Choice of parameters for $t\tau$ deconvolution

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ABSTRACT

The relation between simultaneous $t\tau$ deconvolution, presented by Claerbout in SEP-42, and iterative deconvolution in the t- and τ-domains is illustrated geometrically. One generalization of the $t\tau$ convolutional model which accounts for separate reverberations at shot and geophone locations was implemented. The increase in the number of filter coefficients did not improve the results of the deconvolution.

INTRODUCTION

In "Deconvolution essays" (SEP-41) Claerbout pointed out that deconvolution is aimed at compensating for shot waveform and near surface reverberation. Successive shot pulses ("bubbles") create parallel events on a shot gather (t-domain), while reverberation multiples or peglegs become parallel to primaries only after normal moveout correction (τ-domain refers below to normal moveout and spherical divergence corrected gathers). The effect of offset on multiple events is illustrated with a simple travel time diagram in SEP-41.

The distinctive patterns of bubble or reverberation multiples in the t- and τ-domains can be used to suppress these multiples by predictive deconvolution. The straightforward process of t-decon followed by τ-decon is not theoretically satisfactory, since it fails the "repetition test" defined in SEP-41. The repetition test requires that deconvolution does not modify an already deconvolved gather. In mathematical terms the deconvolution operator should be equal to its square. Linear operators having this property are called projections. The predictive t-decon and τ-decon, defined below as regressions, are projections; their product, however, is not a projection.

An alternative approach, based on a forward convolutional model which includes "bubble" and reverberation multiples was presented by Claerbout in SEP-42. According to that model, a single regression system is set for the simultaneous estimation of "bubble" and reverberation filters. The result of the simultaneous $t\tau$-deconvolution is identical to that obtained by an infinite iteration of t-decon followed by τ-decon. Another original feature of simultaneous $t\tau$-decon is that it solves the regression equation by conjugate gradients, without having to set the normal equations. Simultaneous $t\tau$-decon was applied to several data sets and results for a marine profile from Canada (Yilmaz and Cumro, 1983) are shown in SEP-42. Somewhat disappointingly, $t\tau$-deconvolution did not produce perceptibly better results than simple τ-deconvolution.

REGRESSION MODELS

Orthogonal projections

In this section we present general properties of regression models which apply to the problem of deconvolution as discussed below. In order to simplify the notations a model with only two regression variables is considered:

$$y_i \approx a + bx_i \quad \text{for } 1 \leq i \leq N$$

or, in vector notation:

$$\mathbf{Y} \approx a\,1 + b\,\mathbf{X} \tag{1}$$

In equation (1), $\mathbf{Y}$, 1 and $\mathbf{X}$ are given vectors of length N, and a and b are scalars to be determined. The vector of residuals E is:

$$E = \mathbf{Y} - a\,1 - b\,\mathbf{X}$$

The optimal regression coefficients $\hat{a}$ and $\hat{b}$ are found by minimizing the squared norm of the residuals, $E^t E$, thus defining a vector $\hat{\mathbf{Y}} = \hat{a}\,1 + \hat{b}\,\mathbf{X}$ which is "closest" to the vector $\mathbf{Y}$ in the plane spanned by 1 and $\mathbf{X}$. The residuals $\hat{\mathbf{E}} = \mathbf{Y} - \hat{\mathbf{Y}}$ are orthogonal to the plane spanned by 1 and $\mathbf{X}$ and the mapping P, such that $P(\mathbf{Y}) = \hat{\mathbf{Y}}$ is an orthogonal projection onto that plane (see Figure 1). Vectors in the projection plane are mapped into themselves, therefore a projection operator is equal to its square, $P \circ P = P$.

