APPENDIX A

Effect on Stacking Slowness of a Perturbation in Traveltime

If the interval slownesses are changed by a small amount, the traveltime to a given reflector will change for each offset within a midpoint gather. This will produce the change in stacking slowness given by equation (4.3) of the main text:

$$\Delta w_s = \frac{1}{w_s} \frac{\sum_{j=1}^{n} \left(x_j^2 - \overline{x^2} \right) t_j \ \Delta t_j}{\sum_{j=1}^{n} \left(x_j^2 - \overline{x^2} \right)^2} . \tag{A.1}$$

Thus the stacking slowness response is a weighted sum of the traveltime perturbations. These weights are calculated in this appendix.

$\overline{x^2}$ in equation (A.1) is the mean square offset. This is just a constant that depends on the recording geometry, that is the range of offset values used.

$$\overline{x^2} = \frac{1}{n} \sum_{j=1}^{n} x_j^2. \tag{A.2}$$

The constant n in equations (A.1) and (A.2) specifies the number of offsets. The minimum and maximum offsets are respectively x_0 and L. Thus,

$$\overline{x^2} = \frac{1}{n} \sum_{j=1}^{n} x_j^2 = \frac{1}{n\,\Delta x} \sum_{j=1}^{n} x_j^2 \Delta x$$

$$\approx \frac{1}{L - x_0} \left(\frac{x^3}{3} \right)_{x_0}^{L} = \frac{L^2}{3} \left(\frac{L^3 - x_0^3}{L^2 (L - x_0)} \right) = \frac{L^2}{3} \; b \tag{A.3}$$

where, by definition,

$$b = \frac{L^2}{3} \left(\frac{L^3 - x_0^3}{L^2 (L - x_0)} \right) . \tag{A.4}$$

Note that for $x_0 = 0$,

$$b = 1 . \qquad (A.5)$$

Similarly,

$$\sum_{j=1}^{n}\left(x_j^{\,2} - \overline{x^2} \right)^2 = \sum_{j=1}^{n}\left(x_j^{\,2} - \frac{bL^2}{3} \right)^2 = \sum_{j=1}^{n} x_j^{\,4} - \frac{nb^2L^4}{9}$$

$$= \frac{n}{L-x_0}\left(\frac{L^5}{5} - \frac{x_0^{\,5}}{5} \right) - \frac{nb^2L^4}{9}$$

$$= nL^2\left(\frac{L^3}{5(L-x_0)} - \frac{x_0^{\,5}}{5(L-x_0)L^2} - \frac{b^2L^2}{9} \right) = nL^2\, a \ , \qquad (A.6)$$

where a is defined to be

$$a = \left(\frac{L^3}{5(L-x_0)} - \frac{x_0^{\,5}}{5(L-x_0)L^2} - \frac{b^2L^2}{9} \right) . \qquad (A.7)$$

Note once again that for $x_0 = 0$,

$$a = \frac{4}{45} L^2 \qquad (A.8)$$

Equations (A.3) and (A.6) can now be substituted into equation (A.1) to give:

$$\Delta w_s = \frac{1}{w_s} \frac{\sum_{j=1}^{n}\left(x_j^{\,2} - \frac{bL^2}{3} \right) t_j \ \Delta t_j}{anL^2} . \qquad (A.9)$$

Once again, the constants a and b are defined in equations (A.7) and (A.4) respectively.

For $x_0 = 0$, a takes the simple form given by equation (A.8), and $b = 1$. Thus, the change in stacking slowness given by equation (A.9) reduces to

$$\Delta w_s = \frac{1}{w_s} \frac{\sum_{j=1}^{n}\left(x_j^{\,2} - \frac{L^2}{3} \right) t_j \ \Delta t_j}{\frac{4}{45} nL^4} . \qquad (A.10)$$

Equation (A.10) is identical to equation (4.4) in the main text.

APPENDIX B

Flat Reflectors, Depth-Variable Background Slowness

This appendix derives the linear relation between interval slowness and stacking slowness for a background medium that consists of flat reflectors with depth variable interval slownesses. The development proceeds along the same lines as section 4.3 of the main text.

Equation (B.1) is the equation for the response of stacking slowness to an impulse of anomalous interval slowness (it is the same as equation (4.11) in the main text):

$$G(y, z_r, y_a, z_a) = \int_{x=0}^{x=L} \left(x^2 - \frac{L^2}{3} \right) \frac{t(y, z_r, x)}{w_s(y, z_r)} \times \left[\frac{\delta(y_a - y + \mu(y, z_r, x, z_a)) + \delta(y_a - y - \mu(y, z_r, x, z_a))}{\cos\theta(y, z_r, x, z_a)} \right] . \tag{B.1}$$

For clarity, equation (B.1) includes all of the dependencies for each of the raypath variables. Equation (B.1) is valid, provided the raypaths are symmetric about zero-offset. Thus, it does not depend on an assumption of a constant background slowness. The symmetry of the raypaths will also hold for a laterally invariant background model, provided the reflectors are flat.

