ($t' = i\Delta t$) along the iteration axis n (Figure 1). The RMS velocity obtained from conventional common-midpoint velocity analysis can be used directly as the input migration velocity for cascaded one-step 15-degree migration, without converting it to interval velocity (by Dix's equation), as required by most migration methods. Interpolation between computed values at the i-th column must be applied before computing the values at the (i-1)-th column (Figure 1), if the number of iterations, N, is not sufficiently large.

Figure 5 shows a result of migrating the data set in Figure 3a with RMS velocity. This gives better images of the fault planes than can be seen in any individual constant-velocity migrated section (Figures 3 and 4). The computer CPU time of cascaded one-step migration is $nt/(2N)$ times faster (25 times faster in this field data example) than that of a conventional 15-degree migration (with an extrapolation step, $d\tau$, being a time sampling interval, dt, in the 15-degree migration); yet the cascaded one-step migration gives better results than 15-degree migration.

As mentioned by many authors (e.g., Claerbout, 1985), conventional 15-degree migration usually chooses $d\tau = I \times dt$ (I is an integer). Therefore, the cascaded one-step 15-degree migration may be only $nt/(2N \times I)$ times faster than 15-degree migration. The quantity $nt/(2N \times I)$ is still a significant factor (much larger than one), since

nt is usually much larger than $2N \times I$ (the conventional 15-degree migration of the data set in Figure 3a gives poor results when $I > 6$).

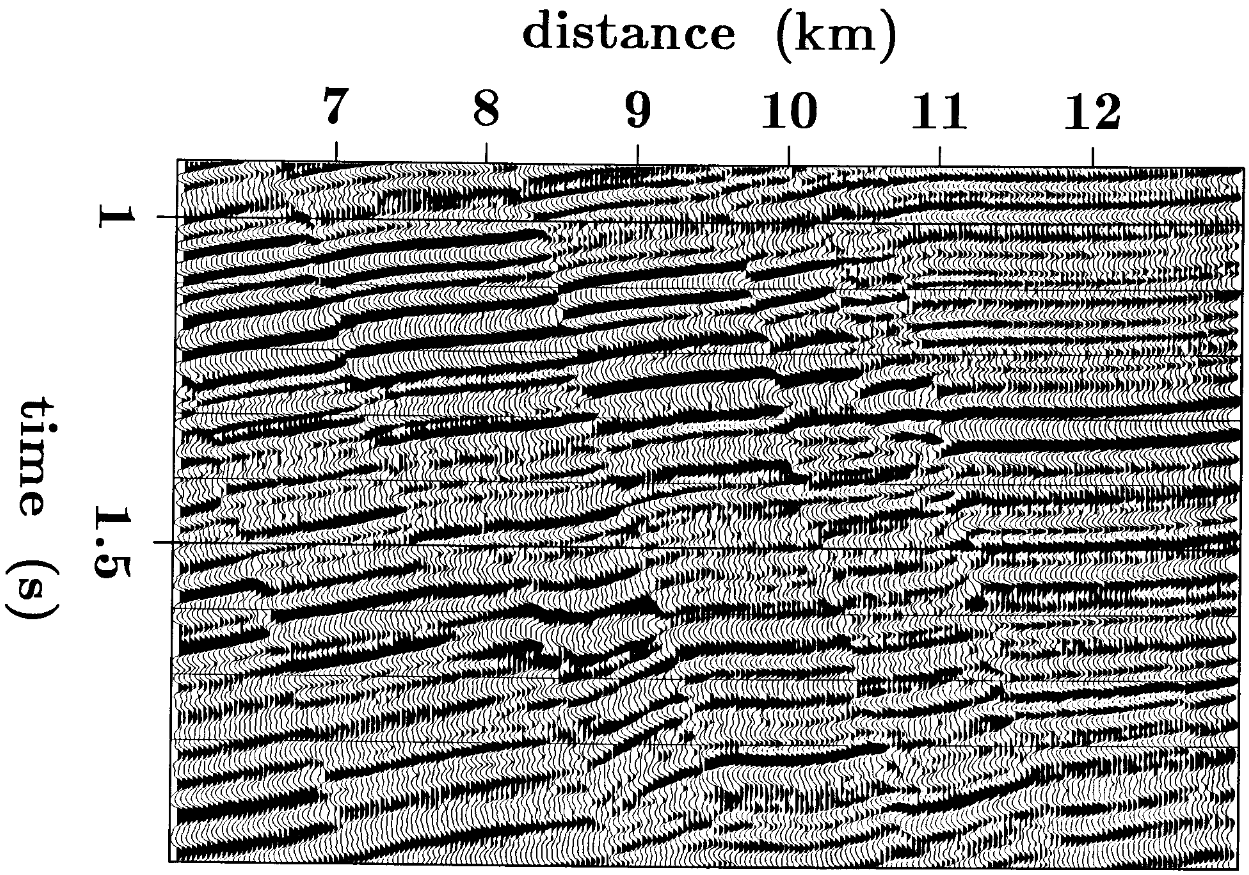

FIG. 5. A section migrated using the RMS velocity as the migration velocity in cascaded one-step 15-degree migration ($N = 10$). All fault planes are imaged well.

