Pseudounitary NMO

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ABSTRACT

The NMO is a linear operator that performs a transformation of variables; in reflection seismology it is used in the moveout of a CDP gather. Stolt's migration is NMO in the frequency domain. In this paper we derive two new NMOs that are unitary operators. We compare the new transformations with the conventional NMO by using the results of a normal moveout of a synthetic CDP gather and the output of a Stolt's migration.

INTRODUCTION

To perform the NMO transformation of variables of a function defined on a discrete domain we need to interpolate the data and to evaluate it on a new grid. The NMO operator can be represented, for any fixed offset or wavenumber, by a matrix containing all zeros except an interpolation operator centered along a hyperbolic trajectory (Claerbout, 1985). In practice the length of the interpolator is finite, and the applied operator is an approximation of the desired transformation. The quality and cost of the NMO operation depend on the kind of interpolator used. In Figures 1 and 2 two quite different examples of conventional NMO matrix are compared; the former was computed using a short and cheap interpolator as the linear, the latter using an infinite sinc interpolator.

Since a unitary NMO has some theoretical advantages, we want to discover whether a unitary operator is a good approximation of the desired transformation of variables. We derive a unitary NMO starting from a conventional NMO, aiming to get an operator better than the conventional one from which we started. In particular, we derive the unitary transformation from a conventional NMO computed with a short and cheap interpolator, in an attempt to get a good NMO without using a long and expensive interpolator.

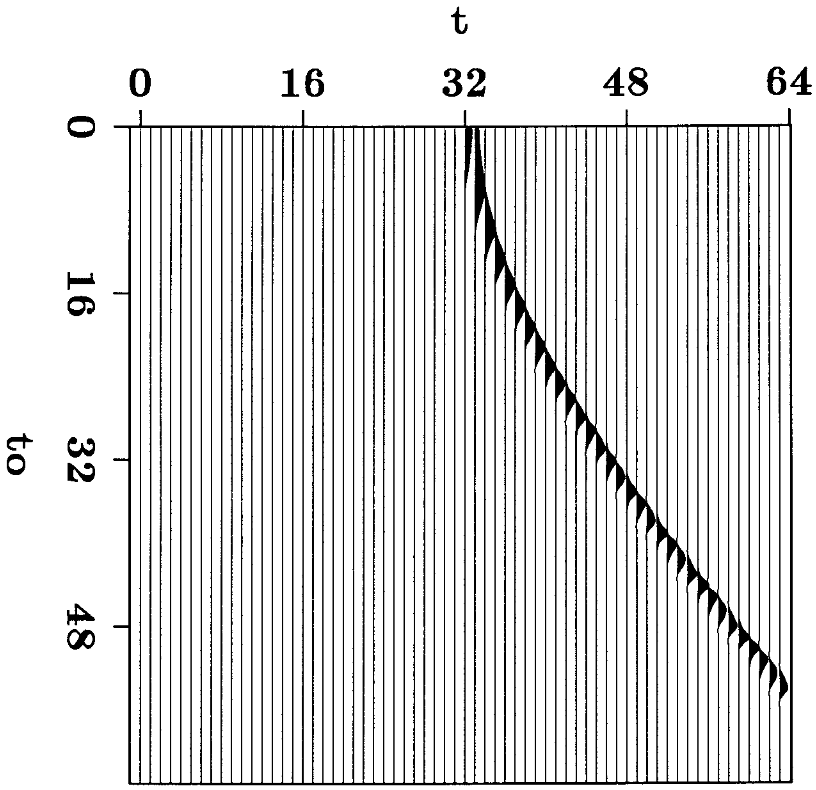

FIG. 1: An example of an NMO matrix using a linear interpolator. All the figures were plotted with a nonlinear gain to better show the particulars of the data; more precisely $plot = data^{\gamma}$. In this figure $\gamma = 1$.

WHY A PSEUDOUNITARY NMO ?

In the time domain NMO is given by the linear transformation of the (t, h) plane into the (t_0, h) plane operating the following change of variables

$$t_0^2 = t^2 - h^2/v^2 \, . \tag{1}$$

The transformation is singular, and its null space is given by the elements in the (t, h) plane for which $t^2 - h^2/v^2 < 0$. The null space corresponds in a CDP gather to the upper part of the trace before the direct arrival, or, in the case of Stolt's migration, to the evanescent energy.