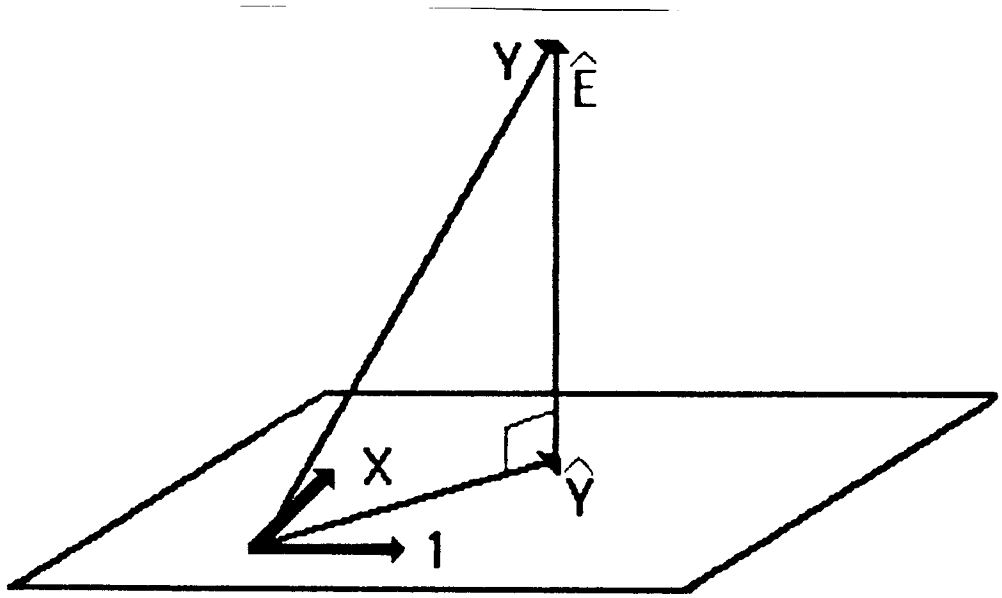

FIG. 1. The vectors 1 and **X** span a plane. The vector $\hat{\mathbf{Y}}$, solution of the regression equation (1), is the orthogonal projection of **Y** on that plane. The vector of residuals $\hat{\mathbf{E}}$ is orthogonal to the projection plane.

Simultaneous versus iterative regression

Assume that instead of solving the regression equation (1) simultaneously for a and b, we decide to solve first a regression equation for a, then another one for b, then again one for a, etc.. Regression for a can be represented as an orthogonal projection P_1, mapping onto the space spanned by the vector 1. Similarly regression for b is represented by an orthogonal projection P_2 mapping onto the space spanned by **X**. The product $P_1 \circ P_2$ is not a projection in general, since it is not equal to its square. However an infinite product of terms $(P_1 \circ P_2)$ does define a projection equal to the projection P representing the simultaneous regression for a and b. A geometrical illustration of this property is given in Figure 2. Figure 2 indicates that the solution of the iterative regression process and the simultaneous regression agree after an infinite number of iterations, except in two important particular cases when convergence requires only one iteration:

(1) for orthogonal regression variables 1 and **X**, that is $1^t X = 0$;

(2) for parallel regression variables, that is $\mathbf{X} = \alpha 1$.

A convenient way to assess the orthogonality of 1 and $\mathbf{X}$ is by comparing their scalar product $1^t\mathbf{X}$ to $\mathbf{X}^t\mathbf{X}$ and $1^t 1$. These products appear in the left-hand side matrix of the normal equations (Luenberger, 1968), also called the correlation matrix. For the regression system (1) the correlation matrix is:

$$(1\ \mathbf{X})^t\,(1\mathbf{X}) = \begin{pmatrix} 1^t 1 & 1^t\mathbf{X} \\ \mathbf{X}^t 1 & \mathbf{X}^t\mathbf{X} \end{pmatrix} \tag{2}$$

When two regression variables are parallel, the number of unknown regression coefficients can be decreased without changing the result of the regression by dropping one of the variables.