OTHER APPROACHES

Conventional 15-degree migration obtains an image at the diagonal line $t' = \tau'$ in the retarded coordinates. Figure 6 shows an impulse response of a 15-degree migration. The undermigrated upper parts of the distorted "semicircle" correspond to a poor imaging of wide-angle reflections. The undermigrated images at shallow depth imply that we probably should continue wavefield extrapolation beyond the diagonal imaging line to compensate for the effects of undermigration.

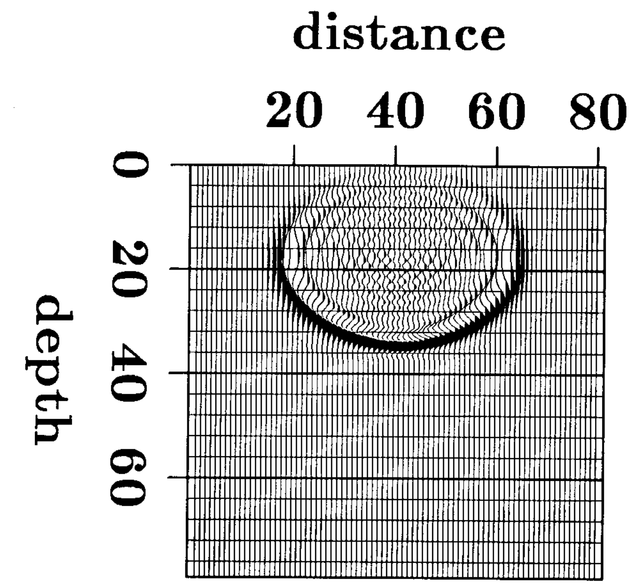

FIG. 6. An impulse response of a conventional 15-degree migration operator. Upper parts of the response are undermigrated and distorted, because of the 15-degree approximation of the one-way scalar wave equation.

However, if we extend the imaging line to the vertical line $\tau' = \tau_{\max}$, overmigration will certainly occur at shallow depth. Hence, it is likely that we should image at a curve (circle, hyperbola, or parabola) between the diagonal line $\tau' = t'$ and the vertical line $\tau' = \tau'_{\max}$.

Figure 7 shows an impulse response of a conventional 15-degree migration with the imaging curve being a parabola $t' = \sqrt{3}\tau'^2/t_{\max}$. This improves the accuracy of imaging the reflector angle up to 45 degrees.

Choosing an imaging curve between the diagonal line and the vertical line becomes difficult when the velocity of the medium is not constant. The imaging curve certainly should depend on the velocity structure.

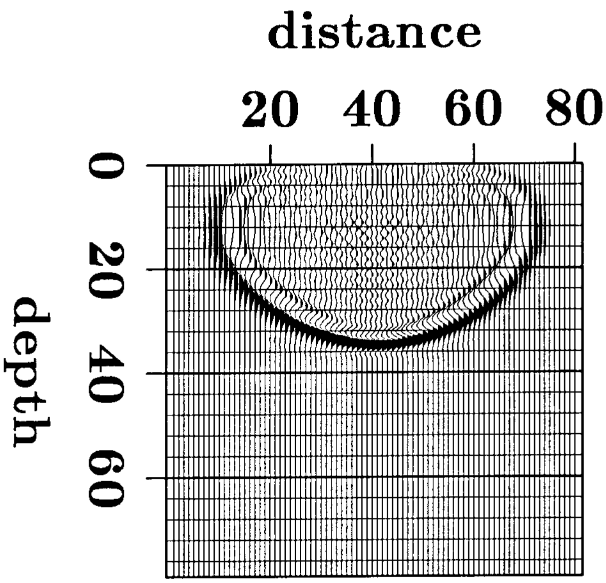

FIG. 7. Impulse response of a conventional 15-degree migration with the parabolic imaging curve $t' = \sqrt{3}\tau'^2/t_{\max}$.

CONCLUSION

Cascaded one-step 15-degree migration is more accurate and faster than conventional 15-degree migration. The migrations of the field data set gave the surprising result that 10 or 20 iterations of cascaded one-step migration could be needed for migrations of moderate-dipping reflections. Because this method is a one-way operator, no reverberations will be generated in the migrated sections even if there are velocity discontinuities. Other advantages in using this method are efficiencies in allocating and saving computer memory and reduced computational cost. RMS velocity should be used in cascaded one-step 15-degree migration when velocity changes.

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