In the practical computation of NMO the null space also depends on whether the samples for which $t^2 - h^2/v^2 < 0$ are used for interpolating the data, even if the interpolated function will be evaluated only at $t_0^2 \geq 0$; this choice depends on the particular application of NMO. For example, in computing the matrix in Figure 2 all the data is used for the interpolation; furthermore, since the sinc is an infinite length interpolator, the null space of this matrix is empty.

A unitary matrix has an empty null space and so cannot be used to approximate a singular transformation as NMO. What we actually derive is a pseudounitary NMO whose null space is

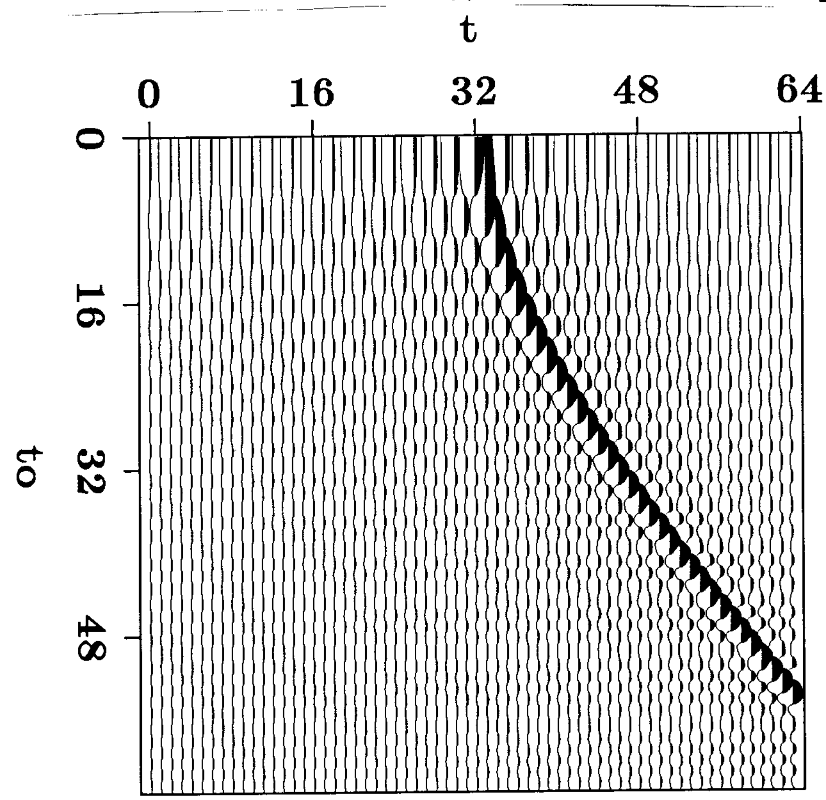

FIG. 2: An example of an NMO matrix using a sinc interpolator. This figure shows the structure of an NMO matrix with an interpolator centered on a hyperbolic trajectory ($\gamma = 0.5$).

the same as the conventional NMO. We will define operatively a matrix as pseudounitary if its transpose is equal to its pseudoinverse.

We have seen that the null space of the **NMO** matrix is the upper part of the data space; consequently, the product of the pseudoinverse $\mathbf{NMO}^{\dagger}$ with the **NMO** itself is $\mathbf{NMO}^{\dagger}\,\mathbf{NMO} = \tilde{\mathbf{I}}$, where $\tilde{\mathbf{I}}$ is a diagonal matrix made out of two blocks. The lower block is an identity matrix; the upper block is a zero matrix and corresponds to the null space of the NMO transformation. We will derive a pseudounitary matrix $\mathbf{NMO}^{\ddagger}$ such that

$$(\mathbf{NMO}^{\ddagger})^T\,(\mathbf{NMO}^{\ddagger}) = \tilde{\mathbf{I}}\,. \tag{2}$$