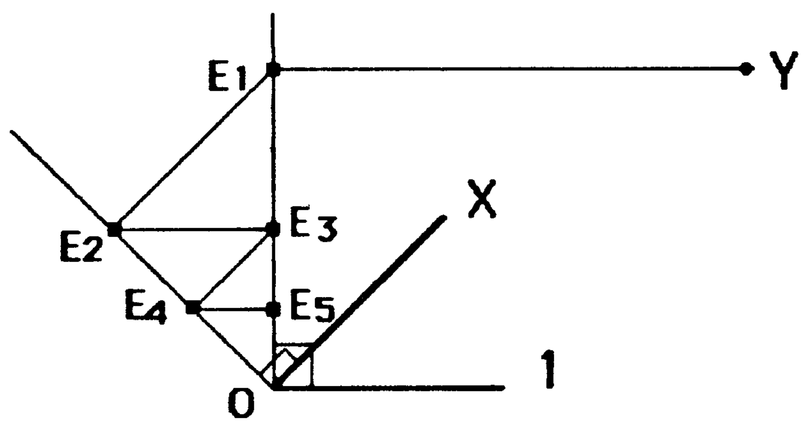

FIG. 2. In a two-dimensional space the vectors 1, $\mathbf{X}$ and $\mathbf{Y}$ are coplanar, hence $\mathbf{Y} = \hat{a}\,1 + \hat{b}\,\mathbf{X}$ and the regression residuals are zero (point O). The sequence of residuals E_1 (projection of $\mathbf{Y}$ on the orthogonal of 1), E_2 (projection of $\mathbf{Y}$ on the orthogonal of $\mathbf{X}$) , E_3 (projection of $\mathbf{Y}$ on the orthogonal of 1 again), etc. , converges to O . When 1 and $\mathbf{X}$ are orthogonal or parallel convergence is attained in one step.

Linear transformation of the regression variables

As mentioned in SEP-42 the regression equations for $t\tau$-decon may be written in the t-domain or in the τ-domain. The regression equations in the τ-domain are obtained from the regression equations in the t-domain by multiplying both sides of the equations by the NMO and spherical divergence operators. Are the solutions to these regression equations related by the same transformation, that is NMO and spherical divergence?

Consider the regression equation (1). Transforming the regression variables to another domain by a linear transformation is done by multiplication with a matrix $\mathbf{A}$. The resulting regression equation in the new domain is (3):

$$\mathbf{AY} \approx (\mathbf{A}\,1)a + (\mathbf{A\,X})b \tag{3}$$

In the original domain the residuals of equation (1) satisfy the condition:

$$\hat{\mathbf{E}}^t\ (\mathbf{Y} - \hat{\mathbf{Y}}) = 0 \tag{4}$$

If the transformed solution of (1), $\mathbf{A}\hat{\mathbf{Y}}$, is also a solution to (3), then

$$(\mathbf{A}\hat{\mathbf{E}})^t\ (\mathbf{AY} - \mathbf{A}\hat{\mathbf{Y}}) = 0$$

that is,

$$\hat{\mathbf{E}}^t\,\mathbf{A}^t\,\mathbf{A}\,(\mathbf{Y} - \hat{\mathbf{Y}}) = 0 \tag{5}$$

If we look for transformations $\mathbf{A}$ such that equation (5) holds for any choice of $\hat{\mathbf{E}}$ and $\mathbf{Y} - \hat{\mathbf{Y}}$ satisfying (4) , a necessary condition on $\mathbf{A}$ is:

$$\mathbf{A}^t\,\mathbf{A} = \lambda\,\mathbf{I}\,,\ \lambda \geq 0$$

This condition states that $\mathbf{A}$ should be proportional to a unitary transformation. Conversely, applying a unitary transformation followed by multiplication by a constant does not modify relative lengths. Therefore $\mathbf{A}\hat{\mathbf{E}}$ is the optimal vector of residuals for the regression (3).

Because the transformation *NMOt* relating the t- and τ-domains is not unitary (Biondi and Claerbout discuss unitary NMO in this report), solving the $t\tau$-decon equations in the t-domain and transforming the solution to the τ-domain will yield a different result from solving the decon equations in the τt-domain. Figure 3 gives a geometrical illustration of this property with an example of a transformation in the plane which is not unitary.