Figure B.1 shows the raypath to a flat reflector at depth z_r, for offset x and midpoint y. Each such raypath can be described by its ray parameter $p(z_r, x)$. Because the medium is laterally invariant, the ray parameter depends only on the offset and reflector depth; it is independent of the midpoint. Thus, most of the variables of the linearization (i.e. in equation (B.1)) can be expressed in terms of the ray parameter $p(z_r, x)$:

$$\cos\theta(y, z_r, x, z_a) = \left[1 - \frac{p(z_r, x)^2}{w_b(z_a)^2} \right]^{1/2} , \tag{B.2a}$$

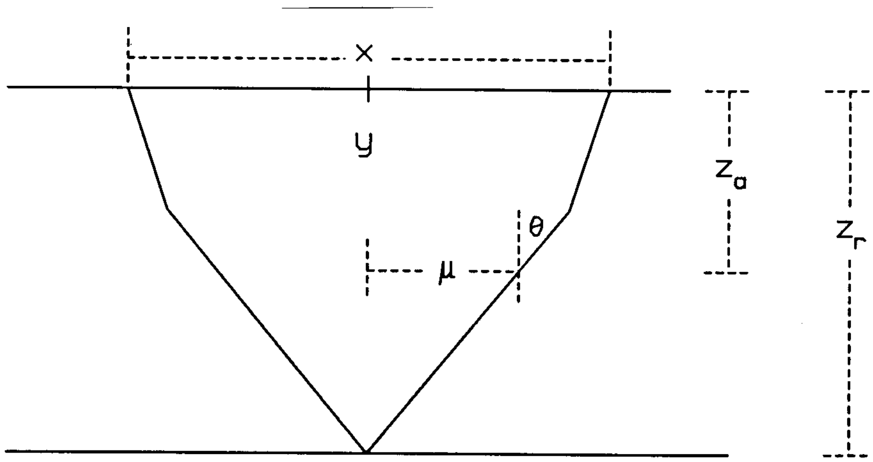

FIG. B.1. Raypath to a flat reflector at depth z_r, for offset x and midpoint y. The raypath is based on the depth variable background slowness, $w_b(z_a)$.

$$\mu(y, z_r, x, z_a) = \int_{z_a}^{z_r} \frac{p(z_r, x)dz_a}{w_b(z_a)\left[1 - \frac{p(z_r, x)^2}{w_b(z_a)^2}\right]^{1/2}}, \tag{B.2b}$$

and

$$t(y, z_r, x) = \int_0^{z_r} \frac{dza}{\left[1 - \frac{p(z_r, x)^2}{w_b(z_a)^2}\right]^{1/2}}. \tag{B.2c}$$

Equations (B.2) express the raypath-dependent variables analytically. The only problem is that determining p for a given z_r and x requires the solution of a two point ray-tracing problem; $p(z_r, x)$ must therefore be determined numerically. The simplification offered by equations (B.2), then, is that the raytracing must be performed at only one midpoint.

For the constant background slowness, a convolutional form of equation (B.1) was derived. The derivation first assumed that the offset was a continuous variable, then changed the argument of the raypath delta function to directly depend on offset. This change of variables was made possible by the simple form of equation (4.14), that is

$$\mu = y - y_a = \pm\frac{x}{2}\left(\frac{z_r - z_a}{z_r}\right) = \pm\frac{x}{2}\left(\frac{L'}{L}\right) \tag{4.14}$$

Unfortunately, for the depth variable case presented here the analogous equation, (B.2b), does not provide an analytic expression between μ and x, because the $p(z_r, x)$ must be determined numerically. This means that equation (B.2b), unlike equation (4.14), cannot be inverted analytically to give x as a function of y_a, z_a, y and z_r. Certainly, the impulse response could be determined numerically, if one were to tabulate the values of μ given by equation (B.2b) for different values of x and z_r. Then, the appropriate value of x to select from the integral in equation (B.1) could be determined from this table, as the value of x corresponding to y_a, z_a, y and z_r.

A simple approximation to the exact equations (B.2) does, however, lead to a convolutional form for the depth variable case. An expression that approximately describes the raypaths is:

$$\mu(y, z_r, x, z_a) \approx \frac{L'(z_r, z_a)}{L} \frac{x}{2} \tag{B.3}$$

Equation (B.3) says that the y coordinate of the raypath for offset x varies with depth exactly as does the raypath for the largest offset L. This results in a value of μ which, although not exactly accurate, differs from the constant background ray in the proper direction. Now only one ray must be traced numerically for each reflector; the result is the function $L'(z_r, z_a)$. Thus all of the detailed depth variation is contained within the variable L'.

As long as the offsets are not too large, or the depth variation of interval slowness not too great, one further approximation can be made: the background traveltimes obey the hyperbolic moveout equation. That is, make the assumption that the standard velocity analysis method is valid for the background medium. Thus,

$$t(y, z_r, x) = w_{rms} \left(x^2 + 4z_r^2 \right)^{1/2} \tag{B.4a}$$

and then

$$w_s(y, z_r) = w_{rms} = \frac{1}{v_{rms}} \tag{B.4b}$$

Equations (B.3) and (B.4) can now replace the appropriate variables in equation (B.1). The result is exactly the same as equation (4.19), with the depth variable version of L' replacing the constant slowness version.

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