A nice property of a pseudounitary NMO is that it does not affect the energy of the data. If we define the energy of a trace $\mathbf{x}$ as $E(\mathbf{x}) = \sum_i x_i^2 = \mathbf{x}^T\mathbf{x}$, and we moveout this trace with a pseudounitary operator, the resulting $\mathbf{y} = \mathbf{NMO}^{\ddagger}\,\mathbf{x}$ will have the same energy as $\mathbf{x}$, less the energy corresponding to the data in the null space. From equation (2) we can derive

$$E(\mathbf{y}) = \mathbf{y}^T\,\mathbf{y} = \mathbf{x}^T\,(\mathbf{NMO}^{\ddagger})^T\,(\mathbf{NMO}^{\ddagger})\,\mathbf{x} = \mathbf{x}^T\,\tilde{\mathbf{I}}\,\mathbf{x}\,. \tag{3}$$

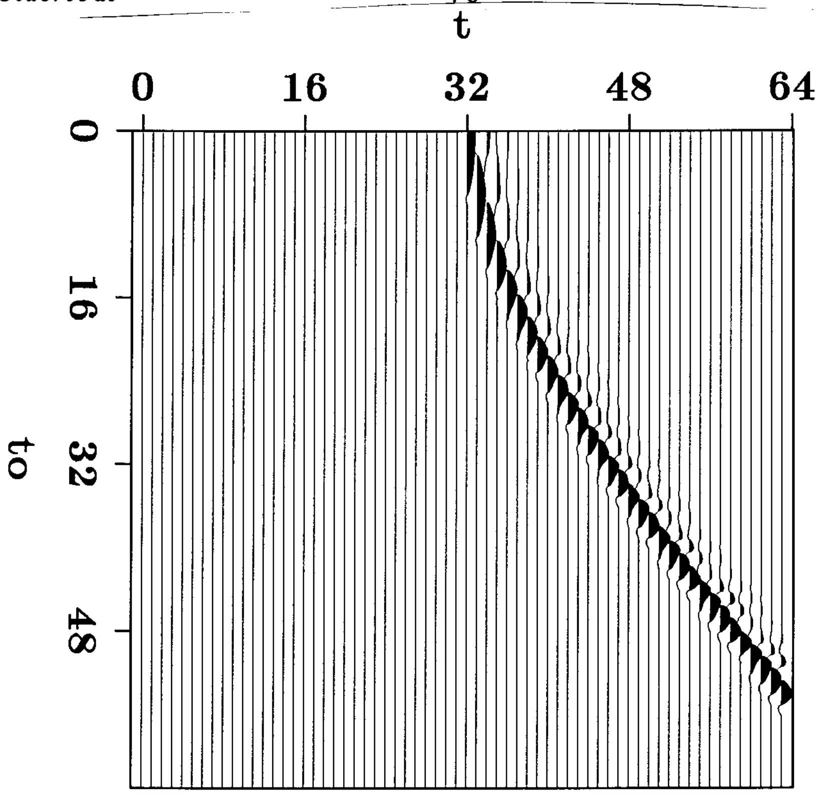

FIG. 3: A pseudounitary NMO matrix derived by a Cholesky decomposition. This operator was obtained from the matrix in Figure 1. ($\gamma = 0.5$)

This relation is valid for every trace at every offset. Therefore it can be said that the energy in a CDP gather is moved, but the total amount of energy is recorded after the first arrival does not change. The wave equation is also an operator that moves the energy but does not affect its total amount. This similarity of behavior in the two operators is a theoretical advantage of the pseudounitary NMO over the conventional NMO.

Furthermore, with the pseudounitary NMO it is possible to go from the (t, h) space to the (t_0, h) space and back from (t_0, h) to (t, h) an infinite number of times without degrading the data and without computing the pseudoinverse of a matrix. This property is also an immediate consequence of equation (2): if $\mathbf{y}$ is the moved-out data and $\tilde{\mathbf{x}}$ is the data back in the (t, h) space, the result is

$$\tilde{\mathbf{x}} = (\mathbf{NMO}^{\ddagger})^T \mathbf{y} = (\mathbf{NMO}^{\ddagger})^T (\mathbf{NMO}^{\ddagger}) \mathbf{x} = \tilde{\mathbf{I}} \mathbf{x} . \tag{4}$$

Then, passing from $\mathbf{x}$ to $\tilde{\mathbf{x}}$, we have lost only the data in the null space of the NMO.