CONVOLUTIONAL MODEL

We recall first the convolutional model presented by Claerbout in SEP-42. At zero offset, or in the τ-domain, a synthetic model for reverberation is simply the convolution of a "reverberation" filter with a white reflectivity sequence denoted below as *random* . Offset and attenuation effects are included by following the reverberation filter with the $\frac{1}{t} NMO^{-1}$ operator. The synthetic model is now in the t-domain and the effect of the bubble is modeled by convolution with a "bubble" filter. Formally,

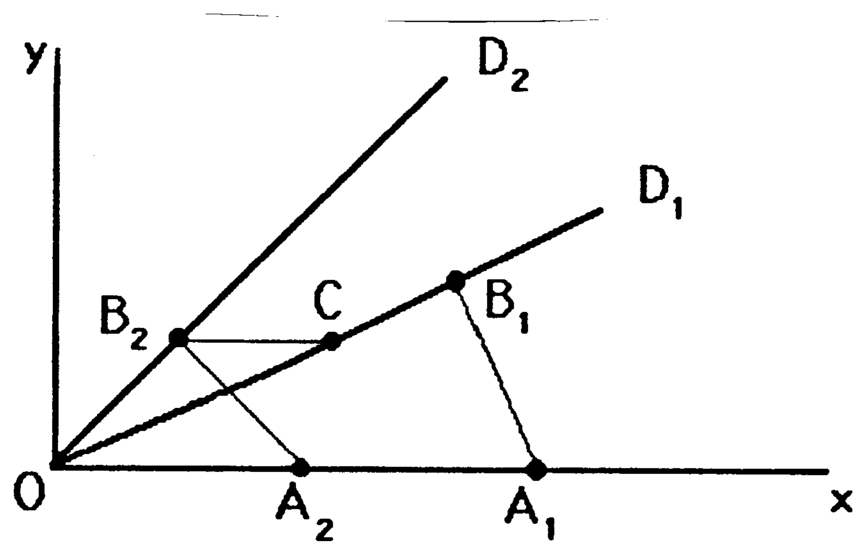

FIG. 3. Regression in the original domain will be represented as an orthogonal projection on the line D_1. The linear transformation shrinks the Ox axis by 0.5 and leaves the Oy axis unchanged. The inverse of this transformation is a stretching of the Ox axis by a factor of 2. Point A_1 projects onto B_1. The linear transformation maps A_1 into A_2, and the first bisector D_1 into the line D_2. A_2 projects then into B_2 on the line D_2. The inverse transformation maps B_2 into C, which is different from the starting point A_1.

$$data \approx \frac{1}{1 - bub} \frac{1}{t} NMO^{-1} \frac{random}{1 - rev} \tag{6}$$

Several possible generalizations are mentioned in SEP-42. One of them, distinguishing reverberation at the shot from reverberation at the geophone locations, was implemented and the results are shown in the next section. Reverberation at the shot is modeled with an offset-independent filter " *revs* ". Reverberation at geophone locations involves an offset dependent filter " *revg* ". Basically the extension amounts to splitting the term $\frac{1}{1 - rev}$ in equation (1), into the product $\frac{1}{1 - revs} \frac{1}{1 - revg}$, leading to:

$$data \approx \frac{1}{1 - bub} \frac{1}{t} NMO^{-1} \frac{1}{1 - revs} \frac{random}{1 - revg} \tag{7}$$

In order to derive a linearized model we follow the same steps as in SEP-42. Equation (7) is multiplied successively by the inverse of of each operator,

$$(1 - revg)(1 - revs)NMO\ t(1 - bub)data \approx random \tag{8}$$

then terms of order higher than one in the unknowns are neglected, and a regression equation is obtained by requiring the norm squared of random to be small:

$$NMO\ t\ data \approx revs*NMO\ t\ data + revg*NMO\ t\ data + NMO\ t\ bub*data \tag{9}$$

Equation (9) can be rewritten with matrix multiplications instead of convolutions as indicated in SEP-42. Equation (8) is expressed in the τt-domain. Another possibility is to set the regression equations in the t-domain as:

$$data \approx NMO^{-1}\frac{1}{t}\ revs*NMOtdata + NMO^{-1}\frac{1}{t}revg*NMOtdata + bub*data \tag{10}$$

As mentioned before, the solution of equation (9) is not the solution of (10) transformed by *NMOt*.

