Velocity Extrapolation by Cascaded 15 Degree Migration

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ABSTRACT

A partial differential equation is derived with velocity squared as the *independent* variable. One application is velocity analysis.

INTRODUCTION

Jakubowicz and Levin [1983] showed that cascaded migrations in a constant velocity media can be analyzed by substituting dispersion relations into one another (see IEI p. 118). Rothman, Levin, and Rocca [1985] showed that residual migrations can be done with the 15° equation, despite possible wave angles much larger than 15°. Larner and Beasley [1985] concluded that cascading 15° migrations leads to wide angle accuracy. By examining cascaded migrations, each with an infinitesimal velocity, we will come to a new partial differential equation that extrapolates in velocity space. The ability to change the velocity of seismic events leads to new possibilities in data processing.

CASCADED 15-DEGREE NORMAL MOVEOUT

A truncated Taylor series of the normal moveout equation is given by the equation:

$$\tau \quad = \quad t - \frac{x^2/v^2}{2t} \tag{1}$$

Define the N^{th} partial normal moveout by

$$\tau_N(t) \quad = \quad \tau_N(t\,,\, x/v) \quad = \quad t - \frac{x^2/v^2}{2tN} \tag{2}$$

Let the N^{th} partial normal moveout be applied N times, i.e.

$$T_N(x) = \tau_N(\tau_N(\tau_N(\tau_N(\cdots \tau_N(t))))) \tag{3}$$

I can't prove it, but $T_\infty = \sqrt{t^2 - x^2/v^2}$. This assertion is confirmed by the plot in Figure 1 which shows the first 100 values of N. The curves tend to a semicircle. The plot program is:

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do n= 1,100 {
      for( x=-4.; x≤4.; x=x+.05) {
            tau = 4.
            do i=1,n
                  if( tau ≠ 0.)
                        tau = tau - x*x / (2 * tau * n)
            call plot ( tau, x)
            }
      }
```

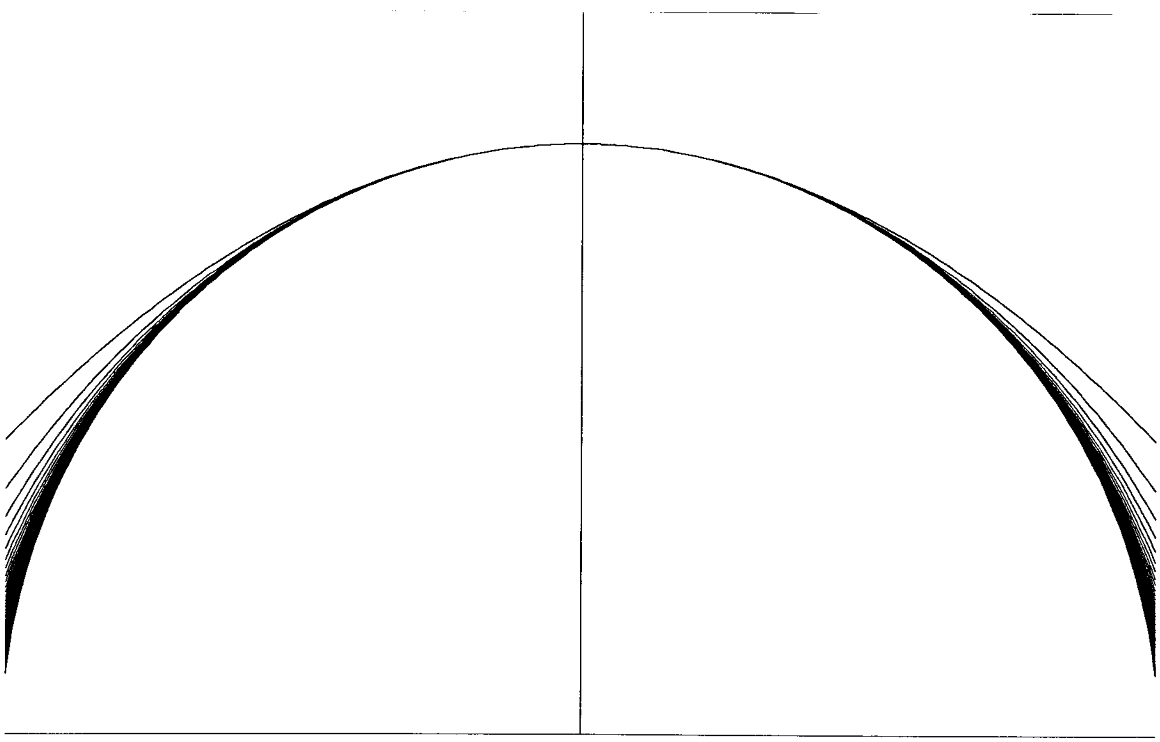

FIG. 1. Semicircle approximations of cascaded NMO or of cascaded 15° migration.

You see that the convergence is not rapid, but by the one hundredth iteration, the accuracy is probably beyond any seismological requirement.

Equation (1) becomes the dispersion relation of the 15° nonretarded wave extrapolation equation by the substitutions $t \rightarrow \omega$, $\tau \rightarrow k_\tau$, $v \rightarrow v^{-1}$.

CASCADED MIGRATION

Ordinarily the input of the migration process is a field data set and the output is an earth model. So, ordinarily it does not make sense to migrate the *output* of a migration. However the forementioned workers in residual migration did just that, namely, took an output of the migration process and used it as an input of a subsequent migration process. This is called *cascaded migration.* Here we will take the cascading to its logical limit, an infinite number of infinitesimal migrations. An infinitesimal migration is one with an infinitesimal velocity.

Time domain 15° migration is ideally suited for migration with low velocities. Recall the basic principle of this migration. Retarded time is used so that the image at $t=0$ is found along the diagonal line $t'v = z'$ (see IEI p.6,133). Consider an infinitesimal velocity Δv. Dropping primes, the image line of an incremental migration becomes

$$z \quad = \quad \Delta v \ t \tag{4}$$

If Δv is small enough compared to the usual migration depth step then the migration requires just one depth step Δz where

$$\Delta z \quad = \quad \Delta v \ t \tag{5}$$

Repeated use of such "single depth step" migration makes a continuum of infinitesimal migrations.

It has long been noted that the 15° program contains depth and velocity only in the product $v \ \Delta z$ so a rescaling of depth is like a rescaling of velocity. Equation (5) gives Δz proportional to time t. So, given a 15° program with Δz proportional to t, an alternate interpretation of the program is that Δz is constant but the migration velocity has been replaced by time t. Let us express this strange interpretation as a differential equation.

Starting from IEI equation 2.7.4 but omitting primes, we have

$$\frac{\partial^2 U}{\partial z \ \partial t} \quad = \quad -\frac{v}{2} \frac{\partial^2 U}{\partial x^2} \tag{IEI-2.7.4}$$

Time migrations do not convert to depth, but to travel-time depth. Converting depth

z to travel time depth $\tau = z/v$ yields:

$$\frac{\partial^2 U}{\partial \tau \, \partial t} \quad = \quad -\frac{v^2}{2} \frac{\partial^2 U}{\partial x^2} \tag{6}$$

The difference between extrapolation and migration is that in extrapolation, equation (6) projects a wavefield a constant distance in τ, whereas migration projects the wavefield to the image line