TWO PSEUDOUNITARY NMOs

We have tested two kinds of unitary NMOs. Each of them was derived from a conventional NMO computed using a linear interpolator, but the way of determining them is general, and may be applied to an NMO computed with any interpolator.

A pseudounitary NMO by a Cholesky decomposition

A family of different pseudounitary NMOs may be derived from all the possible decompositions of the product $\mathbf{NMO}^T\mathbf{NMO}$ in two factors $\mathbf{F}$ and $\mathbf{F}^T$, such that $\mathbf{NMO}^T\mathbf{NMO} = \mathbf{F}^T\mathbf{F}$. All the resulting matrices $\mathbf{NMO}\ \mathbf{F}^{-1}$ will be pseudounitary and will approximate the NMO transformation of variables. A fast matrix decomposition is the Cholesky factorization.

A symmetric positive definite matrix $\mathbf{A}$ may be factorized by a Cholesky decomposition in the product of two triangular matrices, such that $\mathbf{A} = \mathbf{U}^T\mathbf{U}$, where $\mathbf{U}$ is upper triangular (Golub, 1983). Actually $\mathbf{NMO}^T\mathbf{NMO}$ is not positive definite, and we cannot directly compute its Cholesky decomposition. But it is a block matrix, and its lower block $\mathbf{B}$ is a symmetric positive definite matrix; therefore $\mathbf{B}$ may be decomposed as $\mathbf{B} = \mathbf{U}^T\mathbf{U}$. Adding zeroes on the top of $\mathbf{U}^{-1}$ we get a $\tilde{\mathbf{U}}^{-1}$ that is full dimensioned. Now

$$\widetilde{\mathbf{NMO}} = \mathbf{NMO}\ \tilde{\mathbf{U}}^{-1}. \tag{5}$$

will be the pseudounitary matrix that we wanted.

This way of computing the pseudounitary NMO is quite inexpensive. The cost of the Cholesky decomposition is of the order of $O(n)$, when the dimension n of the matrix that must be decomposed is decreasing with the offset as the null space of NMO increases. Furthermore, $\tilde{\mathbf{U}}$ is upper triangular, and thus its inverse may be computed easily with a back substitution.

This computational economy allows the $\widetilde{\mathbf{NMO}}$ transformation to be suitable for practical use; on the other hand, as we will show , the results of the tests do not demonstrate any improvements on the conventional NMO. The operator derived by the matrix in Figure 1 is shown in Figure 3; one of its characteristics is that it is a causal operator.

A pseudounitary NMO by singular value decomposition

By a singular value decomposition (SVD) of a square matrix it is possible to derive the unitary matrix that is the closest to it, in the Euclidean norm, from the space of all the unitary matrices (Ben-Israel et al., 1980).

If we compute the singular value decomposition $\mathbf{NMO} = \mathbf{U}\,\mathbf{\Lambda}\,\mathbf{V}^T$ and we substitute for the diagonal matrix of the singular values $\mathbf{\Lambda}$ the identity matrix $\mathbf{I}$ the resulting matrix $\mathbf{UNIT} = \mathbf{U}\,\mathbf{I}\,\mathbf{V}^T$ will be the closest unitary matrix to $\mathbf{NMO}$.