EXAMPLES

The data set used in the examples is a marine profile from Canada (Yilmaz and Cumro, 1983). The same data were used in SEP-42, (Claerbout, 1985). In all examples, the filters have a 60ms gaps between the leading spike and the next non-zero coefficient. The NMO velocity increases linearly with depth from 1.5km/s to 3km/s.

Figure 4 shows deconvolved gathers with offset independent filters. One gather is deconvolved with the reverberation filter only, one with the bubble filter only, and one with the bubble and the reverberation filters. All time samples are used in the estimation and the lengths of the filters are 30 time samples as in SEP-42.

Figure 5 shows the correlation matrix for the regression system (9). The matrix is symmetric and composed of four approximately Toeplitz matrices. The upper left block is the correlation matrix for the reverberation filter, the lower right matrix is the correlation matrix for the bubble filter, and the off diagonal block matrices are cross-correlation matrices between data in the t- and the τ-domains. The strong cross-correlation is an indication that bubble and reverberation filters are "parallel".

Figure 6 presents results in the τ-domain for offset dependent reverberation filters, estimated in windows along the geophone axis. We used a constant reverberation filter across the gather, which models the reverberation at the shot location. No bubble filter was included. The lengths of the filters are 30 time samples. The numbers of estimation windows for the offset dependent filters are 1, 4 and 8. The increased number of filter coefficients brought only a slight improvement in the deconvolution.

Figure 7 shows the data in time domain and their decomposition in three frequency bands. The low-passed gather shows very interesting events with linear moveout which seem evidence for bubble multiples. Are such multiples present for reflection events? Why was the bubble filter not successful at removing this low-frequency noise?

CONCLUSION

This paper provides a better understanding of the idea of simultaneous $t\tau$-deconvolution, and its relation to iterative t- and τ-decon. A good forward model, including the effect of bubble and reverberation multiples is a helpful tool for improving deconvolution. However, examples better deconvolved gathers are yet to come. The present reverberation and bubble filters appear statistically redundant and a more specific description of the events to be deconvolved will certainly help.

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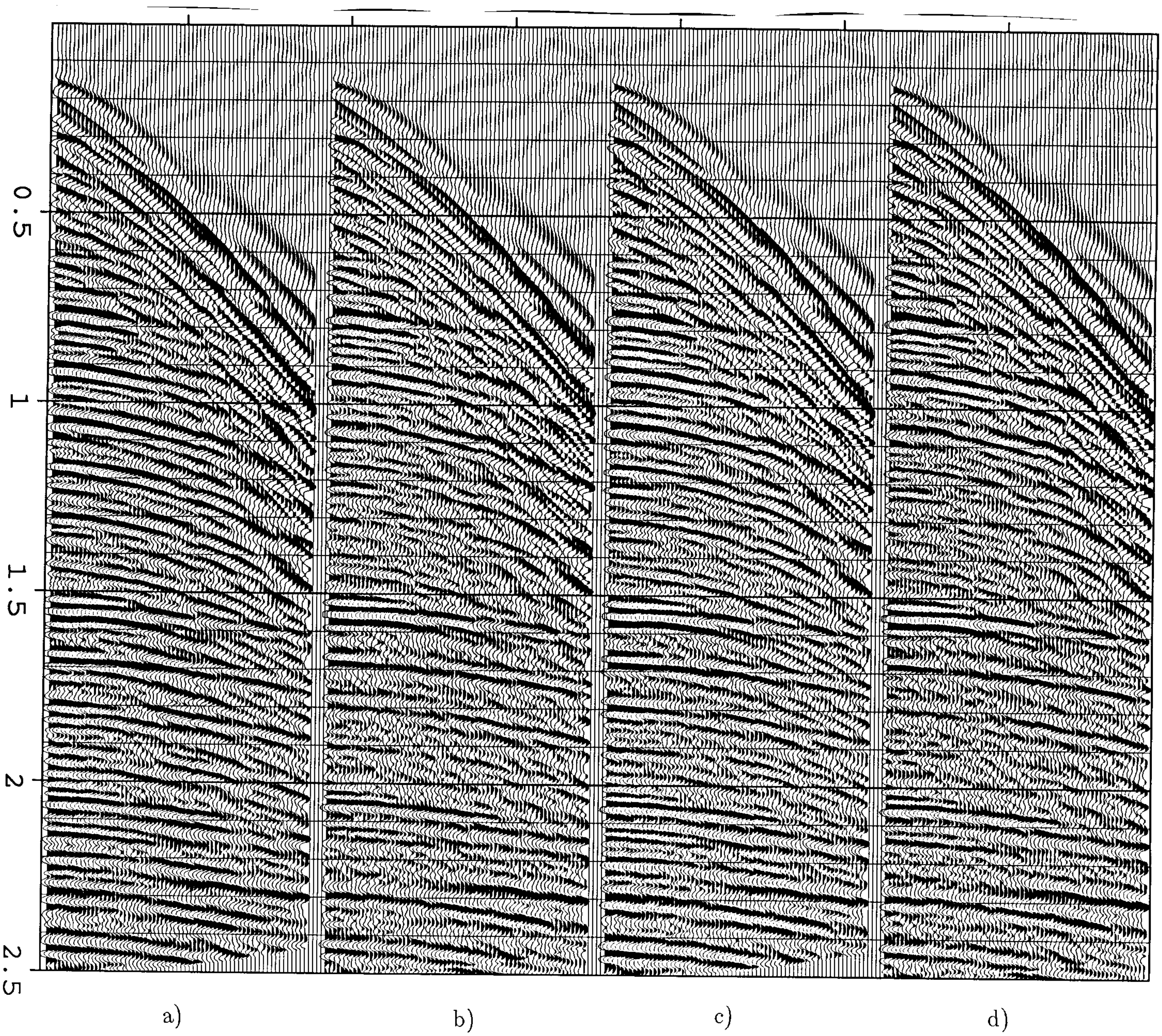

