A Practical Anisotropic System

Francis Muir and *Joe Dellinger*

INTRODUCTION

We are interested in extending the repertory of modeling, estimating, correcting, and imaging programs to allow for the effects of anisotropy, and to this end we have developed a framework of equations that honor two constraints:

1) Not committed to any particular anisotropic model.

We wish to accomodate both intrinsic and extrinsic forms of anisotropy, and, in the latter case, both low-frequency, Backus models and high-frequency, Dix models. In the end, of course, it is likely that all three, and possibly other, models will play a role in unraveling the earth's lithology from seismic and other data.

2) A straightforward connection between wave and ray equations.

This is in contrast to the classical elastic anisotropic model, where there is no direct path between waves and rays, but only through parametric relationships.

A Uniform Structure

All the equations have the same rational, multinomial structure:

$$f = \frac{x^2 + axy + y^2}{x + y} \tag{1}$$

where, if $a = 2$, we return to a simple, linear form:

$$f = x + y \tag{2}$$

The plane wave equation

$$W = \frac{(W_z \cos^2)^2 + (1 + q_w) W_z W_x (\cos \sin)^2 + (W_x \sin^2)^2}{W_z \cos^2 + W_x \sin^2} \tag{3}$$

where W is the alacrity (velocity squared) normal to the wave, W_z and W_x are the vertical and horizontal alacrities, the circular functions are with respect to the angle from vertical, and q_w is the anelliptic factor, which, set equal to unity gives:

$$W = W_z \cos^2 + W_x \sin^2 \tag{4}$$

a simple, elliptic form.

The dispersion relation

$$\omega^2 = \frac{(W_z k_z^2)^2 + (1 + q_w) W_z W_x k_z^2 k_x^2 + (W_x k_x^2)^2}{W_z k_z^2 + W_x k_x^2} \tag{5}$$

following directly from the plane wave equation.

The differential operational constraint

$$D_t^2 = \frac{(W_z D_z^2)^2 + (1 + q_w) W_z W_x D_z^2 D_x^2 + (W_x D_x^2)^2}{W_z D_z^2 + W_x D_x^2} \tag{6}$$

again returning to a familiar elliptic form when $q_w = 1$:

$$D_t^2 = W_z D_z^2 + W_x D_x^2 \tag{7}$$

The ray equation

$$M = \frac{(M_z \cos^2)^2 + (1 + q_m) M_z M_x \cos^2 \sin^2 + (M_x \sin^2)^2}{M_z \cos^2 + M_x \sin^2} \tag{8}$$

where M is the ray sloth (inverse velocity squared) for some angle, M_z and M_x are, respectively, the vertical and horizontal ray sloths, the circular functions are with respect to the ray angle from the vertical, and q_m is the anelliptic factor. Again, if $q_m = 1$ then:

$$M = M_z \cos^2 + M_x \sin^2 \tag{9}$$

Of course, we can always write down similar forms for the wave and ray equations. What is perhaps surprising is that the two forms are consistent to fourth-order paraxial approximation and to second-order at horizontal, provided that:

$$M_z = 1/W_z, \quad M_x = 1/W_x, \quad \text{and} \quad q_m = 1/q_w \tag{10}$$

For example, if we fit our model dispersion relation to some "real" dispersion relation at vertical and horizontal, and also fit the curvature at vertical, then our form of the impulse response will fit the "real" impulse response also at vertical and horizontal, and the curvature at vertical. The reasons for this are discussed in the companion paper in this report "Two Domains of Anisotropy".

The other surprise is how well the rational polynomial form fits away from the axial control points - providing that we recognize that these new forms can never model the sometimes triplicating behaviour of real elastic anisotropy. Figure (1) illustrates how closely the impulse response can model elastic behaviour, even in the (for most people) somewhat extreme anisotropy of the Greenhorn Shale of Jones and Wang.

Modeling NMO equation

This follows directly from the ray equation:

$$T(x)^2 = \frac{T(0)^4 + (1+q_m)T(0)^2 M_x x^2 + (M_x x^2)^2}{T(0)^2 + M_x x^2} \tag{11}$$

reducing to the familiar form:

$$T(x)^2 = T(0)^2 + M_x x^2 \tag{12}$$

in case $q_m = 1$. Note also that M_z is not involved in the equation. As with the conventional hyperbolic form, move-out on a record contains, intrinsically, no information on time/depth conversion.