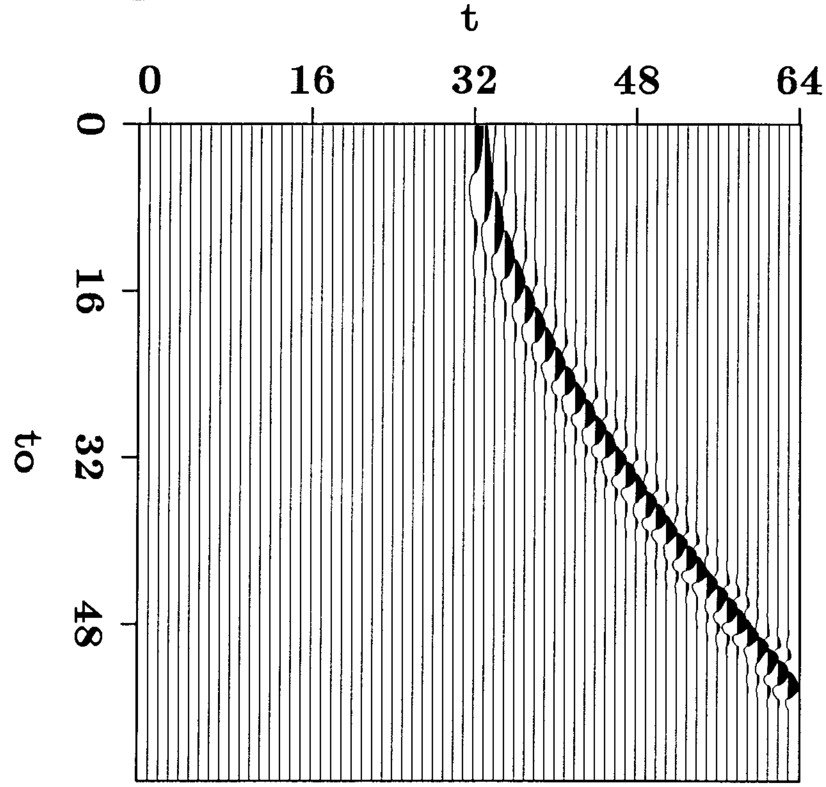

FIG. 4: A pseudounitary NMO derived by a SVD of the matrix in Figure 1 ($\gamma = 0.5$).

But, as we have seen, what we actually want is a pseudounitary NMO. To compute it we substitute for $\mathbf{\Lambda}$ a diagonal matrix $\hat{\mathbf{I}}$ whose elements are either equal to zero, when the correspondent singular values are equal to zero, or equal to one. The resulting matrix

$$\widehat{\mathbf{NMO}} = \mathbf{U}\,\hat{\mathbf{I}}\,\mathbf{V}^T \tag{6}$$

is now a pseudounitary matrix. Leaving unaltered the singular values that are equal to zero means that we leave unchanged the null space of the transformation, that is, the null spaces of **NMO** and $\widehat{\mathbf{NMO}}$ are the same.

While this $\widehat{\mathbf{NMO}}$, shown in Figure 4, gave better results in the tests than the pseudounitary NMO derived by the Cholesky decomposition, it is only of theoretical interest. It is of no practical use because of the cost of computing it using a singular value decomposition in addition to the cost of applying it to the data as a matrix times vector multiplication.

SYNTHETIC TESTS

We have tested the performances of the two new pseudounitary NMOs versus the conventional NMO. Our tests were the stacking of a synthetic CDP gather and the Stolt's migration of a time section.