FIG. 4. Simultaneous $t\tau$-deconvolution with offset independent filters. From left to right: a) NMO and spherical divergence corrected data; b) bubble and reverberation filters each 30 time samples long; c) bubble filter only, length 30 time samples; d) reverberation filter alone, length 30 time samples.

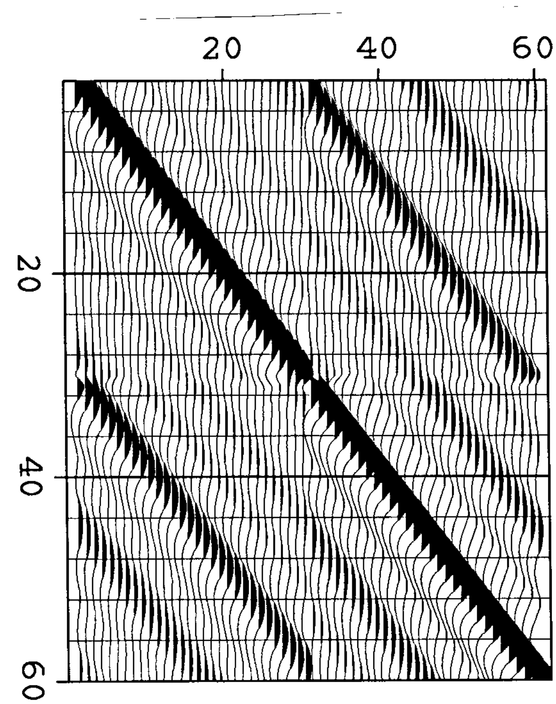

FIG. 5. Correlation matrix for 30 reverberation filter coefficients and 30 bubble filter coefficients. Each block of the matrix is approximately Toeplitz. The upper left block is the correlation matrix for the data in t-domain up to a lag of 30. The lower right block is the correlation matrix for data in τ-domain up to a lag of 30. The off diagonal blocks are the cross-correlation matrices between data in t- and τ-domains. High values of off-diagonal terms indicate parallel regression variables.

