Seismic modeling in medium with linear velocity gradient

Zhiming Li

INTRODUCTION

In a medium with a linear velocity gradient, rays travel along circular arcs (Slotnick, 1959). We define a normal reflection as a reflection from the top of a reflector and an overturned reflection as a reflection from the underside of a reflector.

The travel times and the receiver positions for both the normal and the overturned reflection from a particular reflector can be computed by a ray tracing program. The computation presented in this paper shows these two reflections have opposite curvatures in the common-shot gather and have opposite moveouts in the common-midpoint gather.

COMPUTATION OF NON-ZERO OFFSET SEISMOGRAMS

Figure 1 shows the geometry of a non-zero offset recording. Normal reflections are obtained when the angle of reflector α is from 0 to 90 degrees, while overturned reflection modeling corresponds to an α between 90 and 180 degrees. The propagating angle θ of a ray is defined as the angle between the ray's direction and the vertical axis. θ is between –180 and +180 degrees, and is positive when the ray travels to the right, negative when the ray travels to the left.

In a medium with velocity described by

$$v(z) = v_0 (1 + \beta z) , \qquad (1)$$

rays travel along circular arcs. The underground position (x, z) through which a particular ray passes, and the travel time of the ray at (x, z) are given below (Li et al, 1984).

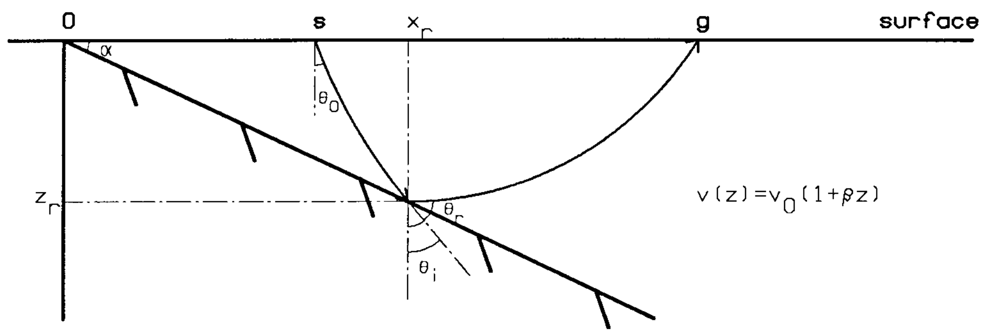


FIG. 1. A non-zero offset recording geometry. θ_i is the propagating angle of the incident ray at the reflector, while θ_r the propagating angle of the reflecting ray at the reflector.

$$\begin{cases} x = s + \dfrac{1}{v_0 \beta p}\left[\sqrt{1-p^2 v_0^2} - \sqrt{1-p^2 v^2(z)}\right] \\ t = \dfrac{1}{v_0\beta}\ln\left\{\dfrac{(1+\beta z)\left[1+\sqrt{1-p^2v_0^2}\right]}{\left[1+\sqrt{1-p^2v^2(z)}\right]}\right\} \end{cases}, \tag{2}$$

$$\begin{cases} x = s + \dfrac{1}{v_0 \beta p}\left[\sqrt{1-p^2 v_0^2} + \sqrt{1-p^2 v^2(z)}\right] \\ t = \dfrac{1}{v_0\beta}\ln\left\{\dfrac{\left[1+\sqrt{1-p^2v_0^2}\right]\left[1+\sqrt{1-p^2v^2(z)}\right]}{p^2 v_0 v(z)}\right\} \end{cases}, \tag{3}$$

where $p=\sin\theta/v(z)=\sin\theta_0/v_0$ is the ray parameter. Equation (2) is used if the ray reaches (x,z) before it turns around its turning point (Li et al, 1984), otherwise equation (3) should be used.

The reflector's position (x_r, z_r) corresponding to a given ray parameter $p=\sin\theta_0/v_0$ is obtained by solving the following system of equations:

$$\begin{cases} x_r = s + \dfrac{1}{v_0\beta p}\left[\sqrt{1-p^2v_0^2} \pm \sqrt{1-p^2v_0^2(1+\beta z)^2}\right] \\ x_r = \dfrac{z_r}{\tan\alpha} \end{cases}. \tag{4}$$

The propagating angle of the reflecting ray at (x_r, z_r) is given by:

$$\theta_r = 180^o - 2\alpha - \theta_i \ . \qquad (5)$$

Having determined θ_r, we obtain the ray parameter for the reflecting ray. Thus, the receiver position and the travel time for the reflecting ray can be calculated by equation (2) or (3), depending on whether the propagating angle of the ray at the reflector position is larger than 90 degrees or smaller than 90 degrees.