$$\tau \quad = \quad t \tag{7}$$

Define $w = v^2$, sometimes called the "levocity." The "strange interpretation" now takes the form

$$\frac{\partial^2 U}{\partial w \;\; \partial t} \quad = \quad -\frac{t}{2} \frac{\partial^2 U}{\partial x^2} \tag{8}$$

Equation (8) is amazing. Given initial conditions $U(t, x, w=0)$ with superposed hyperbolic events of various velocities (asymptotes), extrapolation along w causes the hyperbolas to change the slope of their asymptotes!

Initially I was unsure of the constant of proportionality in equation (8). The physical dimensions are correct. I confirmed the 1/2 by using (8) to generate some hyperbolas, measuring the asymptotes, and testing the results with an NMO program. It bears repeating, that as in Chapters 1 and 2 of IEI, t is the two-way travel time, v is half the rock velocity, so w is a quarter the rock velocity squared.

Since doing this work I find myself disappointed by my inability to create a clear derivation. But then, I don't understand the convergence of (3) either. Zhiming Li suggested that (8) might be derived from (6) by the substitution $w \, \Delta\tau = t \, \Delta w$. Stew Levin suggested a derivation (which should appear elsewhere in this report) based on the Stolt integral. Independently, Dave Hale suggested a similar proof. Li subsequently claimed that the solution speed of (8) is about that of the Stolt integral, without the wraparound problems. I imagined that the number of required τ steps would be comparable to the number of t steps, but Li got good results with 10-20 steps. I continue to point out to the young theoreticians that a defect of (8) is that depth-variable velocity is not included, and it needs to be.

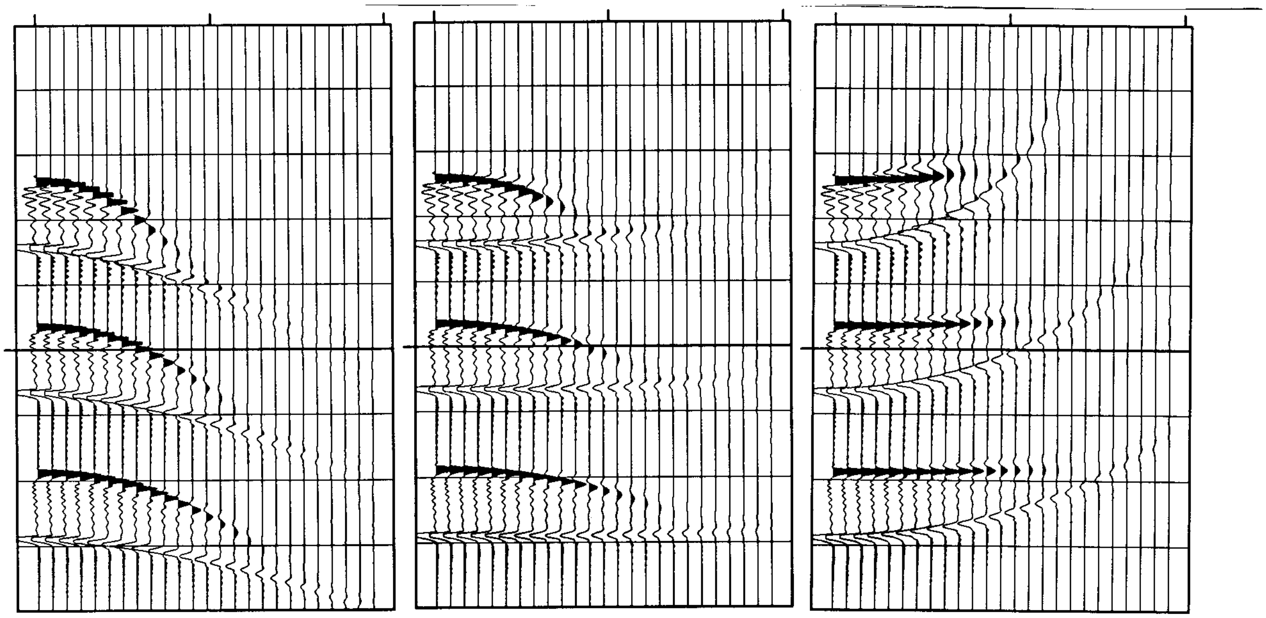

FIG. 2. Left, synthetic hyperbolas superposed. Other panels are after the two theoretical moveout velocities.

VELOCITY ANALYSIS

Hyperbola superposition (of various velocities) is the mathematical transpose of velocity analysis. Ordinarily these are done by hyperbolic superposition and summation. I hope a lower noise level and better invertibility can be achieved by using equation (8).

In classical migration, equation (6) is stepped downward into the earth from the earth's surface. In cascaded migration, equation (8) is initialized with surface data to which we associate $w=0$. The equation is continued until the desired w (velocity squared) is reached. Alternately, a hyperbola synthesis program can begin with a zero valued field $U(w_{\max}, t)=0$. As U is projected toward $w=0$, sources are added in at any desired locations in (t, w)-space. This is how figure 2 (left) was generated.

REFERENCES

Jakubowicz, H., and Levin, S., 1983, A simple exact method of 3-D migration: *Geophys Prosp.,* vol 31, pp 34-56.

Larner, Ken, and Craig Beasley, 1985, Cascaded migrations: a way of improving the accuracy of finite difference time migration. *Expanded Abstracts of the 1985 meeting of the SEG.*

Rothman, Dan, Stew Levin, and Fabio Rocca, 1985, Residual migration: Applications and limitations, *Geophysics,* Vol. 50, no. 1, p.110-126