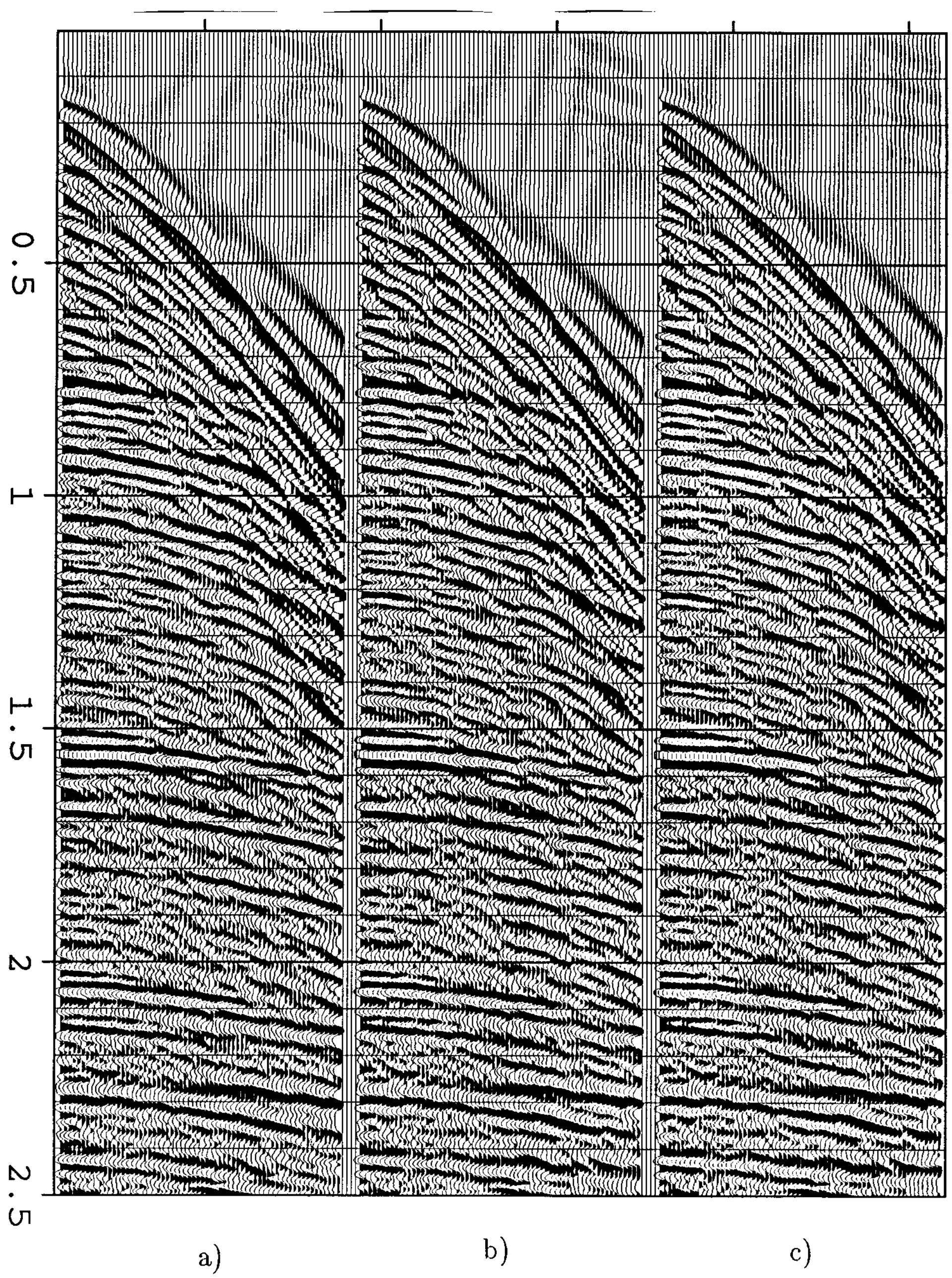

FIG. 6. Results in τ-domain for $t\tau$-decon with shot reverberation filter and offset dependent geophone reverberation filters. The lengths of the filters are 30 time samples. a) one geophone reverberation filter for all offsets; b) four windows for geophone reverberation filters; c) eight windows for geophone reverberation filters. A slight improvement is noticeable in frames b) and c) at 1.3 and 1.6 s near zero offset.