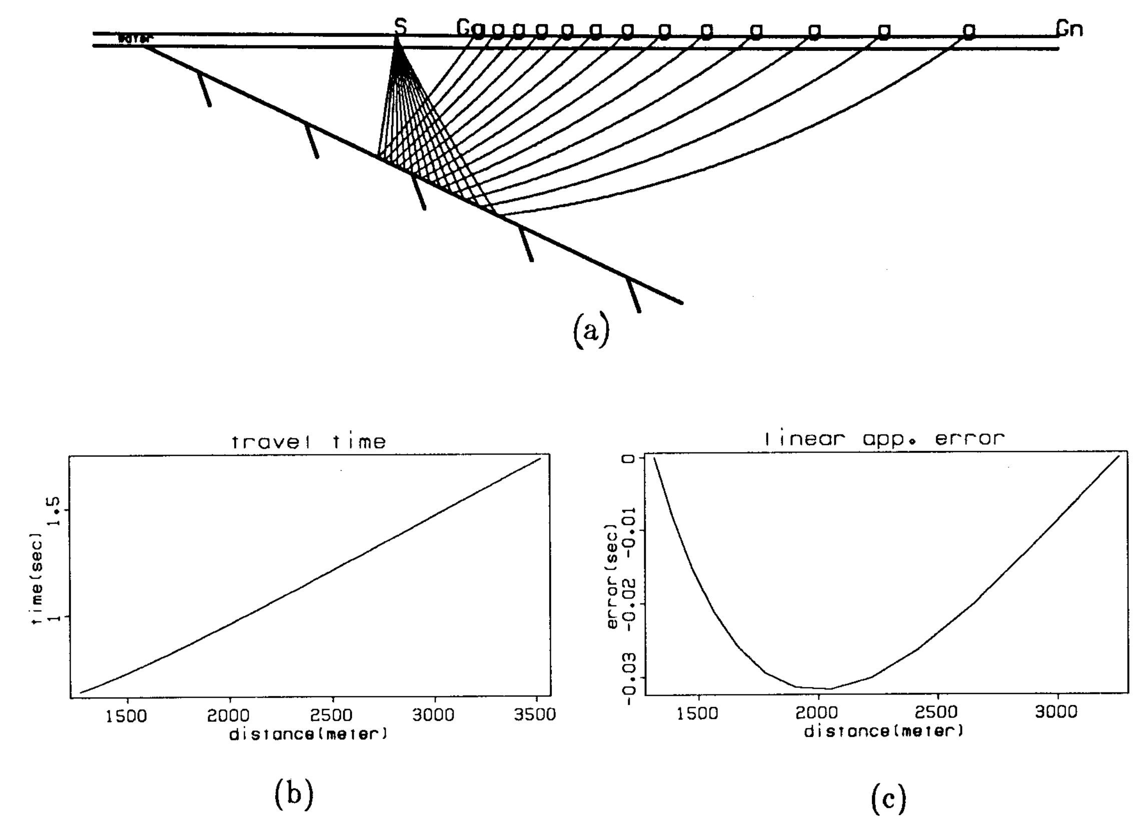


FIG. 2. (a) Recording geometry for normal reflections. The dip of the reflector, α is 25 degrees. The water depth is 50 m and the water's velocity 1500 m/sec. The sediment velocity is given by $v(z) = 1600(1 + .0003z)$. (b) Travel time curve of the normal reflections. The shot position $s = 1000$ meters, the offset 250 m and the cable length 2375 m. (c) Linear approximation error obtained by the subtraction of the travel time in (b) from the straight line connecting the end points of the travel time curve. This error diagram shows negative error because of the hyperbolic feature of the travel time curve of normal reflection.

Figures 2 and 3 show the geometries and the travel time curves for both the normal and the overturned reflections. The normal reflection and the overturned reflection have opposite curvatures in the common-shot gathers. The opposite curvatures (or the second derivatives) of the two travel time curves of the normal reflection and the overturned reflection are not clearly seen in Figures 2(b) and 3(b), because of the relatively short cable length. Subtracting these curves by the straight lines connecting the end points of the curves revives the difference of curvatures between the two travel time curves. The travel time curve for the normal reflection has a positive second derivative, while that of the overturned reflection is negative.