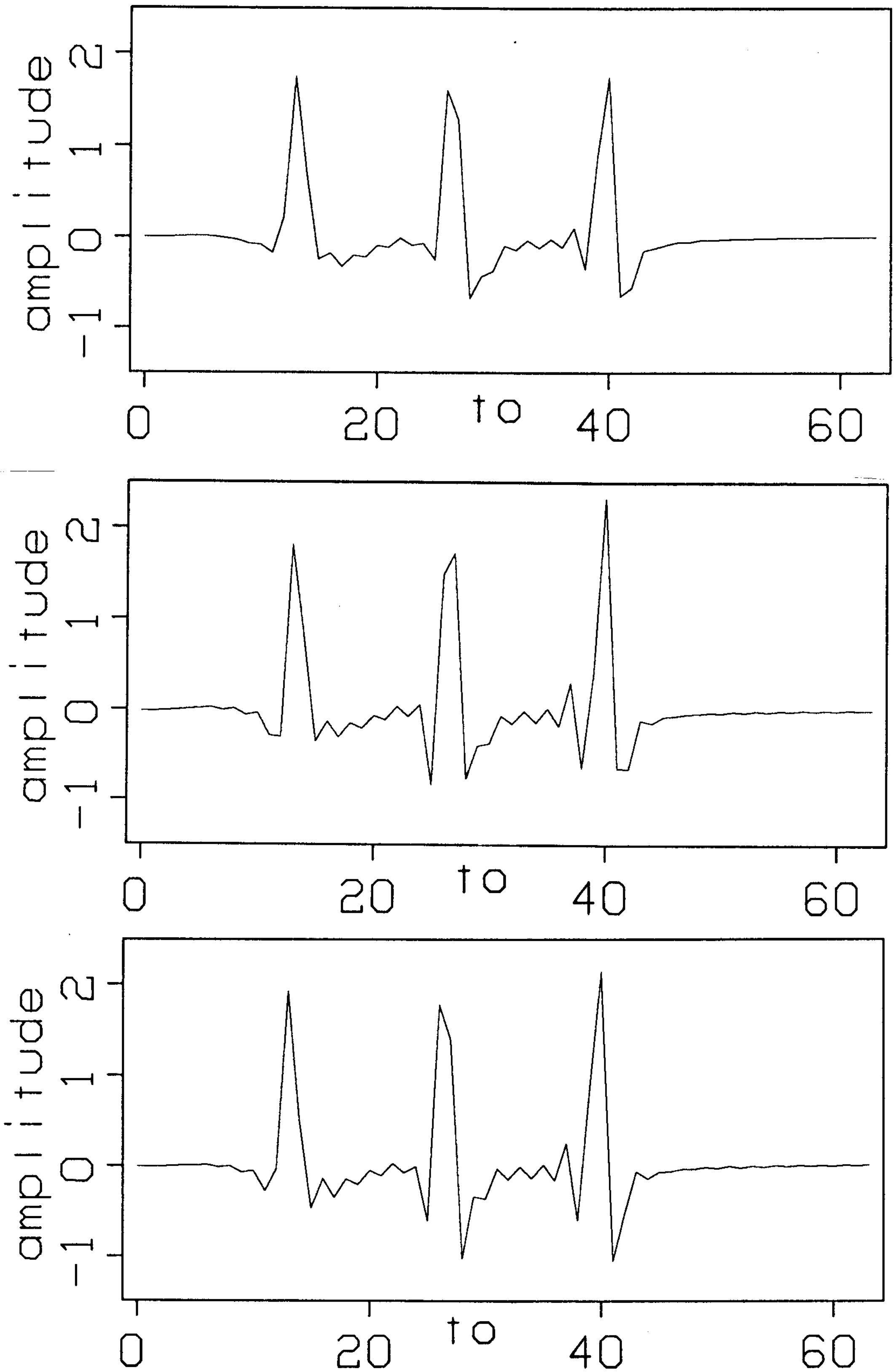

FIG. 5: Stacked traces from a synthetic CDP gather using the three different NMOs: (a) conventional NMO with a linear interpolator; (b) pseudounitary NMO derived by a Cholesky factorization; (c) pseudounitary NMO computed by a Singular Value Decomposition.