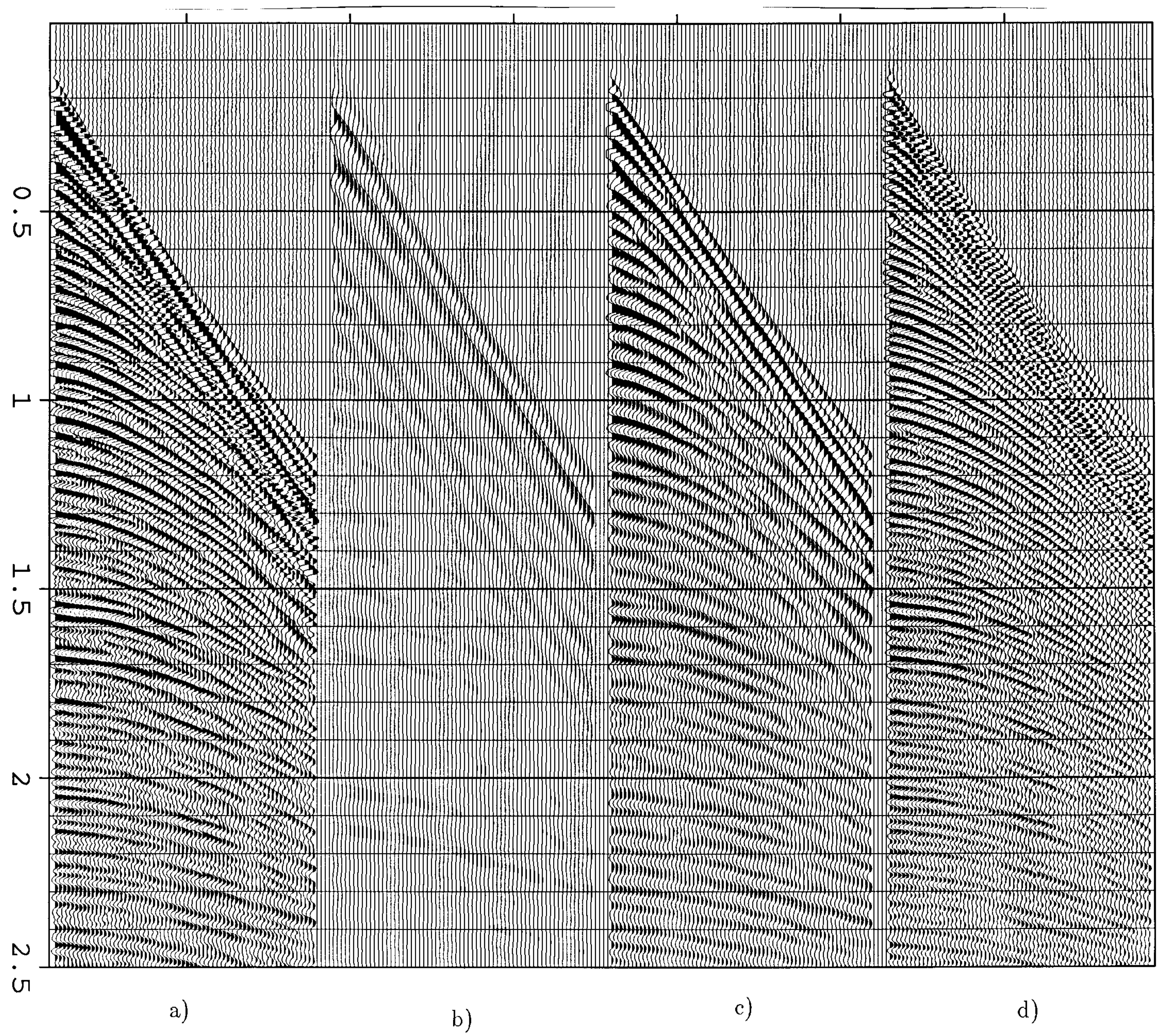

FIG. 7. Frequency decomposition of t-domain data: a) field data; b) frequency band 0-10Hz; c) frequency band 10-25Hz; d) frequency band 25-60Hz. Notice the event with linear moveout on the low passed gather b).