Because of the opposite curvatures of the normal reflection and the overturned reflection, they have different slopes dt/dx within certain range of traces, especially near traces.

COMMON-MIDPOINT GATHER

The seismogram is calculated after scanning over a number of initial angle θ_0's. It is not evenly sampled in the g coordinates. In order to obtain the common-midpoint gather, interpolation (e.g., spline interpolation) is needed. Figure 4 shows the common-midpoint gathers for normal reflections and overturned reflections. The figure shows the opposite moveouts for normal and overturned reflections. The travel time of the overturned reflections in a common-midpoint gather deceases as the offset increases!

The differences between normal reflection and overturned reflection in common-midpoint and common-shot gathers are important in the identification and the stacking of both normal reflection and overturned reflection into the stacked sections, if both of them are recorded in seismograms.

It is interesting to note that the common midpoint gather for the reflections from a vertical reflector in a medium of linear velocity gradient shows a constant travel time for all offsets, as it is shown for a vertical reflector in a constant velocity medium. The concept of imaginary source can be applied to explain this effect, i.e., flipping the sources and the downgoing ray paths to the other side of the reflector makes the total ray paths for all the rays be the circular arcs of the same radius with their centers shifted along the horizontal axis.

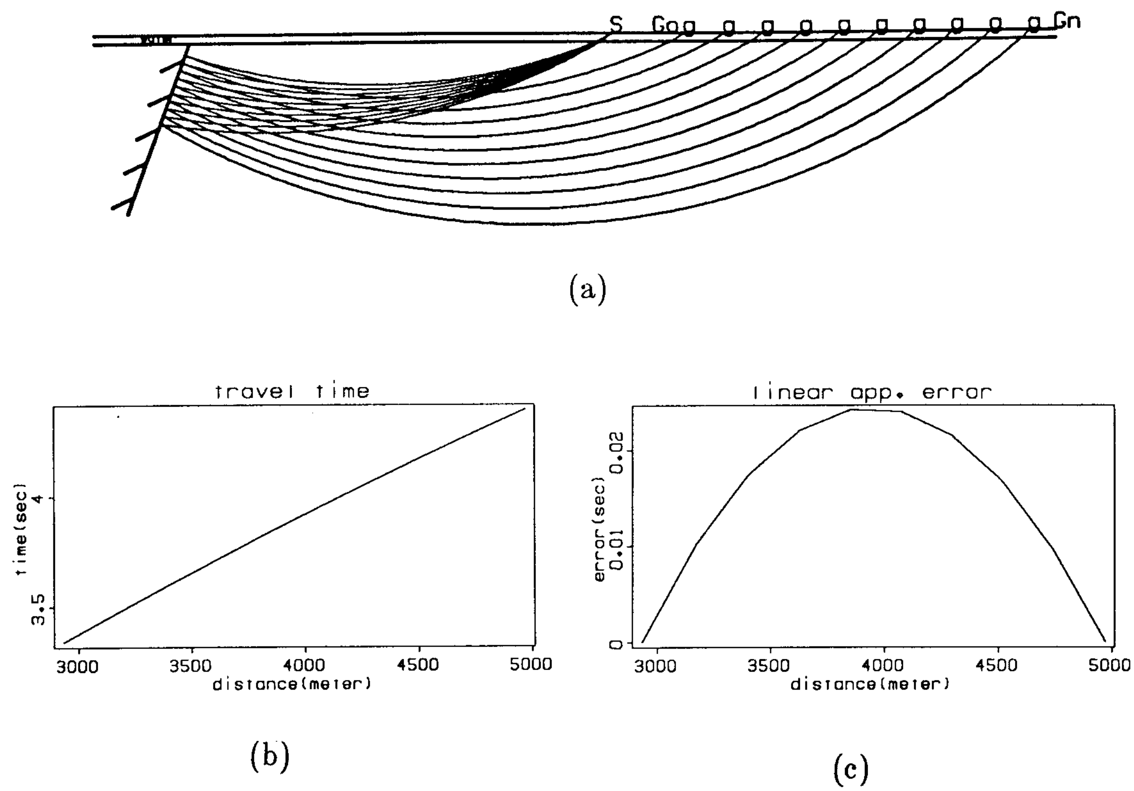
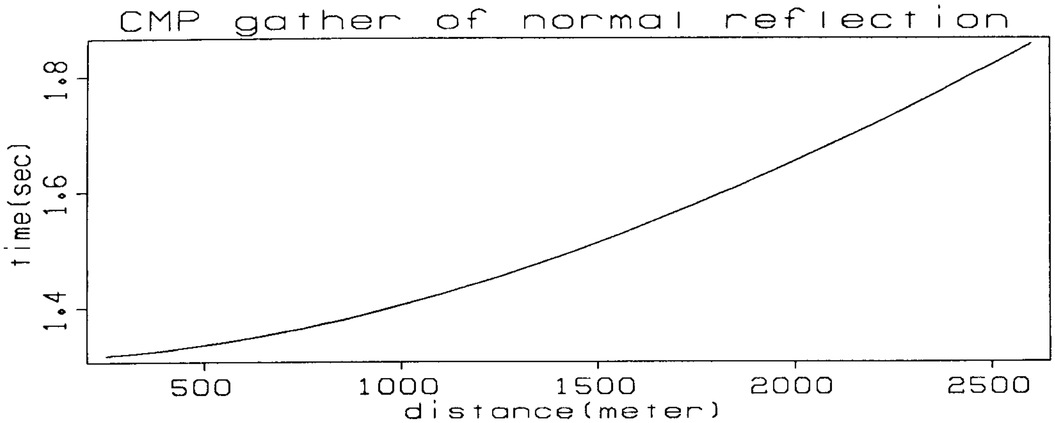