Processing NMO equation

In case of a practical NMO routine, it makes sense to parameterise in terms of the apparent horizontal sloth, corresponding to the paraxial fitting ellipse, rather than the true horizontal sloth. In this case, M_x is replaced by $q_w M_{ex}$, where M_{ex} is the horizontal sloth of the paraxial ellipse, and we have:

$$T(x)^2 = \frac{(T(0)^2)^2 + (1+q_w)T(0)^2 M_{ex} x^2 + (q_w M_{ex} x^2)^2}{T(0)^2 + q_w M_{ex} x^2} \tag{13}$$

This NMO equation forms the basis for an NMOR routine, NEMESIS, which is used by Marta Woodward in a companion paper in this report.

Parameter Bounds

The principal restriction on sensible behaviour of the model is that the phase slowness surface and the discontinuity surface of the impulse response are convex. In turn this means:

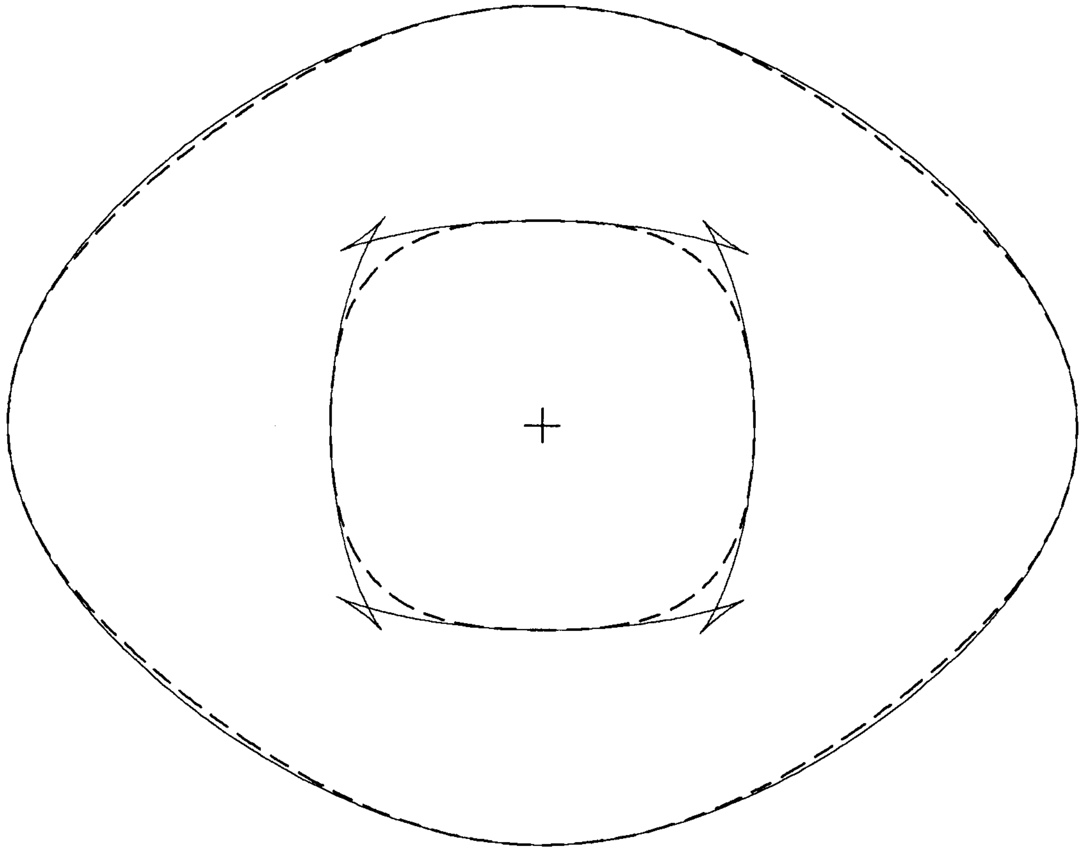

FIG. 1: Our model (dashed line) compared with the classical elastic model (solid line) for the ray surface of Greenhorn Shale. The inner pair of curves are for S_V, the outer for P.

$$\begin{aligned} \text{all } W,\ M &\geq 1.0 \\ 3/7 \leq qw,\quad qm &\leq 7/3 \end{aligned} \tag{14}$$

Areas of Non-application

As mentioned previously, this system is inappropriate for studying or processing data within a region of triplication. In this case there is no choice but to work in a double fourier or a p-tau space. Otherwise it seems that all phases of processing can be handled, although clearly some work remains to be done on one-way approximations to the anelliptic differential forms outlined in this paper. It may be worth remarking that although we have paid attention to kinematic approximations alone, nevertheless, as in the case of all non-dispersive forms, amplitudes are strictly controlled by geometry, and if we get the travel times right the amplitudes will fall into proper place.