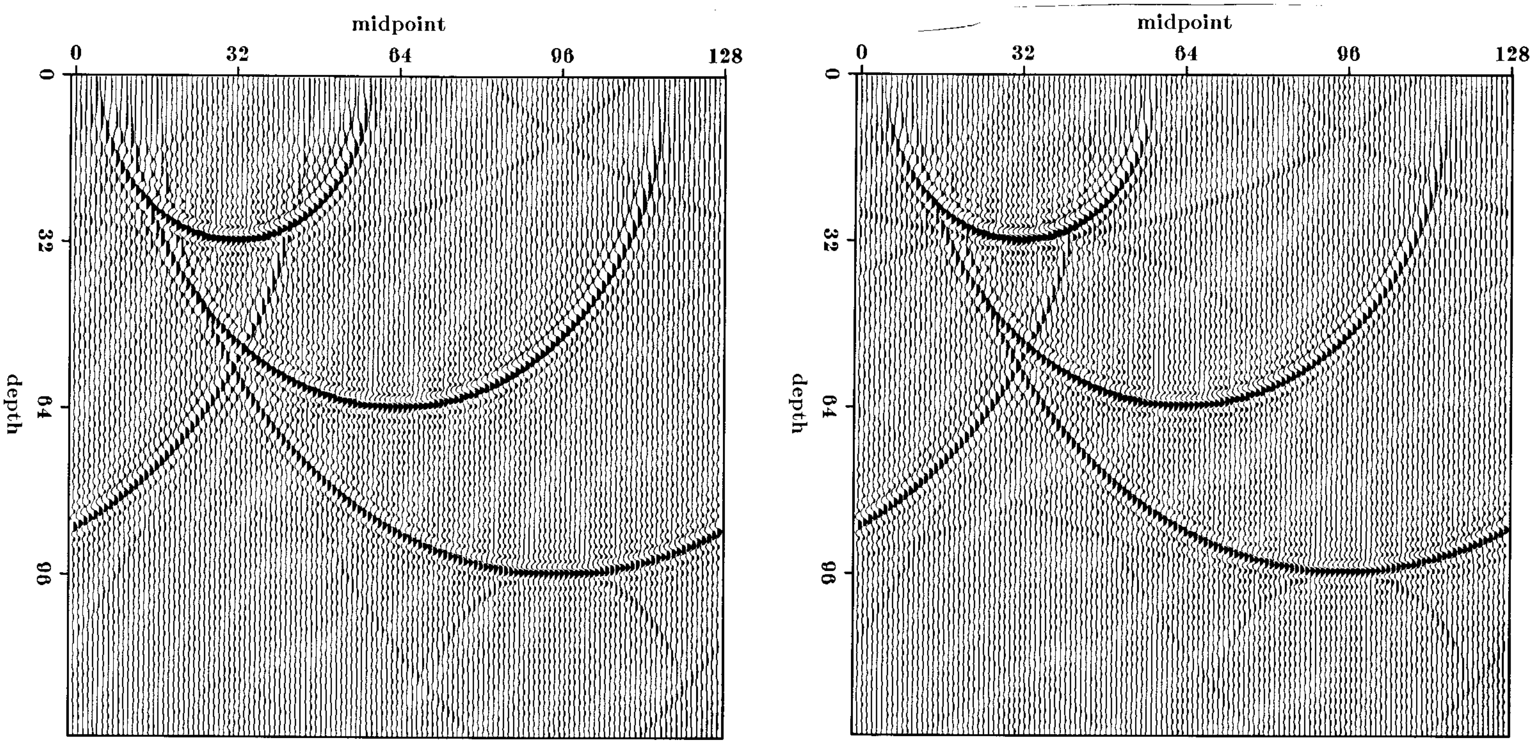

FIG. 6: Migrated sections using the conventional NMO with a linear interpolator, on the left, and the pseudounitary NMO derived by a Cholesky decomposition. The use of the Cholesky NMO does not decrease the interpolation artifacts ($\gamma = 0.5$).

The synthetic CDP gather was modeled by a phase-shift program assuming as a geological model three flat layers in a homogeneous medium. The resulting moved-out sections do not show an appreciable variation of behavior of the three different NMOs. Also the stacked traces shown in Figure 5 are quite similar; the only evident differences are the higher positive spike and the more important negative sidelobes in the stacked traces given by the new pseudounitary NMOs.

In Figures 6 and 7 are shown the results of the Stolt's migration of a time section composed of three points at different travel times. An algorithm with demodulation was applied to decrease the interpolation artifacts near the center of the section (Ronen, 1982). The differences in the results are much more evident in the migrated sections than in the moved-out CDP gathers.

In Figure 6 we can compare the outcomes of migrations using the conventional NMO with linear interpolator and the Cholesky pseudounitary NMO: it is clear that the artifacts don't decrease using the latter. On the contrary the final result applying the SVD pseudounitary NMO shows significant improvements and is comparable with the one computed using a sinc interpolation in the frequency domain.

We can find a possible explanation of these results by looking, in the precedent sections, at the figures showing the examples of the different matrices (Figure 1, 2 and 3). The pseudounitary