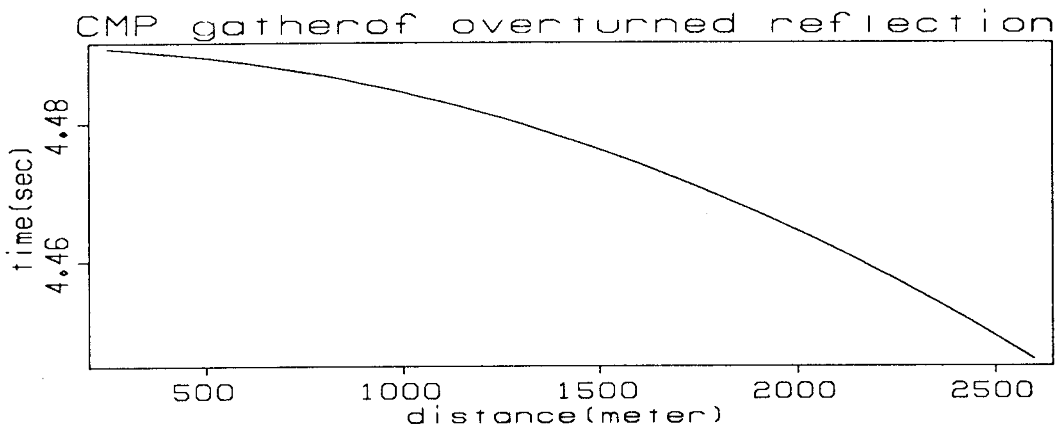
FIG. 3. (a) Recording geometry for overturned reflections. The dip of the reflector α is 110 degrees. The water depth is 50 m and the water velocity 1500 m/sec. The sediment velocity is given by $v(z)=1600(1+.0003z)$. (b) Travel time curve of the normal reflections. The shot position $s=2500$ m, the offset 250 m and the cable length 2375 m. (c) Linear approximation error obtained by the subtraction of the travel time in (b) from the straight line connecting the end points of the travel time curve. This error diagram shows positive error because of the non-hyperbolic feature of the travel time curve of overturned reflection.

SUMMARY

The normal reflections and the overturned reflections have different travel time behaviors in both the common-shot and the common-midpoint gathers; this characteristics makes the identification of overturned reflections possible.



(a)



(b)

FIG. 4. (a) Common midpoint gather for normal reflections. (b) Common midpoint gather for overturned reflections.

REFERENCES

Li, Z., Claerbout, J.F. and Ottolini, R., 1984, Overturned-wave migration by two-way extrapolation: SEP-38, pp 141-149.

Slotnick, M.M., 1959, Lessons in seismic computing: Wisconsin, George Banta Company.