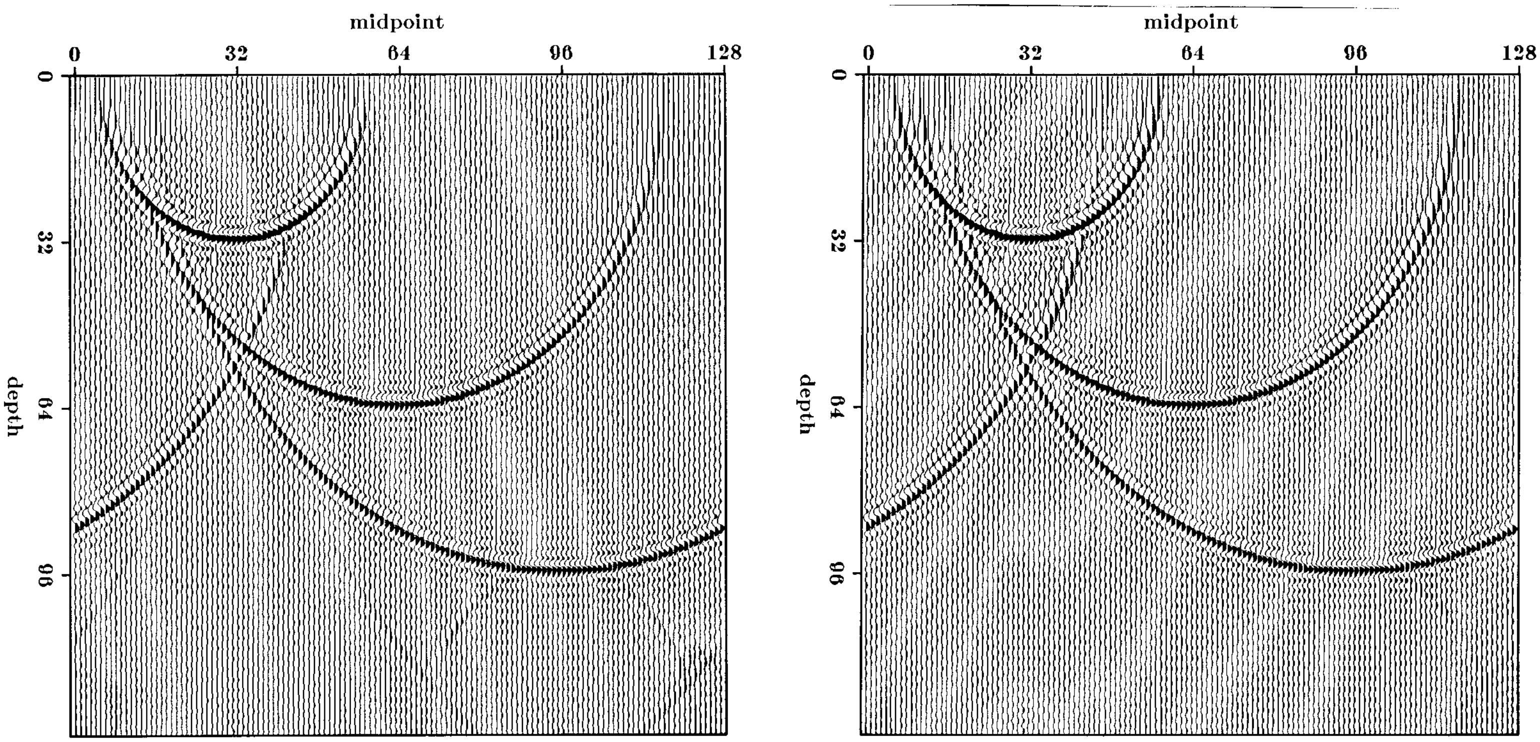

FIG. 7: On the left is the section migrated using the SVD pseudounitary NMO. The quality of the result is comparable with the the migrated section on the right; which was obtained with a conventional NMO using a sinc interpolator. The left section is is distinctly better than the migrated sections in Figure 6 ($\gamma = 0.5$).

NMO derived by a singular value decomposition is a symmetric and long operator. The Cholesky pseudounitary NMO is, on the other hand, a causal operator and is sometimes even shorter than the ordinary linear interpolator.

CONCLUSIONS

The unitary NMO is an appealing idea from the theoretical point of view. The good results given by the pseudounitary transformation derived by the singular value decomposition of a conventional NMO testify to the validity of the unitary NMO. Unfortunately, the Cholesky pseudounitary NMO, which is the only one suitable for a practical application, did not prove to be better than the conventional NMO. Of course, the possibility still exists that another inexpensive way can be found of deriving a pseudounitary NMO that gives better results than the conventional one.

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